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Modeling of forming technological errors in processing by gear shaping machine

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Abstract. In the paper, the mathematical model for evaluation the accuracy of the cut gear depending on the initial error of the technological system, is presented. The model is based on the coordinate transformation matrices, variations matrices and considers the deviations of the base and the real surfaces. The technique allows evaluating the possibilities of the technological system to achieve the specified accuracy.

Keywords: parametric failure, technological reliability, gear, mathematical model, error.

1. Introduction. The main purpose of mathematic modeling of technological systems is to evaluate their performance, as well as the possibility of providing the specified parameters [1,2]. When considering the processing of gear wheels, especially by the generating method, in the first place stands the problem of evaluating the output accuracy of the technological system [3]. This problem is exacerbated by the fact that it is difficult to reveal any clear correlation between geometric and kinematic errors of the technical system, and indicators of accuracy of the machined gear wheel [4,5,6]. To solve this problem it is necessary to apply a direct mathematical modeling of the formation of errors.

2.Modeling of generation of geometry of evolvement surface. Generation of geometry in this case can be represented as the transformation of movements of the tool through the corresponding coordinate transformation matrices of the technological system's elements [7]. The matrix-column that describes a perfect treated surface can be written in the form:

$$r_0 = A^D \cdot A^X \cdot A^Z \cdot A^C \cdot r_{cs} \qquad (1),$$

where$r_0$is the coordinate of the perfect (nominal) surface of the work piece; A^D – transformation matrix from the tool rotation around its axis, A^X - transformation matrix on the X-coordinate (distance between the tool axis and the workpiece), A^Z - transformation matrix on the Z- coordinate (vertical movement of ram),A^C - transformation matrix from the rotation of the workpiece around its axis; r_{cs}- vector of coordinates of the surface of the cutting tool [8].

Calculating the product of the above matrices, we get a vector of the perfect surface:

$$\text{where } r_0 = \begin{bmatrix} X\cdot\cos(C) - r_b\cdot\sin(C) - C\cdot i\cdot r_b\cdot\cos(C) \\ X\cdot\sin(C) + r_b\cdot\cos(C) - C\cdot i\cdot r_b\cdot\sin(C) \\ z \\ 1 \end{bmatrix} \qquad (2).$$

3. Modeling of errors of the technological system. During operation, the machine-tool is influenced by lots of geometric, force, temperature factors. This leads to disruption of the relative position and movement of machine units, including working bodies. Consequently, the actual coordinates of the point of the treated surface are different from the position established by equation (2). Simulation of the resultant error on the output of the machine-tool is done by adding to the equation a perfect surface of the matrices describing the errors of each element of the machine-tool:

$$\Delta r = (\delta A^D)A^D\cdot A^X\cdot A^Z\cdot A^C\cdot r_{cs} + A^D\cdot(\delta A^X)A^X\cdot A^Z\cdot A^C\cdot r_{cs} + A^D\cdot A^X\cdot(\delta A^Z)A^Z\cdot A^C\cdot r_{cs} + A^D\cdot A^X\cdot A^Z\cdot(\delta A^C)A^C\cdot r_{cs} \quad (3),$$

whereδA^D – the matrix describing the effect of the error of the workpiece's rotation; δA^X-the matrix describing the effect of the error on the relative positions of the centerlines of spindle of the tool and spindle of the workpiece; δA^Z- ;the matrix describing the effect of the movement of the ram along the Z axis; δA^C-the matrix describing the effect of the error of the drive of the shaping cutter rotation.

The output of the model is obtained the matrix-column that describes the deviation of parameters of the treated surface.

4. Bringing the output error of the machine tool to the geometry of the formed surface. The errors obtained by forming the gear can be regarded as the error of shape and position of the involute surface. For their definition it is necessary to define the base surface $q_0$and form a vector of dimensional parameters of this surface. Properties of involute are adequately defined by a single parameter – a diameter of the base circle, for convenience, we reduce it to the parameter of the radius of the base circle $r_{b.}$

The offset of the radius of the base circle of the workpiece Δr_b is defined as the sum of the error vectors of position and size [7]:

$$\Delta r_b = \varepsilon_b\cdot r_0 + dr_0 \qquad (4),$$

whereε_b is the matrix of error of the position of the coordinate system associated with the base surface relatively the system in which the equation was set.

$$\varepsilon_b = \begin{pmatrix} 0 & -\delta B_b & 0 & 0 \\ \delta C_b & 0 & -\delta A_b & 0 \\ 0 & \delta A_b & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (5).$$

The components of this matrix are small displacements of the coordinate system of the base surface relative to the coordinate system of the nominal surface for the corresponding axes.

dr_o - the total differential of the radius vectorr_0, taken for the components of the vector q_0 of dimensional parameters of the surface:

$$dr_0 = \frac{\partial r_0}{\partial r_b} \qquad (6),$$

where- $\frac{\partial r_0}{\partial r_b}$ - the vector of the partial derivative of the matrix nominal surface (2) with variable r_b.

