The influences of load mass changing on inverted pendulum stability based on simulation study

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Abstract. An inverted pendulum has nonlinear dynamic, so it is not easy to do in analysis to see its behavior. From many observations which have been made, there are two things that need to be added on the perfection of inverted pendulum. Firstly, when the pendulum has a large mass, and the second when the pendulum is given a load mass much larger than mass of the inverted pendulum. There are some question, first, how big the load mass can be given so that the movement of the inverted pendulum stay stable is. Second, how weight the changes and moves of load mass which can be given. For all the changes, it hopes the inverted pendulum is stay stable. Finally, the final result is still expected to be as stable, it must need conclude what kind of controller is capable of carrying such a mass burden, and how large the mass load limit can be given.

1. Introduction

A behaviour of nonlinear plant on an inverted pendulum can allow the system to become unstable, if its load mass changes suddenly occur on inverted pendulum. The load mass can change with constant values, or it will change depend on time, there is also the burden of mass change all at once moving. The load on cart mass can be made by giving additional load mass after the system is stable. For load mass changes and moves, the movement of that load mass can be done by giving disturbance to the mass of the cart, and it changes depend along with time changes.

If the mass of the cart has changed we can observe the changing of occurring on the position of pendulum, how is that possible to affect the stability of the inverted pendulum. The magnitude of the load mass changes can be made gradually to see the stages of the process, so at last we will find the limit of the mass imposition to the inverted pendulum.

There're so many researchers have designed the stability of an inverted pendulum by using different controllers. One of the techniques was standard pole-placement which proposed by Hari Vasudevan [1]. The controller design was done with the approach of linear system, it has realized to a nonlinear inverted pendulum. The other one has designed the stability of an inverted pendulum by using fuzzy controller which proposed by Ahmad M. El-Nagar [2]. Ronzhina M N [3], have designed the stability of an inverted pendulum by using full feedback from state space techniques, and Sorokin V S [4] has designed stability an inverted pendulum by using swing-up controller techniques. In all these cases, the behavior of the system stability has observed with well, but none of them observe the effect of changing load mass on system stability.

In the other case, the behavior of pendulum's response was observed through giving disturbance and changing the load mass, but the effect of movement of load mass still yet to be observed by them. All those cases have proposed by Prasad at all [5] and Arda and Kuscu [6].

Finally, it needs to observe the effect of mass moving on the stability of the pendulum and analyse the limit of mass on inverted pendulum with a large mass. This paper present a discussion to observe by the influence of changing the load mass on inverted pedulum stability based on simulation study, and expected will give a good contribution to the fields of control systems.

1.1 System Details

In addition, there are two purposes of the paper presented. First, the inverted pendulum's dynamic has been relegated separately by author, it has been also controlled and gave the result with a stable response. With reference to Figure 1, pendulum is assumed has a limit of movement from $-\theta_L$ to $+\theta_L$, and the action of the cart x is free in movement to stabilise the pendulum with apply the control force F. The action of control force generated by the optimal control techniques, and all the works described here based on simulation. In the beginning of the simulation, the cart mass, m_c is **8** kg, the pendulum mass m_p is **2** kg, the pendulum length, L is **1** m. The numerical work was performed by using a special program which developed by the author on using matlab software.
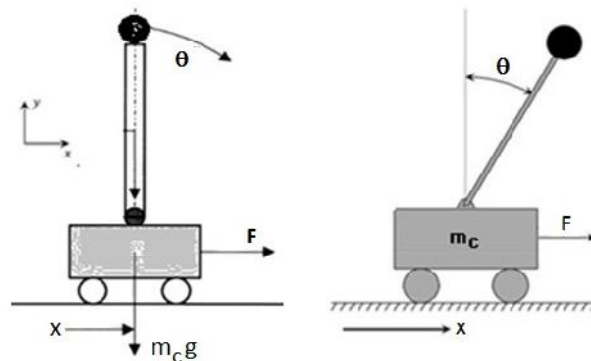

