

# Simulation Research on Vehicle Active Suspension Controller Based on G1 Method

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**Abstract.** Based on the order relation analysis method (G1 method), the optimal linear controller of vehicle active suspension is designed. The system of the main and passive suspension of the single wheel vehicle is modeled and the system input signal model is determined. Secondly, the system motion state space equation is established by the kinetic knowledge and the optimal linear controller design is completed with the optimal control theory. The weighting coefficient of the performance index coefficients of the main passive suspension is determined by the relational analysis method. Finally, the model is simulated in Simulink. The simulation results show that: the optimal weight value is determined by using the sequence relation analysis method under the condition of given road conditions, and the vehicle acceleration, suspension stroke and tire motion displacement are optimized to improve the comprehensive performance of the vehicle, and the active control is controlled within the requirements.

## 1. Introduction

Suspension system is one of the most important components in the vehicle, it can buffer the impact force on the vehicle that caused by the uneven ground, which have a significant effect on the ride comfort, vehicle handling and safety. The traditional passive suspension system stiffness and damping is not adjustable and its own range of changes is small, it is difficult to adapt to different road conditions. The active suspension system can adjust the suspension stiffness and damping automatically according to the different road conditions and the current vehicle movement condition, so that the vehicle can reach the optimal driving state. The determination of the weighting factor is most important when designing an active suspension using the LQG (Linear Quadratic Gaussian) controller. In the literature [2], the weighting coefficients of the performance indexes are determined by constructing judgment matrix and the original suspension controller is optimized, which improves the suspension performance to a certain extent. In literature [3], based on the genetic algorithm, the error correction factor is introduced and the optimal control rules of the weighting coefficient are determined by combining the fuzzy control strategy. The performance of the system is improved to a large extent, and the stability of the optimized system is further improved. In literature [4], a method of determining the weighting coefficient based on the local elite strategy artificial bee colony algorithm is proposed to improve the ride comfort of the vehicle.

In this study, MATLAB/Simulink is used as the simulation platform, aiming at the main passive suspension model of single-wheel vehicle, selecting the system input signal model and combining the sequence relation analysis method to determine the weight value of each performance index, using vehicle dynamics and optimal linear control theory of LQG controller design. According to the

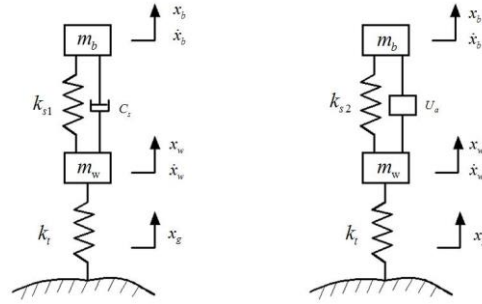


simulation results, the performance of the active suspension controller is analyzed to verify whether the design method is correct and effective.

## 2. Model Establishment

### 2.1 Active and passive model

After a reasonable simplification, the model of single-wheel vehicle passive suspension is established, as shown in figure 1.



(a) Passive suspension model (b) Active suspension model

$m_b$  - A quarter of the body quality;  $m_w$  -Tire quality;  $k_t$  -Tire stiffness;

$k_{s1}$  -Passive suspension spring stiffness coefficient;

$k_{s2}$  -Active suspension spring stiffness coefficient;

$C_s$  -Damping coefficient;  $U_a$  -Active control force;

$x_b$  -Body displacement;  $\dot{x}_b$  -Body speed;  $x_w$  -Tire displacement;  $\dot{x}_w$  -Tire speed;

$x_g$  -Pavement displacement;

**Figure 1.** Single-wheel vehicle active suspension model

### 2.2 Pavement incentive model

According to the requirements of the optimal linear control theory, the filtered white noise of the Gaussian distribution is chosen as the system input [5], as in equation (1):

$$\dot{x}_g(t) = -2\pi f_0 x_g(t) + 2\pi \sqrt{G_0 U_0} w(t) \quad (1)$$

Where  $x_g$  is the road vertical displacement;  $G_0$  is the road roughness coefficient;  $u$  is the vehicle speed;  $w$  is the Gaussian white noise Which mathematical expectation is zero and  $f_0$  is the cut-off frequency.