Due to the small values of errors of moving machine units' position in a shape-generating system of the machine, Δr_b there is a total differential r_b for the components of the vector q, and therefore the formula (9) can be represented as:

$$\Delta r_b = G\cdot\Delta q \qquad (7),$$

Where $G$is the matrix of the 4-th order composed of the matrix-columns of partial derivatives $\frac{\partial r_b}{\partial q_i}$

$$G=\begin{bmatrix} C\cdot i\cdot r_b\cdot\sin(C)-r_b\cdot\cos(C)-X\cdot\sin(C) & 0 & -\sin(C)-C\cdot i\cdot\cos(C) \\ X\cdot\cos(C)-r_b\cdot\sin(C)-C\cdot i\cdot r_b\cdot\cos(C) & -z & \cos(C)-C\cdot i\cdot\sin(C) \\ 0 & r_b\cdot\cos(C)+X\cdot\sin(C)-C\cdot i\cdot r_b\cdot\sin(C) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (8).$$

The next phase of construction of the base surface is the calculation of the components of the vector Δq of errors in position and size. For the average quadratic base surface, the parameters are found by minimizing the sum of squares of deviations of the actual surface *r* from the base surface r_b.

The components of the vector *Δq* can be found from the matrix equation [7]:

$$\Delta q=\frac{d}{H} \quad (9),$$

where *H*is the matrix of 4-th order with the elements h_{ji}:

$$h_{ji}=\int_0^C\int_0^z f_j f_i\cdot n\,\mathrm{dzdC} \quad (10),$$

fk, fi – k and *I* are the coordinates of the vector *f* of normal transfer coefficients.

The normal vector to the nominal surface is determined by the equation:

$$n=\frac{\partial r_0}{\partial C}\times\frac{\partial r_0}{\partial z}=\begin{bmatrix} \dfrac{r_b\cdot\sin(\mathrm{C})-\mathrm{X}\cdot\cos(C)+i\cdot r_b\cdot\sin(C)+C\cdot i\cdot r_b\cdot cos(C)}{\sqrt{r_b^{\,2}\cdot(i+1)^2+(X-C\cdot i\cdot r_b)^2}} \\ \dfrac{r_b\cdot\cos(\mathrm{C})+\mathrm{X}\cdot\sin(C)+i\cdot r_b\cdot\cos(C)-C\cdot i\cdot r_b\cdot\sin(C)}{\sqrt{r_b^{\,2}\cdot(i+1)^2+(X-C\cdot i\cdot r_b)^2}} \\ 0 \\ 0 \end{bmatrix} \quad (11).$$

Determine the vector of normal transfer coefficients:

$$f(z,C)=G^T\cdot n \quad (12).$$

Using the components of the vector *f(z,C)* we calculate the components of the matrix*H*.

Define Δr_n- the projection of the vector *Δr* on the normal to the involute surface:

$$\Delta rn(z,C)=\Delta r(z,C)^T\cdot n \quad (13).$$

Determine the matrix-column *d* according to the formula:

$$d(z,C)=\begin{bmatrix} \int_0^C\int_0^z f(z,C)_0\cdot\Delta rn(z,C)dzdC \\ \int_0^C\int_0^z f(z,C)_1\cdot\Delta rn(z,C)dzdC \\ \int_0^C\int_0^z f(z,C)_2\cdot\Delta rn(z,C)dzdC \\ \int_0^C\int_0^z f(z,C)_3\cdot\Delta rn(z,C)dzdC \end{bmatrix} \quad (14).$$

On the basis of calculated values, determine the vector *Δq* of deviations of the base surface relative to the nominal [7]:

$$\Delta q(z,C) = H^{-1} \cdot d(z,C) \qquad (15).$$

As estimates of the errors in the size and position of the treated surface we use elements of the vector *Δq* determined from the formula (15). These components of the vector can be counted into standard indicators of accuracy of the cut gear wheel.

5. Conclusion. Thus, by using this mathematical model one can calculate the deviation of the real surface of the cut gear from nominal considering the initial errors of the technical system.

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