

Generating elastic band gaps in square lattice grid by periodical cut-off operations

Lin Han^{1,*}, Xiao-mei Li² and Yan Zhang¹

¹ College of Mechanics and Materials, Hohai University, 211100, Nanjing, P. R. China

² Department of Geotechnical Engineering, Nanjing Hydraulic Research Institute, 210024, Nanjing, P. R. China

*corresponding author, hanlin@hhu.edu.cn.

Abstract. From periodically cutting off rods in square lattice grids, we give an easy and feasible way to generate elastic band gaps (BGs) in it. We define the model of the periodical cut-off (PCO) square lattice grid, and study the frequency dispersion relation and the motion modes of rods, as well as the frequency responses. On this basis, we confirm that the PCO operation is feasible to generate BGs in square lattice grids. Furthermore, the BGs generated in PCO square lattice grids are basically locally resonant BGs with low frequency ranges which attributes to the motions of exposed cantilevers.

1. Introduction

Lattice grids are cellular structures composed of periodical cells, and usually used in solar panels, interstage of spacecraft launcher [1], aircraft fuselage and wing [2], for the merits of high specific stiffness and specific strength. Several types, including square lattice grid, triangular lattice grid, hexagonal lattice grid, kagome lattice grid, reentrant lattice grid, etc. are often adopted [3]. For now, the static properties of lattice grids are studied a lot, including various material properties depending on periodical structures [4], buckling [5], fracture [5], etc. Besides, the versatility of lattice grids in vibration insulation, energy absorption [7], heat dissipation, etc. also has been borne in upon researchers recently, for wide prospect in applications. However, the study on the versatility of lattice grids is still rare, and should be paid more attention. We mainly focus the characteristic of controlling elastic waves.

Recently, the propagation of elastic waves in periodical structures or materials has been received much attention [8-14]. In specific periodical structures or materials, elastic waves fall into some frequency ranges, which are called band gaps (BGs), cannot propagate. The periodical structures or materials are the so-called phononic crystals (PCs). Due to the BG characteristics, PCs could have many applications such as high-precision mechanical systems [9], noise control [15], vibration control [16, 17], vibration reduction [18], transducers and wireless communications [19]. Clearly, lattice grids with periodical structures could bring BG characteristics for elastic wave control. Based on the PC concept, the propagation characteristics of elastic waves in lattice grids has been studied, including the BG range [20], BG control [3, 21], as well as directional propagation of elastic waves [22, 23]. However, the current studies are still not enough to support actual applications, for the BG ranges are hard to meet the actual requirements. So the BGs regulation and control method urgently need breakthrough.



Actually, for general PCs, BGs regulation and control have some measures, which are mainly based on the changes of material properties and distribution, lattice type and size, etc. Specifically, by means of some materials which could easily control properties, such as piezoelectric material [24], electrorheological material [25], magnetostriction material [26], BGs could be controlled. Furthermore, rotating the scatterers or adding scatterers in cells [28] could help to changing the material distribution. Additionally, changing the deformation form by external effects could change the lattice size or even lattice type, so the BGs could be changed [29, 30]. While the most of these measures are hard to implement in lattice grids, because one kind of actual lattice grid is generally made by just one material, and the BGs characteristics are just determined by the periodical geometry. So when a kind of certain lattice grid is given, how can we change the propagation characteristics of elastic waves in it? Specifically, BGs are hard to show for square lattice grids. And how can we do to obtain BGs in square lattice grids by using simple methods? We think periodically cutting off particular rods in square lattice grids is an easy and feasible way to realize. Cutting off rods in square lattice grids in periodically pattern, could not only reduce the stiffness of structure, but also change the mass distribution of structure due to the exposed cantilevers, so the frequency dispersion relation would be changed, and BGs might be obtained.

In this paper, firstly, we define the model of the periodical cut-off (PCO) square lattice grid, and study the elastic BGs characteristics in it. Then we determine the BGs mechanism from analysing the frequency dispersion relation and motion modes in it. The influence of the cantilever length on BGs is also illustrated. Finally, we give the frequency response analysis of the corresponding finite specimen to support our conclusions.

