

Investigation of reliability indicators of information analysis systems based on Markov's absorbing chain model

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Abstract. In the result of study of the algorithm of the functioning of the early detection module of excessive losses, it is proven the ability to model it by using absorbing Markov chains. The particular interest is in the study of probability characteristics of early detection module functioning algorithm of losses in order to identify the relationship of indicators of reliability of individual elements, or the probability of occurrence of certain events and the likelihood of transmission of reliable information. The identified relations during the analysis allow to set thresholds reliability characteristics of the system components.

An integral part of modern energy supply systems are energy consumption accounting systems. In most cases, their functionality is expanded by monitoring and monitoring functions. The monitoring and control function carries in deep analytical potential of revealing of extraordinary situations at power supply objects and detection of excessive losses in networks. The economic effect of timely, early detection of emergency situations and localization of areas of excess losses is hard to overestimate. Detailed problem-oriented monitoring and control of energy supply provides an increase in reliability of power supply. To this end, analytical systems for early detection of losses are integrated into the systems for commercial accounting of energy consumption. In the course of studying the algorithm for the functioning of the module for the early detection of excess losses, as well as individual characteristics of extraordinary situations, the question arises of assessing the degree of influence of the latter on the quality of the transmitted information. It is necessary to identify the relationship between the probabilities of occurrence of extraordinary events and the probability of transfer of full-fledged data. The study of signs of extraordinary situations allows us to assert the stochastic character of the latter.

A detailed investigation of the algorithm for the operation of the early loss detection module allows one to refer it to the Markov processes [1, 2, 3]. That, in turn, creates prerequisites for the application of existing methods of analysis of the algorithm as the Markov process [6, 8].



The analysis of the algorithm of functioning as the Markov process is reduced to three stages. At the first stage, the process is formalized in terms of the mathematical apparatus of the theory of Markov processes, a state graph and a transition probability matrix are constructed [1]. At the second stage, proceeding from a specific task, necessary mathematical operations are performed, the interrelation of process parameters is derived. At the third stage, the final calculation of the required parameters is carried out.

We represent the stages of the data analysis process by positions, and the probability of transition to the next stage is denoted by the corresponding variables. Thus, the probability of transition to the next stage will be uniquely determined by the value of the corresponding index. The investigated Markov chain will consist of the following positions:

S_6 - characterizes the receipt of data from the meter;

S_5 - investigation of data for the detection of energy carrier leakage;

S_4 - the stage of complex analysis of the readings of the meters for the detection of incorrect work;

S_3 - analysis of energy quality indicators for compliance with the standard;

S_2 - final conclusion on the successful passage of all stages of control;

S_1 - detection of the deviation of the investigated data from the set value.

And the states S_6, S_5, S_4, S_3 – are transient states that form a non-returnable set, and S_2 and S_1 – are absorbing and create an absorbing set. Consequently, the Markov chain, which simulates the algorithm for the functioning of the module for the early detection of excess losses, is an absorbing Markov chain.

By G.I. Novikov's definition, the absorbing Markov chain contains a state that the process has never reached before. A graphical representation of the Markov chain is the state graph depicted in Fig. 1.

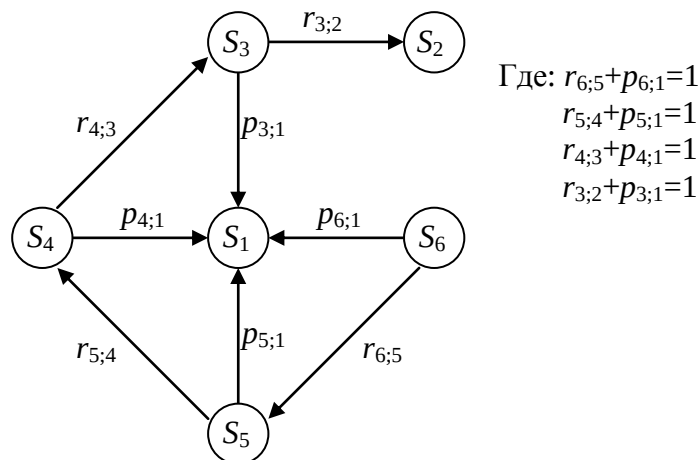


Fig. 1. The graph of the states of the algorithm for the operation of the module for early detection of excess losses

When considering the transition matrix P , it is convenient to give the canonical form by combining all absorbing states into one group and all transient states to another.

Suppose there are s non-return states and r - s absorbing states, then the canonical form of the matrix P takes the following form:

$$P = \begin{pmatrix} S & 0 \\ R & Q \end{pmatrix}$$

Here the region "0" is composed entirely of zeros. The submatrix Q (of dimension $s \times s$) describes the behavior of the process before the exit from the set of non-transient states, the submatrix R (of dimension $s \times (r-s)$) corresponds to transitions from irrecoverable to ergodic states, and the matrix S (dimension $(r-s) \times (r-s)$) refers to the process after reaching the ergodic set.

