

# The algorithm of central axis in surface reconstruction

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**Abstract.** Reverse engineering is an important technique means of product imitation and new product development. Its core technology -- surface reconstruction is the current research for scholars. In the various algorithms of surface reconstruction, using axis reconstruction is a kind of important method. For the various reconstruction, using medial axis algorithm was summarized, pointed out the problems existed in various methods, as well as the place needs to be improved. Also discussed the later surface reconstruction and development of axial direction.

## 1. Introduction

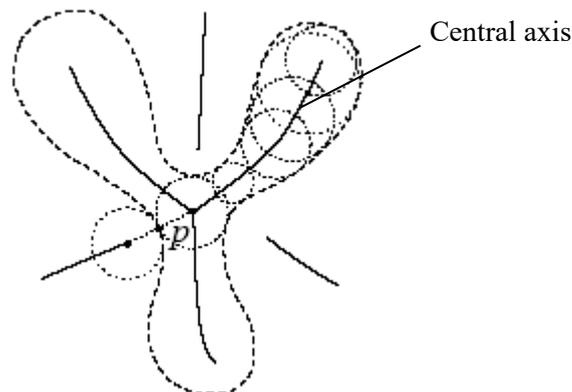
Reverse Engineering is a new emerging discipline with the development of computer development. At the same time, maturity of data measurement technique further promote the development of Reverse Engineering, making this technology a new way for imitation and new product development [1]. This technology has been widely used in the field of machinery, aviation, automobile and shipbuilding. Surface reconstruction is one of the most important and difficult problems, which solves how to construct surface by scattered points. Lots of domestic and overseas scholars pay more attention to surface reconstruction, and rise numerous algorithms and methods, many algorithms revolve around central axis. Central axis is an important tool of describing sampling density and solid shape, which plays an important way in surface reconstruction algorithm. Central axis has been used in many fields, such as picture processing, computer visualization, solid modeling, grid production, kinematics.

## 2. Implication of central axis

There is no single argument about central axis in the sphere of computer graphics and computation geometry. Reaching the definition by consulting related articles [2]. First one is the definition of astrosphere. For one smooth surface named  $S$ , if one ball is tangent to  $S$  and has at least two pointcuts, the ball is called astrosphere. Central axis is the set of center of central axis, as shown in figure 1 in two dimensional state. It is observed that it can capture all characteristic of  $S$  if have enough sampling points, then central axis of  $S$  can be approached by set of sampling points named  $P$ . Central axis is a curve on the plane, the branch of it is different on the basis of different shape of entity.

The density of sampling points is request in the counting process of central axis and in the reconstruction of surface, so sampling should has enough density to contain all characteristic of  $S$ . Here is a definition of local characteristic function  $f(x)$ ,  $f: S \rightarrow R$ ,  $f(x)$  represents distance between point  $x$  and  $f(x)$ . For point  $x$  in the set of sampling point, point  $y$  can be fined, it makes the distance between  $x$  and  $y$  less than  $\varepsilon f(x)$ , the smaller  $\varepsilon$  is, the more closely spaced is the sampling, on that way, it is better for the calculation of central axis. Generally,  $\varepsilon < 0.25$ , small value of  $\varepsilon$  is used under certain circumstances.





**Figure 1.** Central axis in two dimensional state

### 3. The method of approaching to the central axis

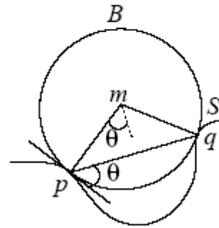
#### 3.1. The rule of surface reconstruction based on Voronoi filtering

Although the central axis plays an important role in the surface reconstruction, but calculating the central axis directly is difficult, it is the key point to find suitable algorithm to approaching the central axis. The study find that all approaches are based on Voronoi and Delaunay triangulation. Voronoi vertices in two dimensions are able to approximate the central axis, which is already achieved, and when the sampling density tends to infinity, the Voronoi vertices converge to the central axis. But this property is broken by the Sliver in three dimensions, because the split in three dimensions, regardless of the density of the sampling, is able to close to the surface. In order to mitigate the effects of the whole surface reconstruction, Amenta and Bern, put forward the concept of pole [3], that is, each sample point corresponds to the Voronoi polygon and two points of Voronoi polygon farthest from the sample point are respectively referred to positive pole and negative pole. It shows that the pole is close to the central axis and converges to it for a continuous smooth entity, when sampling density tends to zero, the sample point tends to infinity.

