IOP Conf. Series: Materials Science and Engineering **241** (2017) 012022 doi:10.1088/1757-899X/241/1/012022

A Gradient Taguchi Method for Engineering Optimization

Shun-Fa Hwang[1, a], Jen-Chih Wu[1] and Rong-Song He [2]

[1]Department of mechanical engineering, National Yunlin University of Science and Technology, 123 University Road, Sec. 3, Touliu, Taiwan 640, R. O. C.

[2]Department of mechanical engineering, WuFeng University, 117 Chian-Kuo Road, Sec. 2, Ming-Hsiung, Chia-Yi, Taiwan 621, R.O.C

[a]hwangsf@yuntech.edu.tw

Abstract. To balance the robustness and the convergence speed of optimization, a novel hybrid algorithm consisting of Taguchi method and the steepest descent method is proposed in this work. Taguchi method using orthogonal arrays could quickly find the optimum combination of the levels of various factors, even when the number of level and/or factor is quite large. This algorithm is applied to the inverse determination of elastic constants of three composite plates by combining numerical method and vibration testing. For these problems, the proposed algorithm could find better elastic constants in less computation cost. Therefore, the proposed algorithm has nice robustness and fast convergence speed as compared to some hybrid genetic algorithms.

1. Introduction

In engineering applications, optimization is often required to enhance the performance of a material or structure and to reduce its weight or cost at the same time. Recently, stochastic methods such as genetic algorithm (GA), simulated annealing (SA), particle swarm optimization, ant colony optimization, and immune algorithm have been widely and successfully applied in various engineering problems. However, one drawback of stochastic methods is the large times of function evaluation. Taguchi method is a statistical methodology to analyze tested data for determining the effects of various factors and optimizing their levels. Taguchi method using orthogonal arrays could reduce the number of tests even when the number of factors is quite large. However, since Taguchi method is concerned with the discrete level of factors, it is troublesome to handle the continuous values of factors.

Among different algorithms for optimization, fast convergence speed and robustness in finding the global optimum are always concerned. However, it is not easy to be satisfied simultaneously by just a single algorithm. Therefore, there is an increasing interest to propose hybrid approaches [1-4] to avoid premature convergence towards a local minimum and to reach the global optimum.

Similar to the above attempts, it is desired to propose a method to balance the robustness and the convergence speed. Hence, a novel hybrid algorithm is proposed in this work. This novel hybrid algorithm consists of Taguchi method and the steepest descent method. Three composite plates discussed by other researchers are chosen to test the applicability of the proposed algorithm for the inverse determination of elastic constants of materials by combining numerical method and vibration testing.

2. Gradient Taguchi Method

By using orthogonal arrays, Taguchi method could quickly find the optimum combination of the levels of various factors, even when the number of level and/or factor is quite large. In real applications, the

IOP Conf. Series: Materials Science and Engineering **241** (2017) 012022 doi:10.1088/1757-899X/241/1/012022

number of level is common to be two or three. Taguchi orthogonal array is usually represented as $L_a(Q^b)$, where a denotes the number of experiments, Q is the number of levels, and b is the number of factors. In the present study, the number of level is always chosen as two. For example, the Taguchi orthogonal array for three factors is chosen to be $L_4(2^3)$. The main effect of each level j of each factor i, f_{ij}, is calculated according to Eq. (1), and the difference of the main effect of each factor i, fd_i, is calculated according to Eq. (2).

$$f_{ij} = \frac{1}{N}\sum_{k=1}^{a} F_k \delta_k \tag{1}$$

$$fd_i = \left| f_{i1} - f_{i2} \right| \tag{2}$$

In these equations, F_k is the experiment value. The Kronecker delta $\delta_k = 1$ when the F_k value is from the factor i and the level j, otherwise its value is zero. Also, N is the number of experiments that are accounted during the summation. In general, when the minimum is the better, smaller f_{ij} represents better level of j for factor i, and larger fd_i denotes that factor i has profound effect as compared to other factors.