## 3. LQG Controller Design

### 3.1 System state equation

From the kinetic knowledge we can know that:

Passive suspension system vehicle model state space equation, as shown in equation (2):

$$\begin{aligned} \dot{X}(t) &= A_1 X(t) + B_1 W(t) \\ Y &= C X(t) + D W(t) \end{aligned} \quad (2)$$

According to Newton's second law, active suspension system motion as in equation (3) (4):

$$m_b \ddot{x}_b(t) = U_a(t) - K_s [x_b(t) x_w(t)] \quad (3)$$

$$m_w \ddot{x}_w(t) = -U_a(t) + K_s[x_b(t) - x_w(t)] - K_t[x_w(t) - x_g(t)] \quad (4)$$

Obtained by the equation (1) (3) (4), the state space equation of the active suspension vehicle model system is rewritten into the form of matrix, as in equation (5):

$$\begin{aligned} \dot{X}(t) &= A_2 X(t) + B_2 U(t) + F W(t) \\ Y &= C X(t) \end{aligned} \quad (5)$$

Where:  $X(t)$  is the system state vector;  $W(t)$  is the system input matrix;  $U(t)$  is the control input matrix;  $A_1, A_2$  is passive and active system matrix;  $B_1, B_2$  is passive and active input matrix;  $C$  is output matrix;  $D$  is direct transfer matrix.

$$A_1 = \begin{bmatrix} -\frac{C_s}{m_b} & \frac{C_s}{m_b} & -\frac{K_{s1}}{m_b} & \frac{K_{s1}}{m_b} & 0 \\ \frac{C_s}{m_w} & -\frac{C_s}{m_w} & \frac{K_{s1}}{m_w} & \frac{-K_{s1} - K_t}{m_w} & \frac{K_t}{m_w} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\pi f_0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & -\frac{K_{s2}}{m_b} & \frac{K_{s2}}{m_b} & 0 \\ 0 & 0 & \frac{K_{s2}}{m_w} & \frac{-K_{s2} - K_t}{m_w} & \frac{K_t}{m_w} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\pi f_0 \end{bmatrix}$$

$$B_1 = F = \begin{bmatrix} 0 & 0 & 0 & 0 & 2\pi\sqrt{G_0 U_0} \end{bmatrix}^T, \quad B_2 = \begin{bmatrix} \frac{1}{m_b} & -\frac{1}{m_w} & 0 & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

### 3.2 Controller design

In the vehicle suspension design, the main of the three factors is considered. The tire displacement reflects the ability of the tire to contact the ground; the acceleration of the body reflects the ride comfort; the suspension stroke reflects the smoothness of the vehicle. Among them, the size of the body acceleration and actuator output force is proportional to the size of the relationship.

Definition: the LQG optimization index function of the active suspension is the integral value of the weighted square sum of the tire dynamic load, the vertical acceleration of the vehicle body and the suspension stroke, as in equation (6).

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \{q_1[x_w(t) - x_g(t)]^2 + q_2[x_b(t) - x_w(t)]^2 + \rho \ddot{x}_b(t)\} dt \quad (6)$$

Where:  $q_1, q_2, \rho$  are the weighting factors of each performance index [6]. For the convenience of research, take the vehicle body vertical acceleration of the weighting factor  $\rho = 1$ .

To facilitate the calculation, the equation (6) is rewritten as a matrix form, as in equation (7):

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (X^T Q X + U^T R U + 2X^T N U) dt \quad (7)$$

Where:  $X$  is state variable,  $Q$  is state variable weighting matrix,  $R$  is control variable weighting matrix,  $N$  is weighting matrix of the cross term.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_2 + \frac{K_{s2}^2}{m_b^2} & -q_2 - \frac{K_{s2}^2}{m_b^2} & 0 \\ 0 & 0 & -q_2 - \frac{K_{s2}^2}{m_b^2} & q_1 + q_2 + \frac{K_{s2}^2}{m_b^2} & -q_1 \\ 0 & 0 & 0 & -q_1 & q_1 \end{bmatrix}, \quad N = \frac{1}{m_b^2} \begin{bmatrix} 0 \\ 0 \\ -K_s \\ K_s \\ 0 \end{bmatrix}, \quad R = \frac{1}{m_b^2}$$

When the vehicle parameters and the weighting coefficients are determined, the optimal control feedback gain matrix can be obtained by the Riccati equation [7-8], as in equation (8).

