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Stabilization of nonlinear system using uniform eigenvalue assignment in linear model

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Abstract. Some plant in control system has nonlinear dynamic, so it is not easy to do in analysis to see its behavior using eigenstructure assignment. From many observations which have been made, some literature give methods to design nonlinear control system. The modern control theory uses state-space method to explain the behaviour on stability of a plant. To improve the stability of the closed-loop system, designer commonly use the state feedback control law. For the case inverted pendulum plant with the nonlinear dynamics, its need to perform the nonlinear control law with the concepts of modern control theory to satisfy the closed-loop system characteristic, and all the behaviour of the closed-loop system only determined from the given linear pole specifications.

1. Introduction

For the class of linear control systems, the performance of the systems is essentially determined by using method the eigenstructure assignment in the closed loop systems. It is also empirically true that the performance of a non linear system, is determined by the eigenstructure of the linearized system around certain nominal conditions. In this paper, the closed loop system will solve the uniform eigenvalue placement problem. That is, designing a single nonlinear state feedback or output feedback control law such that the linearized closed-loop system around the whole equilibrium manifold has the given set of eigenvalues [1]. Clearly, this objective is motivated by the intuition that performance of the resulting closed-loop system is less sensitive to variations of the equilibrium point.

2. System Details

Consider a general nonlinear system described [1] by equations,

$$\dot{x}(t) = f\{ x(t), u(t)\}$$
$$y(t) = h\{ x(t), u(t)\} \tag{1}$$

where $x(t)$ is the n-dimensional plant state, $u(t)$ is the m-dimensional plant input, $y(t)$ is the p-dimensional plant output. The equilibrium set of system (1) is defined as,

$$E = \{(x,u) \in R^n \times R^m \mid f(x,u) = 0\}$$

For x is the function of α as below,

$$f\{x(\alpha), u(\alpha)\} = 0 \tag{2}$$

then the equilibrium set given by,

$$E = \{(x(\alpha), u(\alpha)) \in R^n \times R^m \mid f\{x(\alpha), u(\alpha)\} = 0\} \tag{3}$$

Now consider the following parameterized system that is obtained by linierizing the plant around each point of the equilibrium manifold E [2].

$$\dot{x}_\delta = A(\alpha)\, x_\delta + B(\alpha)\, u_\delta$$

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$$y_\delta = C(\alpha)\, x_\delta \tag{4}$$

where

$$x_\delta = x - x(\alpha)$$
$$u_\delta = u - u(\alpha)$$
$$y_\delta = y - h\{x(\alpha), u(\alpha)\} \tag{5}$$

and

$$A(\alpha) = \frac{\partial f}{\partial x}\{x(\alpha), u(\alpha)\}$$
$$B(\alpha) = \frac{\partial f}{\partial u}\{x(\alpha), u(\alpha)\}$$
$$C(\alpha) = \frac{\partial f}{\partial x}\{x(\alpha), u(\alpha)\} \tag{6}$$

The fundamental idea can be well characterized in terms of the state feedback control and the design of nonlinear state feedback control law [4] of the form,

$$u(t) = K\{x(t), v(t)\} \tag{7}$$

where $v \in R^s$ represent an exogenous input, and $K\{\, x(t), v(t)\}$ is a smooth function satisfying $K(\,0, 0\,) = 0$ such that the closed loop system has the following properties:

(1) There exists a smooth function $v(o)$ satisfying $v(o) = 0$ such that,

$$f[x(\alpha),\ K\{x(\alpha), v(\alpha)\}] = 0 \tag{8}$$

(2) The linearized state feedback control law around the whole equilibrium manifold of the closed loop system [3] is given by,

$$u_\delta = K_1(\alpha)\, x_\delta + K_2(\alpha)\, v_\delta$$
$$v_\delta = v - v(\alpha) \tag{9}$$

where $K_1(\alpha)$, $K_2(\alpha)$ are smooth function synthesized according to the specific task such as output regulation. The above idea can be illustrated using the block diagrams shown in Figure 1.