From the values of f_{ij}, the optimum combination of the level of each factor could be obtained. However, in some circumstances, this combination is not the best, and some modifications need to be done to get the optimum combination. By using the two levels of factor i, its gradient $\nabla f_{i,0}$ is calculated as

$$\nabla f_{i,0} = \frac{f_{i2} - f_{i1}}{x_i^{0,U} - x_i^{0,L}} \tag{3}$$

where $x_i^{0,L}$ and $x_i^{0,U}$ denote the first and second levels. Hence, the gradient of all factors could be represented as

$$\nabla f_0 = \left\{ \nabla f_{1,0}, \nabla f_{2,0}, \ldots\ldots, \nabla f_{n,0} \right\} \tag{4}$$

The index 0 in Eqs. (3) and (4) is used to represent the number of iteration. Then, the point of next generation is obtained as

$$\tilde{x}^{i+1} = \tilde{x}^{i} \pm \lambda \nabla f_i \tag{5}$$

where λ is the step length, "+" is for the maximum problem, and "-" is for the minimum problem. To obtain the appropriate step length, the steepest descent is used. An initial step length, H, could be guessed or set to be 1 in this work. Then, three different step lengths are created as 0, 1, and 2 times of H, and the corresponding values of objective function are Y_0, Y_1, Y_2, respectively. Also, define a value D as

$$D = 4Y_1 - 2Y_0 - 2Y_2 \tag{6}$$

If $D \geq 0$, let λ be equal to H/3. Otherwise, for the maximum problem,

$$\lambda = \frac{H\left(4Y_1 - 3Y_0 - Y_2\right)}{D} \tag{7}$$

For the minimum problem,

$$\lambda = \frac{H\left(4Y_1 - Y_0 - 3Y_2\right)}{D} \tag{8}$$

From the Taguchi method, the best combination of the level of each factor, $\tilde{b}$, could be obtained according to the main effects and the modification. However, there is one point named base point for each generation, $\tilde{x}$, which could be further used to accelerate the searching. At this point, the value of each factor is equal to the average of the two levels. In some circumstances, the base point may provide even better choice than the best value from main effect analysis. By considering both the best point from

IOP Conf. Series: Materials Science and Engineering **241** (2017) 012022 doi:10.1088/1757-899X/241/1/012022

Taguchi method and the base point for iteration number i, the possible values for the next iteration are $\tilde{b}^i \pm \lambda \nabla f_i$ and $\tilde{x}^i \pm \lambda \nabla f_i$. Here, "+" is for the maximum problem, and "-" is for the minimum problem.

3. Inverse Determination Examples

To apply the present optimization method to engineering problems, the inverse determination of elastic constants of materials by combining numerical method and vibration testing is considered. Three composite plates discussed by other researchers are chosen. Plate A is a unidirectional glass/epoxy laminate with 12 layers, its dimensions are 268x150x1.43 mm and its density is 1932 kg/m^3 [5]. Seven natural frequencies measured are 72.3, 97.6, 176.7, 197.4, 238.3, 277.5, 348.0. Plate B is a symmetric cross-ply laminate with 14 layers, its dimensions are 191x282x2.87 mm and its density is 1624 kg/m^3 [6]. Eight natural frequencies measured are 72.2, 108.6, 182.7, 293.9, 308.7, 343.5, 369.4, 442.6. Plate C is a commercially pressed, woven-glass epoxy board having 18 fabric layers. Its dimensions are 120x90x1.19 mm and its density is 1755 kg/m^3 [7]. Eight natural frequencies measured are 138.7, 261.1, 374.3, 463.0, 536.5, 713.3, 771.0, 834.1.

For all these three plates, the optimization problem could be stated as follows:

$$\text{Minimize } F(X) = \sum_{i=1}^{n_0} \left(\frac{f_i - \overline{f_i}}{\overline{f_i}} \right)^2 \quad (9)$$

where F is the objective function that is a function of the design variable set $X = [x_1, x_2, x_3, ..., x_m]$. Here m is the number of the design variables x_i. The natural frequency obtained by vibration testing is denoted as $\overline{f_i}$ and that obtained by finite element analysis is represented as f_i. The number of natural frequencies used in the objective function is n_0. The elastic constants are assigned as design variables and restrained to be positive. Since all these three plates are considered to be thin and each layer is transversely isotopic, only four elastic constants are necessary and they are the design variables. In the finite element analysis to calculate the natural frequencies, 100 shell elements were used for all three plates.