$$AK + KA^T + Q - KBR^{-1}B^TK + FWF^T = 0 \quad (8)$$

And then according to any moment of feedback state variables, according to equation (9) can be obtained at any time the optimal active control force.

$$U_a(t) = -KX(t) \quad (9)$$

### 3.3 Determination of weighting factor

**3.3.1 Quantitative factor.** As the difference between the units and value of performance indicators is too large, which need to be quantified before compared. In this paper, the LQG controller design of the vehicle active suspension is carried out on the basis of the original passive suspension [9], and the RMS (Root mean square) value of the vehicle statistic is compared with the vehicle body acceleration  $\delta_{BA}^2$ , the tire moving displacement  $\delta_{DTD}^2$  and the suspension stroke  $\delta_{sws}^2$ .

The quantitative factor of the active suspension is 1, according to equation (10) to determine the quantitative factor of the tire movement and suspension of active suspension.

$$\delta_{BA}^2 \cdot 1 = \beta_1 \cdot \delta_{DTD}^2 = \beta_2 \cdot \delta_{sws}^2 \quad (10)$$

**3.3.2 Subjective weighting factor.** Order relationship analysis method (G1 method), all indicators are sorted according to the importance of the evaluation criteria for the different degree, and then in the two comparison at the same time, given different weight values. Compared with the fuzzy comprehensive evaluation method and the tomographic analysis method, there is no need to construct the judgment matrix, and it is not necessary to carry out the consistency test. It has the advantages of small computation and intuitive results [10-11].

#### 1. Determine the order relationship

Order relationship analysis method defined: relative to the user's subjective evaluation criteria, if the importance of indicators  $x_i$  greater than or equal to the index  $x_j$ , there are  $x_i \succ x_j$ .

For the index set  $\{x_1, x_2, \dots, x_m\}$ , the order relation  $x_1^* \succ x_2^* \succ \dots \succ x_m^*$  is established according to the importance of each indicator relative to the evaluation target.  $x_i$  represents the evaluation index that  $\{x_i\}$  is sorted by the relationship " $\succ$ ".

The steps to establish the order relationship are as follows:

(1) The decision maker selects one of the most important indicators in the index set  $\{x_1, x_2, \dots, x_m\}$  as  $x_1^*$ ;

(2) The decision maker chooses the most important one in the remaining  $\{x_1, x_2, \dots, x_m\}$  indicators as  $x_2^*$ ;

And so on, until after  $m-1$  times to select the rest of the evaluation index recorded as  $x_m^*$ ;

This uniquely identifies an order relationship:  $x_1^* \succ x_2^* \succ \cdots \succ x_m^*$ .

So we determined: body acceleration  $\succ$  suspension moving stroke  $\succ$  tire displacement.

2. Comparison between the relative importance of  $x_{k-1}$  and  $x_k$ .

The degree of importance of an evaluation index  $x_{k-1}$  and  $x_k$ , the rationality of the ratio  $\omega_{k-1} / \omega_k$  is  $r_k$ . As shown in table 1.

**Table 1.** Assignment table of  $r_k$

$r_k$	Description
1.0	Indicator $x_{k-1}$ are just as important as indicator $x_k$
1.2	Indicator $x_{k-1}$ are slightly important than indicator $x_k$
1.4	Indicator $x_{k-1}$ are obvious important than indicator $x_k$
1.6	Indicator $x_{k-1}$ are more important than indicator $x_k$
1.8	Indicator $x_{k-1}$ are most important than indicator $x_k$

Also stipulated that  $r_{k-1} > 1/r_k$  must be satisfied.

According to the assignment table we give: body acceleration is more important than the tire moving displacement,  $r_2=1.2$  tire moving displacement is extremely important than the suspension moving stroke,  $r_3=1.8$ .

3. Calculation of weight factor

For the sorting completion of the evaluation index set can be calculated according to equation (11), (12) weight coefficient.

$$\omega_m = \left( 1 + \sum_{k=2}^m \prod_{i=k}^m r_i \right)^{-1} \quad (11)$$

$$\omega_{k-1} = r_k \omega_k, k = m, m-1, \dots, 3, 2 \quad (12)$$

4. Subjective weighting factor

The subjective weighting factor of the body acceleration is 1, and the subjective weighting factor  $x$  and  $x$  of the other two indicators are determined according to equation (13).

$$\frac{\omega_1}{1} = \frac{\omega_2}{\gamma_1} = \frac{\omega_3}{\gamma_2} \quad (13)$$

**3.3.3 Determination of final weighting factor.** Determine the final weighting factor according to equation (14).

$$\begin{aligned} q_1 &= \beta_1 \gamma_1 \\ q_2 &= \beta_2 \gamma_2 \end{aligned} \quad (14)$$

## 4. Model Simulation and Analysis

### 4.1 InVehicle model parameters

The relevant vehicle model parameters are shown in table 2.