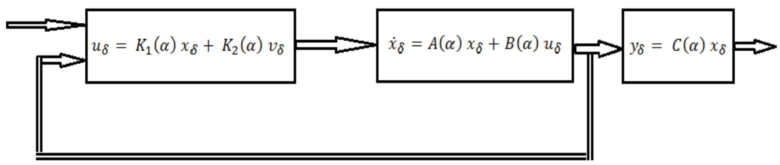

Figure 1. Parameterized Linear System

3. Design Of Nonlinear Controller For An Inverted Pendulum

For the asymtotic tracking, supposefor each α that the pair of equation below:

$$\frac{\partial f}{\partial x}\{x(\alpha), u(\alpha)\}\ ,\ \frac{\partial f}{\partial u}\{x(\alpha), u(\alpha)\} \tag{10}$$

Is controllable and has constant controllability indexes and has constant observability indexes. Under the additional requirement that the equilibrium manifold satisfy the static decoupling condition below for convenience.

$$f\{x(\alpha), u(\alpha)\} = 0$$
$$h\{x(\alpha), u(\alpha)\} = 0 \tag{11}$$

For under the class of either state feedback control laws, the closed loop satisfies, in addition to the local stability around each point in the equilibrium manifold,

$$\alpha = h\,[\{x(\alpha), u(\alpha) + K_1(\alpha)\langle v(\alpha) - x(\alpha)\rangle\}] \tag{12}$$

which complies, in term of state feedback for simplicity, that, given any constant exogenous input $v(t) = \alpha$. The output trajectory $y(t)$, asymtotically approach $v(t) = \alpha$; that is, we have,

$$\lim_{t \to \alpha} y(t) = v(t) \tag{13}$$

Consider a problem of balancing an inverted pendulum on a cart as a nonlinear plant. The dynamics equation of motion for the inverted pendulum shown in equation (14).

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{g\ \sin(x_1) - a\,m\,L\,x_2^2\ \sin\frac{2\,x_1}{2} - a\ \cos(x_1)\,u}{\frac{4}{3}L - a\,m\,L\ (\cos\ (x_1))^2}\end{bmatrix} \tag{14}$$

where x_1 is the angle (in radians) of the pendulum from vertical axis, and *u* is the force applied to the cart (in Newtons). The various parameters and values are,

m = pendulum mass = 2.0 kg.
M = cart mass = 8.0 kg.
2L = pendulum length = 1.0 m
G = 9.8 m/sec^2
$a = \frac{1}{m+M}$

It is straighforward to verify that equilibrium manifold,

$$x(\alpha) = \begin{bmatrix}\alpha\\ 0\end{bmatrix}\ ,\ u(\alpha) = \frac{g}{a}\tan\alpha \tag{15}$$

Is such that equation ( ) we take calculation and gives two parameters below.

$$A(\alpha) = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{\left[\frac{4}{3}L - a\,m\,L\ \cos\alpha^2\right]\ \cos\alpha} & 0\end{bmatrix}\ ,\ B(\alpha) = \begin{bmatrix} 0 \\ \dfrac{-a\ \cos\alpha}{\frac{4}{3}L - a\,m\,L\ \cos\alpha^2}\end{bmatrix} \tag{16}$$

To satisfy the closed-loop system specification according to Figure 1, the effect of feedback controller from (9) and (4) will give equation below.

$$\dot{x}_\delta = A(\alpha)\,x_\delta + B(\alpha)\,[K_1(\alpha)\,x_\delta + K_2(\alpha)\,v_\delta]$$

This equation give result,

$$\dot{x}_\delta = [A(\alpha) + B(\alpha)\,K_1(\alpha)\,]\,x_\delta + B(\alpha)\,K_2(\alpha)\,v_\delta \tag{17}$$

The solution of equation (17) will give the linear model according to equation below [5].

$$\frac{Y(s)}{V(s)} = \frac{w_n^{\ 2}}{s^2 + 2\,z\,w_n\,s + w_n^{\ 2}} \tag{18}$$

dimana *z* adalah faktor redaman dan w_n adalah frekuensi alamiah, serta *s* adalah variabel Laplace. Given any second order polinomials, the requirement is,

$$F(s) = s^2 + 2\,z\,w_n\,s + w_n^{\ 2} = 0 \tag{19}$$

The closed-loop system (17) has linear system defined below.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -w_n^{\ 2}x_1 - 2\,z\,w_n\,x_2 + b\,v \tag{20}$$

With combination of equation (17) and (20) we found the parameter of controller $K_1(x)$ and $K_2(x)$ below [6],[7] :

$$\frac{g\ \sin(x_1) - a\,m\,L\,x_2^2\ \sin\frac{2\,x_1}{2} - a\ \cos(x_1)\,[K_1(\alpha)\,x_\delta + K_2(\alpha)\,v_\delta]}{\frac{4}{3}L - a\,m\,L\ (\cos\ (x_1))^2} = -w_n^{\ 2}x_1 - 2\,z\,w_n\,x_2 + w_n^{\ 2}\,v \tag{21}$$