Table 1. Elastic constants obtained for composite plate A.

	Number	E_1(GPa)	E_2(GPa)	υ_{12}	G_{12}(GPa)	Obj. Fun.
Ref [5]		44.20	17.70	0.195	7.20	0.000956
ARSAGA	100	45.30	17.91	0.202	7.42	0.000085
GTM	102	45.22	17.83	0.215	7.44	0.000076

Table 2. Elastic constants obtained for composite plate B.

	Number	E_x(GPa)	E_y(GPa)	υ_{xy}	G_{xy}(GPa)	Obj. Fun.
Theory		23.30	13.80	0.217	2.70	0.000513
ARSAGA	97	23.24	14.22	0.229	2.71	0.000101
GTM	171	23.44	14.20	0.241	2.71	0.000030

For plate A and under the condition of plane stress, the obtained elastic constants are listed in Table 1. Since the present algorithm may be dependent on the initial guess, different runs may result in different results. Hence, five runs are executed to check its repeatability, and only the best one is shown. The reference results in the first row were obtained by Rikards et al. [5]. The best results denoted as

ARSAGA were obtained by an adaptive genetic algorithm combined with simulated annealing [1]. The five runs by the present method obtain very close results and its repeatability is excellent. As shown in the table, from the viewpoint of the objective function, the present method has the lowest value. In the table, the column denoted as “Number” represent the generation number for ARSAGA and the iteration number for the present method. Since the population size of the present method is much less than that of ARSAGA, the convergence speed of the present method is much superior that that of ARSAGA.

Table 3. Elastic constants obtained for composite plate C.

	Number	E_1(GPa)	E_2(GPa)	υ_{12}	G_{12}(GPa)	Obj. Fun.
Ref [7]		16.47	16.49	0.207	2.44	0.001197
ARSAGA	71	16.74	16.72	0.039	2.48	0.000759
GTM	59	16.92	16.57	0.174	2.46	0.000798

Since plate B is a cross-ply laminate, the effective elastic constants for the whole plate are sought in this work. Five runs by the present method are executed and the repeatability of the obtained effective elastic constants is excellent. As shown in Table 2, the row denoted by “Theory” represents the results calculated by the classic lamination theory. As the value of the objective function is utilized to compare the present method with the theory and ARSAGA, it is evident that the best result of present method is superior to the others. Table 3 shows the elastic constant obtained for the woven-glass epoxy layer from plate C. The reference values in the first row were from the results of Hutapea and Grenestedt [7]. Similarly, these results indicate that the present method have excellent repeatability as well as fast convergence speed and could find the best results.

4. Summary

In this work, a novel hybrid algorithm consisting of Taguchi method and the steepest descent method is proposed. The inverse determination of elastic constants of three composite plates by combining numerical method and vibration testing is used to test this novel hybrid algorithm. The results imply that the present method have excellent repeatability and fast convergence speed to find the best results as compared to other methods.

References

[1] S.F. Hwang, R.S. He: Adv. Eng. Inform. Vol. 20 (2006), p. 7–21.

[2] J.T. Tsai, T.K. Liu, J.H. Chou: IEEE Trans. Evol. Comput. Vol. 8 (2004), p. 365-377.

[3] A.R. Yildiz: Comput. Ind. Vol. 60 (2009), p. 613-620.

[4] P.W. Tsai, J.S. Pan, S.M. Chen, B.Y. Liao: Expert Syst. Appl. Vol. 39 (2012), p. 6309–6319.

[5] R. Rikards, A. Chate, W. Steinchen, A. Kessler, A.K. Bledzki: Compos. Part B-Eng. Vol. 30 (1999), p. 279-89.

[6] A.L. Araujo, C.M. Mota Soares, M.J. Moreira de Freitas, P. Pedersen, J. Herskovits: Compos. Struct. Vol. 50 (2000), p. 363-72.

[7] P. Hutapea, J.L. Grenestedt: J. Electron Mater. Vol. 32 (2003), p. 221-227.