**Table 2. The relevant vehicle model parameters**

Model Parameters	Symbol	Value	Unit
1/4 Body Quality	$m_b$	320	$Kg$
Wheel Quality	$m_w$	40	$Kg$
Passive Suspension Stiffness	$K_{s1}$	20000	$N \cdot m^{-1}$
Active Suspension Stiffness	$K_{s2}$	22000	$N \cdot m^{-1}$
Tire Stiffness	$K_t$	200000	$N \cdot m^{-1}$
Passive Suspension Damping Coefficient	$C_s$	1000	$N \cdot s \cdot m^{-1}$
Suspension of Work Space	$SWS$	$\pm 100$	$mm$
Actuator Control	$U_a$	$\pm 3000$	$N$
Simulation of Road Roughness Coefficient	$G_0$	$5.0 \times 10^{-6}$	$m^3 \cdot cycle^{-1}$
Vehicle Speed	$u$	20	$m \cdot s^{-1}$
Cut-off Frequency	$f_0$	0.1	$Hz$

In the given road condition, the importance order of the factor is: body acceleration  $\succ$  suspension moving stroke  $\succ$  tire displacement. According to the Eq. (10) (11) (12) (13) (14) calculate the results:

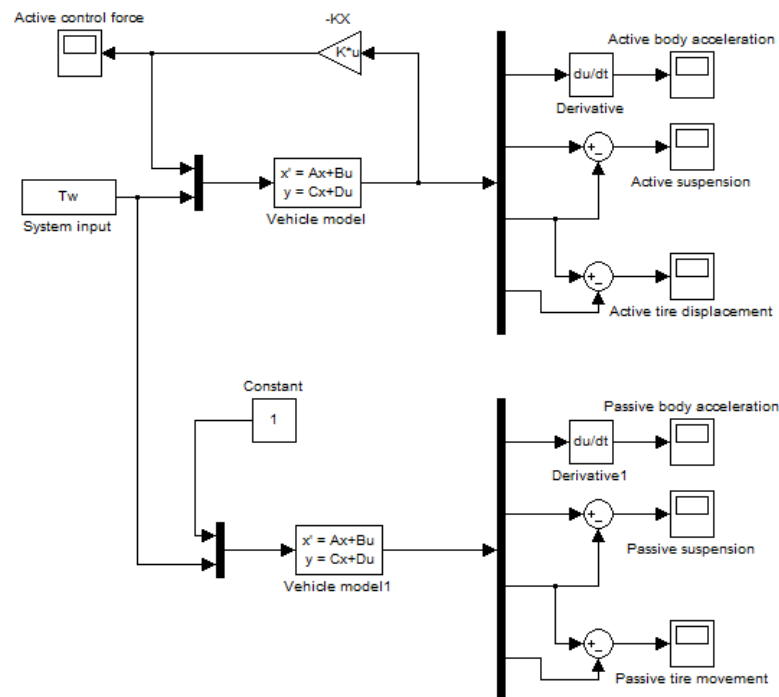
$$q_1 = 72538.17, \quad q_2 = 4842.31$$

Call the linear quadratic optimal controller design function  $[K, S, E] = LQR[A, B, Q, R, N]$  to complete design optimal active controller. The resulting optimal feedback gain matrix result is as follows:

$$K = [3826.54 \quad -1176.67 \quad 2267.75 \quad -11639.95 \quad 10926.46]$$

### 4.2 Establish simulink model

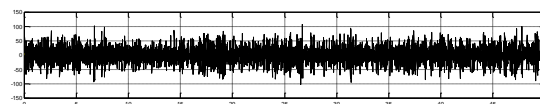
According to the above design scheme, the main passive suspension model is simulated in Simulink environment [12-13], as shown in figure 2. The white noise can be called by the MATLAB function WGN (M, N, P) to produce a M-row N-column Gaussian white noise matrix. The noise intensity is determined by the P value (in unit of DB). In the simulation, take  $M = 10001$ ,  $N = 1$ ,  $P = 20$ , that is, take a total of 10001 sampling points, sampling time is set to 0.005s, when the speed of 20m/s, the equivalent of the simulation road length is 1000m.