This equation (21) gives two parts equation for $K_1(x)$ and $K_2(x)$ as follows:

For $K_1(x_1, x_2)$ the equation is,

$$\frac{g\ \sin(x_1) - a\,m\,L\,x_2^2\ \sin\frac{2\,x_1}{2} - a\ \cos(x_1)\,K_1(\alpha)\,x_\delta}{\frac{4}{3}L - a\,m\,L\ (\cos\ (x_1))^2} = -w_n^{\ 2}x_1 - 2\,z\,w_n\,x_2$$

and give the result,

$$K_1(x_1, x_2) = \frac{\frac{4}{3}L - a\,m\,L\ (\cos\ (x_1))^2}{a\ \cos(x_1)}\left[\frac{g\ \sin(x_1) - a\,m\,L\,x_2^2\ \sin\frac{2\,x_1}{2}}{\frac{4}{3}L - a\,m\,L\ (\cos\ (x_1))^2} + w_n^{\ 2}x_1 + 2\,z\,w_n\,x_2\right] \tag{22}$$

Next, for $K_2(x_1, x_2)$ we make,

$$\frac{a\ \cos(x_1)\ K_2(\alpha)}{\frac{4}{3}L - a\ m\ L\ (\cos\ (x_1))^2} = {w_n}^2$$

and give the result,

$$K_2(x_1, x_2) = \frac{\frac{4}{3}L - a\ m\ L\ (\cos\ (x_1))^2}{a\ \cos(x_1)}\ {w_n}^2 \tag{23}$$

Both equations (17) and (18) show that performance of the closed-loop system only determined by damping factor of z and natural frequency of w_n.

4. Simulation Result

From the dynamics of inverted pendulum equation (14), it needs to generate the linear responses of nonlinear plant with nonlinear control system. For thoose equation from (14) to (23), the dynamics of inverted pendulum have changed into the equation (21). The response of the system before controlled, can be seen in Figure 2.

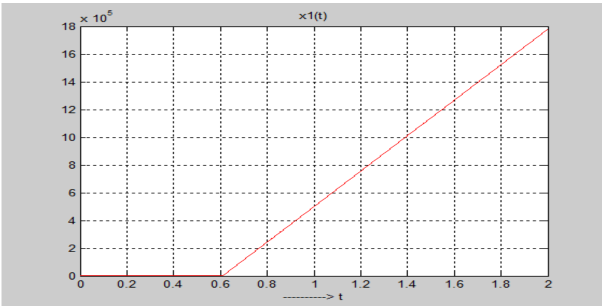

Figure 2. Initial Responses of Inverted Pendulum's Positions

From the dynamics result as shown in Figure 2, the initial position is given with value θ = π/4 rad or 22.5°. The position response of that inverted pendulum is not stable, so its need to stabilize the system.

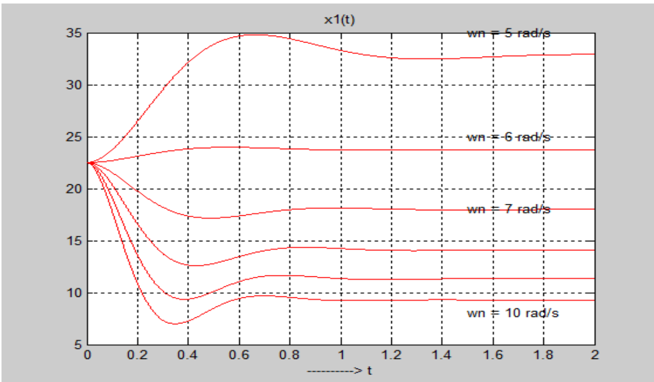

Figure 3. Positions Responses of Inverted Pendulum with Different $\boldsymbol{w_n}$

With apply the feedback control law equation (22) and (23), first we have to test which value of $\boldsymbol{w_n}$ can satisfy the response needed. The natural frequencies $\boldsymbol{w_n}$ is varied from 5 to 6, 7, 8, 9 and 10 rad/s, and all the responses is shown in Figure 3. The good response happen if the response decrease from the given set point toward steady state position. From the result given in Figure 3, the good response start when $\boldsymbol{w_n}$ = 7 rad/s, 8 rad/s, etc. Furthermore, in this case we choose that w_n = 12 rad/s, and the response is shown in Figure 4. For all response in Figure 3 and Figure 4 we choose damping factor z = 0.5, its mean that maximum overshoot is below 20%.