There exists a strict theorem [5], asserting that the powers of Q tend to zero. Consequently, when the matrix P is raised to higher degrees, all elements of the last S columns will tend to zero. This is a matrix version of the corresponding theorem

From the definition of the absorbing Markov chain it follows that $S = I_{(r-s) \times (r-s)}$, i.e. S is an identity matrix of dimension $(r-s) \times (r-s)$. Therefore, the canonical form of the matrix P has the form:

$$P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$$

The matrix of transition probabilities P takes the form:

$$P = \begin{array}{c|cccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ \hline S_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ S_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline S_3 & p_{3,1} & r_{3,2} & 0 & 0 & 0 & 0 \\ S_4 & p_{4,1} & 0 & r_{4,3} & 0 & 0 & 0 \\ S_5 & p_{5,1} & 0 & 0 & r_{5,4} & 0 & 0 \\ S_6 & p_{6,1} & 0 & 0 & 0 & r_{6,5} & 0 \end{array}$$

This matrix already has a canonical form (ergodic and irrecoverable states are grouped together). The first two of its states are absorbing. We divide the matrix P into blocks S , R and Q , dimension 2×2 , 4×2 and 4×4 , respectively.

We obtain three submatrices of the following form:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; R = \begin{pmatrix} p_{3,1} & r_{3,2} \\ p_{4,1} & 0 \\ p_{5,1} & 0 \\ p_{6,1} & 0 \end{pmatrix}; Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ r_{4,3} & 0 & 0 & 0 \\ 0 & r_{5,4} & 0 & 0 \\ 0 & 0 & r_{6,5} & 0 \end{pmatrix}.$$

Moreover, the submatrix Q (of dimension $s \times s$) describes the behavior of the process before the exit from the set of non-transient states, the submatrix R (of dimension $s \times (r-s)$) corresponds to transitions from transient states to absorbing ones, and the submatrix S (dimensions $(r-s) \times (r-s)$) describes the process after reaching the absorbing set.

According to [5], by investigating the Markov chain, one can give a quantitative estimate of the average number of changes in the states of the process prior to absorption. In the simulated process, this value reflects the average number of cycles of the algorithm recalculation.

To estimate the probability of the end of the process when it is in the current state, we use the theorem: If $b_{i,j}$ the probability that the process emerging from the irrecoverable state of S_i stops in the absorbing state S_j , then

$$\{b_{i,j}\} = B = N \cdot R, \quad S_i \in T, \quad S_j \in \hat{T}.$$

Proof: leaving the state S_i , the process can be absorbed in S_j in one or more steps. The capture probability at the first step is $p_{i,j}$. If this does not happen, the process can get into either another absorbing state, or some kind of irrecoverable state of S_k . In the latter case, the wandering particle can be captured by the desired state with the probability $b_{k,j}$. Consequently,

$$b_{i,j} = p_{i,j} + \sum_{\substack{k=1 \\ (S_k \in T)}}^n p_{i,k} \cdot b_{k,j},$$

That in the matrix form takes the following form:

$$B = R + Q \cdot B.$$

Matrix B shows the probabilities of the end of the data processing in one or another absorbing state. In this example, the first column reveals the probability of transition to the state S_1 (detecting the deviation of the data under study from the set value) when it is in the current state of the graph (row of the matrix), and the second - the probability of transition to the state S_2 (the final conclusion about the successful passage of all stages of control).

Thus, the set of obtained data establishes the probabilistic characteristics of the mathematical model of the algorithm for the functioning of the module for the early detection of excess losses. Practical application of the proposed algorithm of information analysis is possible on the basis of statistical data and expert estimates of the probability of occurrence of the corresponding events [4, 5, 7].

As a result of the investigation of the algorithm for the functioning of the module for the early detection of excess losses, it was proved that it can be modeled using absorbing Markov networks. Of particular interest is the investigation of the probabilistic characteristics of the algorithm for the functioning of the module for early detection of losses for the purpose of revealing the relationship between the reliability indicators of individual elements or the probability of occurrence of individual events and the probability of reliable information transmission.

The dependencies revealed during the analysis make it possible to establish threshold values of the reliability characteristics of the nodes of the system. For example, by choosing a certain probability of the transmission of reliable information, it is possible to impose restrictions on the threshold values of the probabilities of occurrence of individual events of the algorithm, which, in turn, imposes limitations on the reliability characteristics of the system hardware. In addition, the results obtained make it possible to assess the influence of reliability characteristics of individual system nodes on the probability of reliable data transmission. In addition to the values themselves, formulas for calculating the variance for all data groups are obtained. The developed algorithms have considerable practical potential both in designing and in evaluating the work of already existing systems.

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