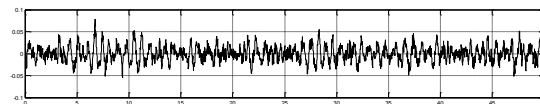
**Figure 2.** Schema diagram of the system simulation in Simulink environment

#### 4.3 Results and analysis

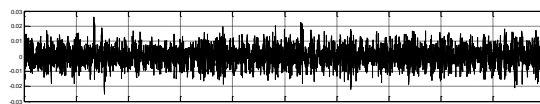
The simulation results are shown in figure 3 and figure 4.



(a) Body acceleration curve

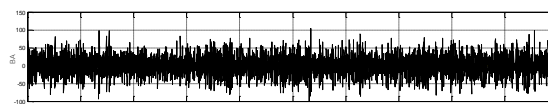


(b) Suspension stroke change curve

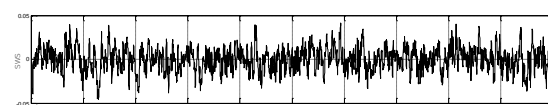


(c) Tire displacement curve

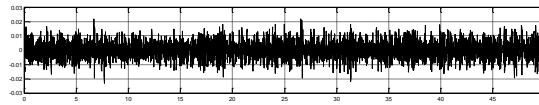
**Figure 3.** Passive suspension system simulation results



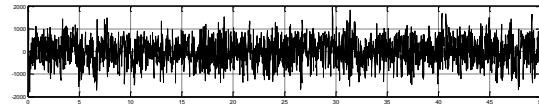
(a) Body acceleration curve



(b) Suspension stroke change curve



(c) Tire movement displacement curve



(d) Active output control curve

**Figure 4.** Simulation results of active suspension system

From the simulation results we can do the following conclusions:

(1) Calculate the RMS value of the performance index of the main passive suspension, as shown in table 3.

**Table 3.** Main passive suspension performance RMS value

		Body Acceleration /(m/s <sup>2</sup> )	Suspension Stroke /(m)	Tire Movement /(m)
Active	Suspension	1.533	0.013366	0.005984
System				
Passive	Suspension	1.850	0.017917	0.006350
System				

In the case of the difference of the displacement of the tire, the optimal active suspension significantly reduces the vehicle acceleration and reduces the suspension stroke. Compared with the passive suspension, the active suspension of the LQG controller based on the G1 method, the root mean square values of each performance index were reduced respectively by 17.14%, 25.4% and 5.76%.

(2) It can be seen from figure 4(d) that the active control force is in the range of -2422.167~1947.846N, and the calculated RMS value of the active control force is 523.565N, which means that the control force of the dominant force is within the required range ( $\pm 3000$ N).

(3) Compared with figure 3 and figure 4, it can be seen that the active suspension can smooth the change of each performance index in the adjacent time compared with the passive suspension, which is beneficial to ensure the ride comfort of the passenger and the ride comfort of the vehicle. Suspension travel is limited to the range of  $\pm 50$ mm, which further improve the occupants ride comfort, but also on the rational design of the suspension played a reference role.

(4) The G1 method is used to determine the weighting coefficient of each performance index, and the appropriate weighting coefficient is selected to get a better comprehensive performance. This suggests that we can design the suspension according to the needs of different users, while the focus can also make the performance indicators to achieve a balance, and then get the best design results.

## 5. Conclusions

The dynamic model of the main passive suspension system of the single-wheel vehicle is established. The weighting coefficient of each performance index is determined by the sequence relation analysis method, the construction of the judgment matrix is avoided, the calculation is simplified and the motion state space is established by using the knowledge of dynamics equation and combined with the optimal control theory to complete the optimal linear controller design, and Simulink in the model was simulated. The results show that: the appropriate weight value is determined by using the sequence relation analysis method, which makes the acceleration of the vehicle body, the suspension stroke and the tire displacement are optimized to a certain extent, and the overall performance of the vehicle is improved.



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