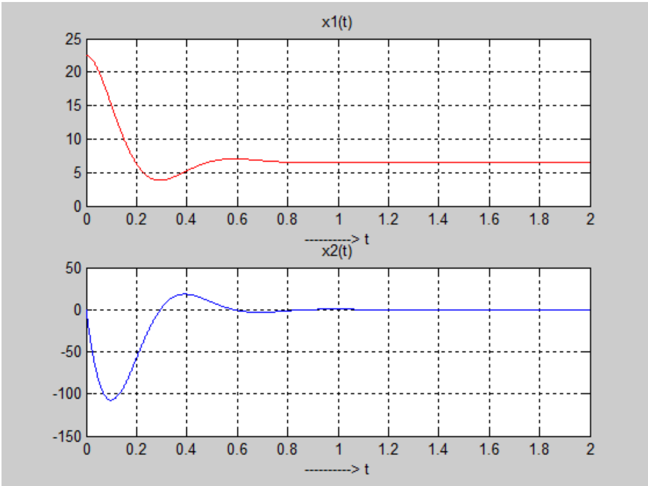

Figure 4. Positions Responses of Inverted Pendulum with v = 0.000 rad

The next problem appear from the result in Figure 4. The movement of the position starts from π/4 rad (22.5°) to 6.56° (not zero) , so the response of inverted pendulum still not adequate.

In the next step of the simulation, we need to satisfy the goal to find the zero position on state state, we try to give the set position from $v(t)$. We choose v(t) = -0.2 rad, and the response its shown in Figure 5. The movement of the position starts from π/4 rad (22.5°) to -4.90° (not zero) , so the response of inverted pendulum still not adequate.

If we do trial and error to find the match set point, we choose v(t) = -0.115 rad. The movement of the position starts from π/4 rad (22.5°) to -0.049° (near zero or ~0) , so the response of inverted pendulum is adequate, as shown in Figure 6.

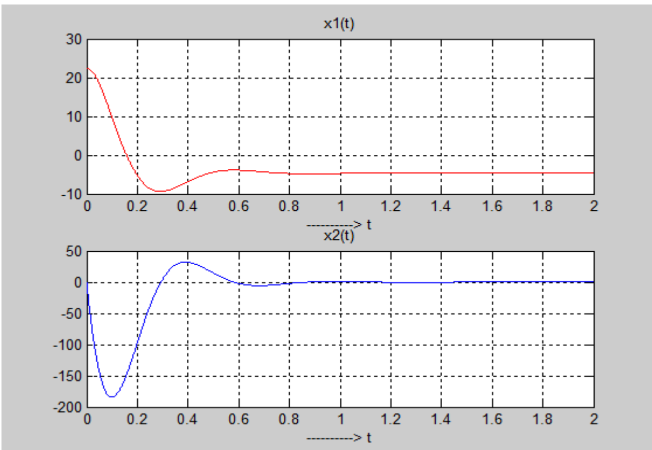

Figure 5. Positions Responses of Inverted Pendulum with v = -0.2 rad

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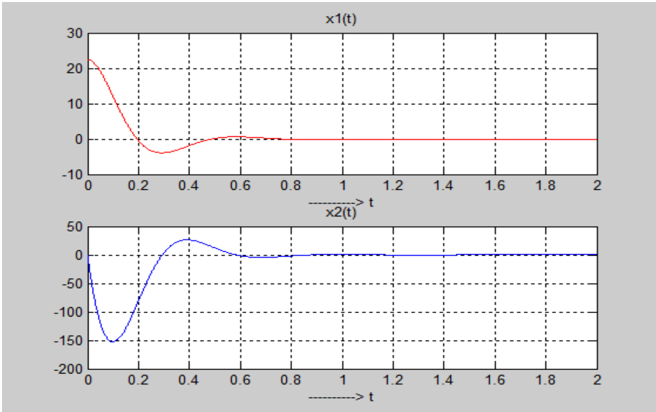

Figure 6. Positions Responses of Inverted Pendulum with v = -0.115 rad

From the last results in Figure 6, we saw the response of the pendulum will be continued toward zero. These all responses mean, the system has stable, the both responses toward equilibrium pont.

5. Conclusion

As we have described in the introduction, we need to observe how to choose feedback control law $\boldsymbol{K}_1(\alpha)$ and $\boldsymbol{K}_2(\alpha)$, and how to choose the eigenvalue needed by choosing $\boldsymbol{z}$ and $\boldsymbol{w_n}$.

Lastly, the controller has been designed to the linear system by using the state feedback nonlinear techniques using eigenstructure assignment for the nonlinear plant.

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