Figure 1. General Arrangement of An Inverted Pendulum

Second, based on the topic above, the load mass of the cart can be changed with the constant mass from 8 kg become 18, 28, 38 or 48 kg. The variable load mass can also be varied by using time, as shown the Figure 2 below. Mass of the cart changed and it changed according to a half sinus with magnitude started from 8 to 18, 20, 38 or 38 48 kg.
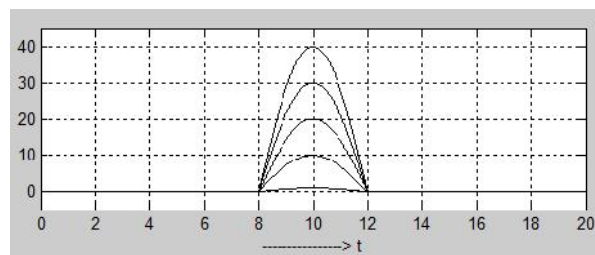

Figure 2. Varies of Pendulum's Mass

1.2 Modelling Inverted Pendulum

The dynamics of an inverted pendulum can be derived from its arrangement as shown in Figure 1. It consists of a cart, a pendulum and a driving force unit. The cart uses four wheels can move left or right on the floor freely. The pendulum is placed on the centre of the top surface of the cart. The cart able to rotate to the left or right as like as the cart movement. If all frictions are neglected on the system, the dynamic equation of the inverted pendulum have two equations as expressed below:

$$\ddot{\theta}(t) = \frac{m_p\,\dot{\theta}^2\,c\quad\theta\,s\quad\theta - (m_p + m_c)\frac{g}{L}\,s\quad\theta - \frac{c\quad\theta}{L}F}{(-1 + c\quad^2\theta)\,m_p - m_c} \tag{1}$$

$$\ddot{x}(t) = \frac{L\,\dot{\theta}^2\,s\quad\theta - (m_p + m_c)\,g\,t\quad\theta - F}{(-1 + c\quad^2\theta)\,m_p - m_c} \tag{2}$$

„ is the the position of pendulum, x is the position of the cart. The initial position of pendulum was „ = „ $_0$ in the unit (rad), the initial position of the cart with $x = x_0 = 0$ in the unit (m). The parameters m_c and m_p are respectively by the mass of the cart and pendulum in the unit (kg), and $g = 9{,}81$ (m/s^2) is the grafity acceleration.

1.3 Optimal Control Design

From the dynamics of inverted pendulum equation (1) and (2), it needs an approach to generate the linear form with asumtions: for small „, the value of sin „ = „, and the value of cos „1N1B?lFor those asumtions, the dynamics of inverted pendulum have changed into the equation (3) and (4) below:

$$\ddot{\theta}(t) = \frac{(m_p + m_c)\frac{g}{L}\theta - \frac{1}{L}F}{m_c} \tag{3}$$

$$\ddot{x}(t) = \frac{m_c\,g\,\theta - F}{m_c} \tag{4}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(m_p + m_c)\,g}{L\,m_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m_p\,g}{m_c} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L\,m_c} \\ 0 \\ \frac{1}{m_c} \end{bmatrix} F \tag{5}$$

Equation (5) has the general form with $\dot{x} = A\,x + B\,u$. By using the Riccati's equation [9] to design the optimal feedback, we can choose the weight Q and R as below:

$$Q = [20\ 0\ 0\ 0;\ \ 0\ 1\ 0\ 0;\ \ 0\ 0\ 20\ 0;\ \ 0\ 0\ 0\ 1]^T \text{ and } R = [1]$$

We found the optimal feedback gain by using the matlab's command,

$$K = [k_1 \quad k_2 \quad k_3 \quad k_4] = \text{lqr}\,(A, B, Q, R) \tag{6}$$

and the result was $K = [32.6220\ 6.8964 - 1.4142\ -2.4721]$.

On insertion of those gain feedback in the form of closed loop system with r(t) is the reference intput, the control force is,

$$F(t) = r(t) - K\,x(t) \tag{7}$$

Finally, the equation (8) and (9) are obtained for the closed loop system dynamics:

$$\ddot{\theta}(t) = \frac{m_p \dot{\theta}^2 c \quad \theta s \quad \theta - (m_p + m_c)\frac{g}{L} s \quad \theta - \frac{c \quad \theta}{L}\left[k_1 \theta + k_1 \dot{\theta} + k_3 x + k_4 \dot{x}\right]}{(-1 + c \quad ^2\theta) m_p - m_c} \tag{8}$$

$$\ddot{x}(t) = \frac{L \dot{\theta}^2 s \quad \theta - (m_p + m_c) g \, t \quad \theta - \left[k_1 \theta + k_1 \dot{\theta} + k_3 x + k_4 \dot{x}\right]}{(-1 + c \quad ^2\theta) m_p - m_c} \tag{9}$$

The examination of equations (8) and (9) have been done with $r = 0$ and the initial conditions $x(0) = \left[\frac{\pi}{8} \quad 0 \quad 0 \quad 0\right]^T$. It shows the state variables of inverted pendulum are coupled.