2. Model

Figure 1 shows the original square lattice grid and the PCO square lattice grid. The PCO operation is implemented according to the square lattice type, so the PCO square lattice grid still maintains the square lattice type. The original square lattice grid has the lattice constant of a , while the lattice constant of PCO square lattice grid is twice as that of the original square lattice grid. Actually, there could be several different kinds of PCO operations which just need to meet at least two requirements: (1) the PCO lattice grids should still maintain some kinds of periodicity, but not necessarily the same as that of the original lattice grids; (2) the PCO lattice grids should not disintegrate into pieces. Here we just consider one kind of PCO square lattice to study the effect and mechanism of PCO operations on square lattice grids.

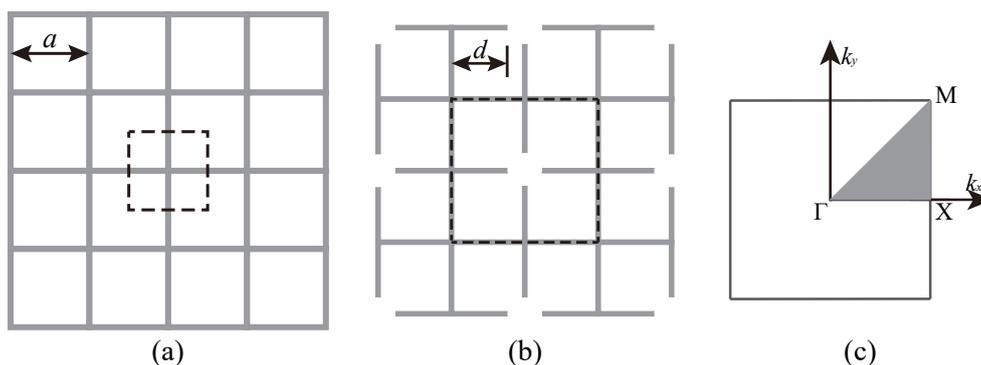


Figure 1. (a) The square lattice grid, (b) PCO square lattice grid and (c) irreducible Brillouin zone

Clearly, the PCO operation generates several cantilevers which vertically stretch from the middle position of each rod. We can change the length of the cantilever d by adjusting the size of cut-off parts. Because the exposed cantilevers lose the load-carrying ability, thus the stiffness of PCO square lattice grids must be lower than the original square lattice grids. In spite of this, the cantilevers still have the mass effect. Moreover, the lattice constant increases after the PCO operation. These three factors make

it possible to change the propagation characteristic of elastic waves in it. Cantilevers must be the key to generate the elastic BGs in PCO square lattice grids.

3. Elastic BGs

The frequency dispersion relations of lattice grids could be calculated by the finite element method, the details could be seen in [31]. We just consider the propagation of in-plane waves. Here, we use the normalized frequency ff_0 to describe the frequency dispersion relation and BGs ranges of PCO square lattice grids, where f_0 is the basic frequency of the simply supported beam constructed by the rod in lattice grid. $f_0 = \pi (EI/\rho l)^{1/2}/2l^2$, where l is the length of the rod, EI is the flexural stiffness of the rod, $\rho l = \rho S$ is the line density, ρ is the material density, S is the cross section area of the rod. Because of using the normalized frequency, the results below could be independent on the material type. In other words, the PCO square lattice grids with identical construction have the same frequency dispersion relations; even they are composed of different materials. So it is more convenient to study the influence of geometry construction on BGs. Here we choose the cross section of rod as the square shape, which is easy to manufacture. Actually, the cross section of the rod could be any shape, and just need to be the same at any position of rod. Let the square cross section width be b , so the slenderness ratio of rod could be defined as $\gamma = l/b$. In this paper, we just analyze the case of $\gamma = 10$. Specifically, we set $l = 0.07$ m and $d = 0.06$ m, in order to more conveniently find the influence of cantilevers and the mechanism based on the case of relatively long cantilevers. The cells and the corresponding Brillouin zone are shown in figure 1.

In order to comprehensively illustrate the changes of frequency dispersion relation and BGs after cut, we calculate the frequency dispersion relations of the PCO square lattice grids with partial cantilevers left ($d = 0.06$ m) and no cantilevers left ($d = 0$), as well as the original square lattice grid. The comparison is shown in figure 2. Clearly, the original square lattice grid and the PCO square lattice grids with no cantilevers left both have no BGs in the concerned frequency range, which also verifies the fact that the square lattice grid is hard to obtain BGs. The difference is the latter one has bigger lattice constant, thus its frequency dispersion curves are compressed to low frequency direction. While the PCO square lattice grid with partial cantilevers generates two BGs, which cover the ranges of $0.39 \sim 0.40$ and $0.48 \sim 0.53$.