The discovery of the pole is a huge progress in 3D reconstruction. The output of most surface is discrete by pure approximation with pole axis, while many continuous approximation are needed in practical application. A series of algorithms arise at the historic moment. Amenta et al, on the basis of the pole and the pole of Delaunay ball, view the radius of Delaunay ball as weight, then weight to the pole of Delaunay triangulation, finally calculate the central axis by the boundary of the balls [4]. It is the first time that the approach method is of continuity for central axis in three dimensions. However, this algorithm requires a second Voronoi diagram to achieve the calculation of the central axis, and the central axis is noisy in some cases.

It is well known that noise has a large effect on surface reconstruction, and it will invalidate the reconstruction algorithm in many cases. The above algorithm is not available because of noise. Tamal k. Dey et al., on the basis of the previous algorithm, propose two independent conditions of the ratio and density, and the Voronoi plane is selected from the Voronoi diagram to approach the central axis. The main idea of the algorithm is to filter the Delaunay edge from the Delaunay triangle in the sample point set and output its dual Voronoi surface as approach of center axis. The Angle condition named  $\theta$  is used to filter the Delaunay.  $\theta$  is the maximum Angle, which the intersection point of the center ball  $B$  and plane  $S$  is intersected to the sectional polygon (sectional polygon refers to the crossing plane by plane passing through point  $P$  with normal vector and Voronoi diagram of point  $P$ ), as shown in figure 2. Angle  $\theta$  needs to increase with the decrease of appropriate sampling density. So we can not find a continuous  $\theta$  to adapt to any model. Proportional conditions are needed. The second is the proportional conditions  $\rho$ . When the side of the Delaunay triangle is basically parallel to the cut polygon, the condition fails and the proportional condition is required to constrain it. The defect of proportional

condition is that one  $\rho$  can not capture the central axis of the whole entity if sampling density is irregular on the entire surface, So, the Angle and proportional conditions are cross-over many times.



**Figure 2.** Angle condition  $\theta$

Specifically, it may lead to empty of reconstructing surface when the scope of  $\theta$  is too large, while it may appear for something looks like nail when the scope of  $\theta$  is too small. The experiment has proved that  $\theta$  for  $\pi/8$  is the most appropriate.  $\rho$  also has a limitation, it is found that eight is the best.

So, in summary, this is the first time that a subset of Voronoi is used to approximate the central axis and the convergence is a guaranteed algorithm. But this algorithm does not prove that the output is consistent with the original topology. Although it is very difficult to prove that the axis of the output is identical to the original axis, it is not impossible. So future studies should focus on the problem of homeomorphism.

### 3.2 Algorithm of surface reconstruction based on Crust

In the algorithm of approaching the medial axis, there is also a classic approach based on the Crust algorithm. The algorithm idea uses a subset of the vertices of the Voronoi to remove the redundant triangles from Delaunay triangulation, and it is proved that the sampling density depends on the local feature surface, the output of this algorithm can also be consistent with the original surface topology and converges [5,6]. Its uniqueness is that after the initial crust, there are two steps of subsequent processing. One is to remove the extra triangles according to the surface normal vector. The specific approach is that  $T$  to be a triangle,  $S$  is its maximum angle vertex. If the angle between the normal vector of  $T$  and the normal vector of  $S$  and  $p$  is too large, more than the input parameters  $\theta$ , then remove the triangle  $T$ . The second is to adjust all the triangles, mainly for Sliver. Starting at any point  $S$  on the convex shell  $S$ , find the positive and negative poles of  $S$ , and the triangle  $T$  near  $S$ , define its outward side to be visible. Then adjust the other vertices of  $T$  also follow this rule, and each triangle having the same vertex with  $T$  follows this rule, and goes on until the end.

## 4. Conclusion

Summarizing the algorithms for approaching the medial axis, most of them are based on Voronoi diagrams, and the effect of most refactoring is better. But in all kinds of algorithms, there are still many problems in 3D reconstruction. For example, the algorithm mentioned above, in some parts of the machine parts of the area of creases or corners, the approximation of the axis will often appear error, resulting in the result error; the other is the presence of lobes (Sliver) in three dimensions. For the characteristic of being tangent and close to the surface makes many algorithms failure, so most of the algorithms only mitigate the impact of the lobes on the reconstruction results to some extent, and not completely eliminated, these are the problems that need to be solved later.

## Acknowledgments

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