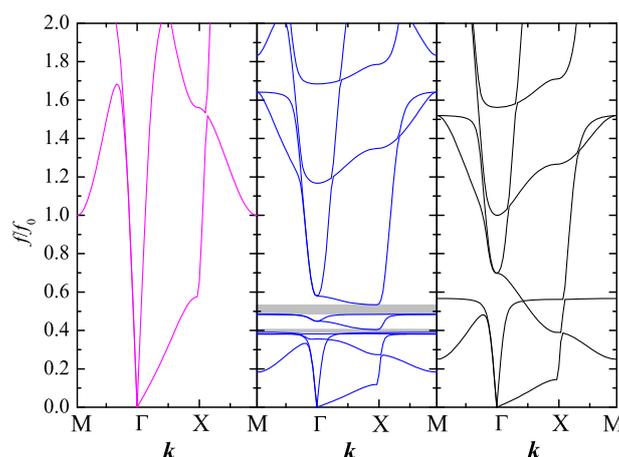
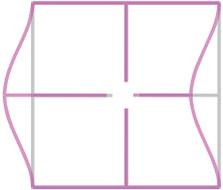
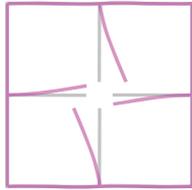
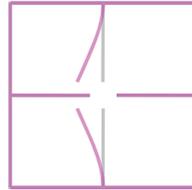
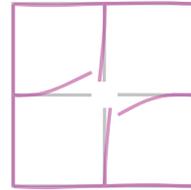


Figure 2. Frequency dispersion relations of the original square lattice grid (left), PCO square lattice grids with partial (middle) and no (right) cantilevers. The shadow zones are the BGs ranges.

Definitely, the PCO square lattice grids with or without partial cantilevers have the similar features in frequency dispersion curves. Because the two cases have the same lattice constant, as well as the

load-carrying skeleton and material. But, several new relatively flat curves occur in the case of PCO square lattice grid with partial cantilevers, which could only be attributed to the existence of cantilevers. The motion modes of a cell of the PCO square lattice grid with cantilevers at certain wave vectors on frequency dispersion curves (table 1) support the conclusion. The 1st BG locates between the 4th and 5th order curves, and the frequencies corresponding to wave vector X determine the 1st BG range. The longitudinal motions of cantilevers and flexural motions of skeleton are shown at wave vector X in 4th order curve. While the flexural motions of cantilevers are shown at wave vector X in 5th order curve. The discrepancy of cantilever motions leads to the 1st BG. Similarly, the 2nd BG is generated. While the motions which determine the 2nd BG are both flexural modes of cantilevers, which but behaves different types. However, the 1st BG is too narrow to work. Actually, when we cut off longer parts, the 1st BG disappears. So the 2nd BG is really worth considering.

Table 1. Motion modes of a cell of the PCO square lattice grid with cantilevers at wave vector X

4 th order	5 th order	6 th order	7 th order
			

Due to the cantilevers motion at the lower limit of the 2nd BG are the pure first flexural mode in the same direction, we think the first flexural motion of cantilevers determines the lower limit of the 2nd BG. The natural frequency corresponding to the first flexural motion of cantilever is given as $f_1=1.875^2(EI/\rho l)^{1/2}/2\pi l^2$. So we get the normalized frequency f_1/f_0 is 0.48, which matches the lower limit of the 2nd BG. Evidently, the cantilevers indeed determine the lower limit of the 2nd BG. The conclusion also support that the 2nd BG is generated by the localized resonance of cantilevers. So the PCO square lattice grid is actually a localized resonant PC, although it is composed of just one material.

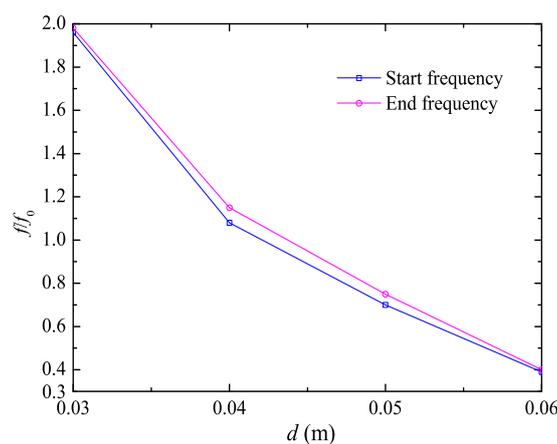


Figure 3. The start (square) and end (circle) frequencies of the BG of the PCO square lattice grid with different lengths of cantilevers.

For verifying the above conclusions, we also calculate other three cases with cantilevers lengths of 0.05 m, 0.04 m and 0.03 m, and give the corresponding BG ranges with different cantilevers lengths,

which are shown in figure 3. Surely, the lower limit of the BG is determined by the first flexural mode of cantilevers. The longer cantilever leads to the lower natural frequency. So the longer cantilever case gives the lower BG frequency. Additionally, the BG range changes with the change of cantilever length, while the difference is little.

After the theoretical analysis of infinite PCO square lattice grids, then we study the practicable finite cases, to find the feasibility of BG regulation and control from PCO operations. We choose the specimen with 8×6 cells which is shown in figure 4 (a). We just focus the wave propagation along the ΓX direction which corresponds to the frequency dispersion curves between ΓX . We apply the harmonic displacement impulses which sweep over the normalized frequency range of 0~0.6 to the left end of specimen, and then get the frequency responses at point A on the other end. Two kinds of impulses which correspond to the in-plane longitudinal and transverse vibrations are applied and marked as I and II in figure 4 (a). Figure 4 (b) illustrates the frequency responses at point A. The shadow area shows the both distinct attenuation areas of the two kinds of impulses, which is the actual BG range. Clearly, just the above mentioned 2nd BG exists in frequency response results. The 1st BG is too narrow to show. The actual BG covers the range of 0.47~0.54, which matches the theoretical BG result very well. The results verify that the square lattice grid indeed generates BGs after PCO operations. Additionally, the PCO square lattice grids could give relatively low frequency BGs based on the locally resonant mechanism, which could be used in many potential applications.

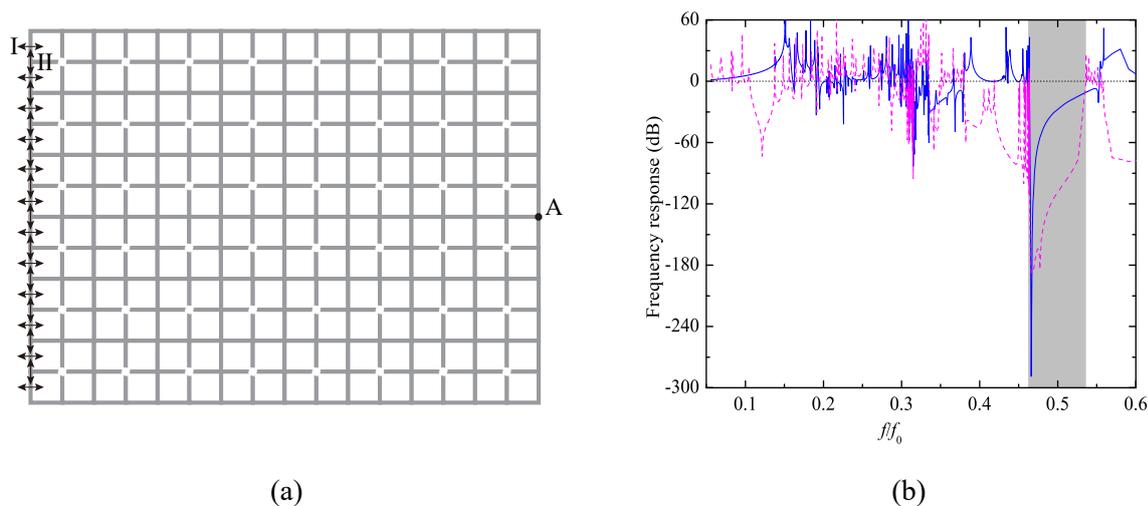


Figure 4. (a) Finite specimen of the PCO square lattice grid. The in-plane longitudinal (I) and transverse (II) vibration impulses are applied. (b) The frequency response is obtained at point A. The overlapping of curves of transverse (dash) and longitudinal (solid) vibrations gives the actual BG.

4. Conclusions

In summary, we mainly define the model of PCO square lattice grid, and study the mechanism and properties of elastic BGs in it. We confirm that the PCO operation is feasible to generate BGs in square lattice grids. Furthermore, the BGs generated in PCO square lattice grids are basically locally resonant BGs with low frequency ranges which attributes to the motions of exposed cantilevers. The lower edge of the locally resonant BG could be predicted by the natural frequency of first flexural mode of cantilevers. One can easily control the BG location from adjusting the size of cut-off parts. All the features make the BGs in PCO square lattice grids more feasible in applications.

Acknowledgement

The authors would like to thank the support of the National Natural Science Foundation of China (Grant No. 51608173), Natural Science Foundation of Jiangsu Province (Grant No. BK20150796) and

Fundamental Research Funds for the Central public welfare scientific research institutes (Grant No. Y316022).

References:

- [1] Vasiliev V V and Razin A F 2006 *Compos. Struct.* **76** 182
- [2] Noor A K, Venneri S L, Paul D B and Hopkins M A 2000 *Comput. Struct.* **74** 507
- [3] Phani A S, Woodhouse J and Fleck N A 2006 *J. Acoust. Soc. Am.* **119** 1995
- [4] Chen Y, Liu X and Hu G 2014 *Comptes Rendus Mécanique* **342** 273
- [5] Vigliotti A, Deshpande V S and Pasini D 2014 *J. Mech. Phys. Solids* **64** 44
- [6] Cui X, Xue Z, Pei Y and Fang D 2011 *Int. J. Solids Struct.* **48** 3453
- [7] Gonella S, To A C and Liu W K 2009 *J. Mech. Phys. Solids* **57** 621
- [8] Sigalas M M and Economou E N 1992 *J. Sound Vib.* **158** 377
- [9] Kushwaha M S, Halevi P, Dobrzynski L and Djafari-Rouhani B 1993 *Phys. Rev. Lett.* **71** 2022
- [10] Liu Z, Zhang X, Mao Y, Zhu Y Y, Yang Z, Chan C T and Sheng P 2000 *Science* **289** 1734
- [11] Shu H, Shi X, Li S, Liu S, Wang W and Shi D 2014 *Int. J. Mod. Phys. B* **28** 1450231
- [12] Han L, Zhang Y, Ni Z, Zhang Z and Jiang L 2012 *Physica B* **407** 4579
- [13] Zhang Y, Han L and Jiang L 2013 *Phys. Status Solidi B* **250** 1245
- [14] Han L, Zhang Y, Jiang L and Zhang Z 2013 *Phys. Status Solidi - R.* **7** 514
- [15] Martinezsala R, Sancho J, Sanchez J V, Gomez V, Llinares J, Meseguer F 1995 *Nature* **378** 241
- [16] Yu D, Wen J, Zhao H, Liu Y and Wen X 2008 *J. Sound Vib.* **318** 193
- [17] Yu D, Liu Y, Zhao H, Wang G and Qiu J 2006 *Phys. Rev. B* **73** 64301
- [18] Farhat M, Guenneau S and Enoch S 2009 *Phys. Rev. Lett.* **103** 24301
- [19] Mohammadi S, Eftekhar A A, Hunt W D and Adibi A 2009 *Appl. Phys. Lett.* **94** 51906
- [20] Liebold-Ribeiro Y and Körner C 2014 *Adv. Eng. Mater.* **16** 328
- [21] Nolde E, Craster R V and Kaplunov J 2011 *J. Mech. Phys. Solids* **59** 651
- [22] Gonella S and Massimo R 2008 *J. Sound Vib.* **312** 125
- [23] Spadoni A, Ruzzene M, Gonella S and Scarpa F 2009 *Wave Motion* **46** 435
- [24] Dai L, Jiang S, Lian Z, Hu H and Chen X 2015 *J. Sound Vib.* **337** 150-160
- [25] Yeh J 2007 *Physica B* **400** 137
- [26] Robillard J, Matar O B, Vasseur J O, Deymier P A, Stippinger M, Hladky-Hennion A, Pennec Y and Djafari-Rouhani B 2009 *Appl. Phys. Lett.* **95** 124104
- [27] Wu F, Liu Z and Liu Y 2002 *Phys. Rev. E* **66** 46628
- [28] Yao Y, Hou Z and Liu Y 2007 *Phys. Lett. A* **362** 494
- [29] Wang L and Bertoldi K 2012 *Int. J. Solids Struct.* **49** 2881
- [30] Bertoldi K and Boyce M 2008 *Phys. Rev. B* **77** 52105
- [31] Han L, Zhang Y, Li X, Jiang L and Chen D 2016 *Crystals* **6** 11