1.4 The Comparison Between Nonlinear and Linear Dynamics

If we applied the optimal feedback gain from the equation (7) for the linear system as given in equation (3) and (4), we can express both equations as below:

$$\ddot{\theta}(t) = \frac{(m_p + m_c)\frac{g}{L}\theta - \frac{1}{L}\left[k_1 \theta + k_1 \dot{\theta} + k_3 x + k_4 \dot{x}\right]}{m_c} \tag{10}$$

$$\ddot{x}(t) = \frac{m_c g \theta - \left[k_1 \theta + k_1 \dot{\theta} + k_3 x + k_4 \dot{x}\right]}{m_c} \tag{11}$$

The responses of the linier system $\theta(t)$ and $x(t)$ from equation (10) and (11) must resemble to the nonlinear system from equation (8) and (9), then the error between the responses nonlinear and linear system must be as small as possible.

2. Simulation
In the first step of the simulation, the initial position responses of pendulum and cart showed by the Figure 3. The simulations carried out by using Runge-Kutta integration.
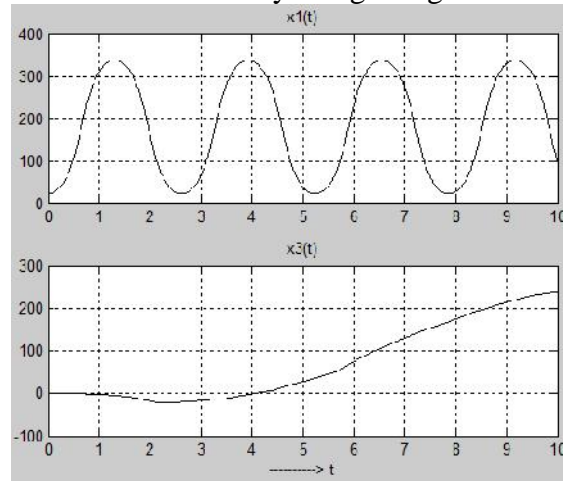

Figure 3. Initial Responses of Inverted Pendulum's Positions

$x_1(t)$ is $\theta(t)$ in degree (°), $x_3(t)$ is $x(t)$ in (m). From the results, it can be seen that the pendulum will continue oscillating from 22.5° to 337.5°. The movement of the cart starts from zero to infinity, so the pendulum and cart position become unstable.

In the second step of the simulation, it shows the optimal respond the linier and nonlinear system according to the Figure 4.
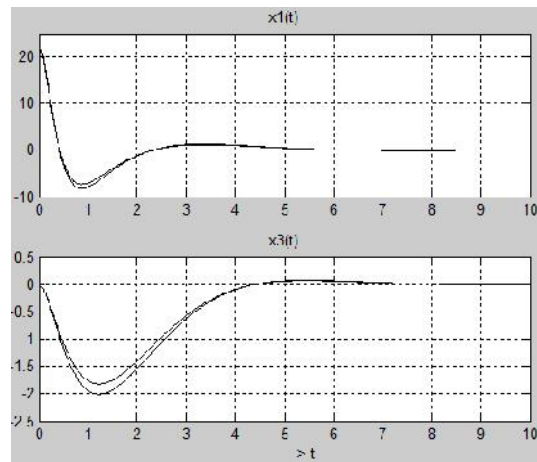
Figure 4. Comparison of Responses of Linear and Nonlinear System

From the results, we saw the response of the pendulum and the cart will be continued toward zero. These all responses mean, the system has stable, the both responses toward equilibrium pont. The position error between nonlinear and linear response are calculated by sum of square error, and those equal to 0.4865 % and 0.9815 % for each for the pendulum error and the cart error. Both the error are small enough, and all the responses are eligible.

3. The Influence of Load Mass Changes
The change of the load mass from 8 kg to 18, 28, 38 and 48 kg was appiled as given in Figure 5. It seems the response of the pendulum and cart for the different mass was stay stable.
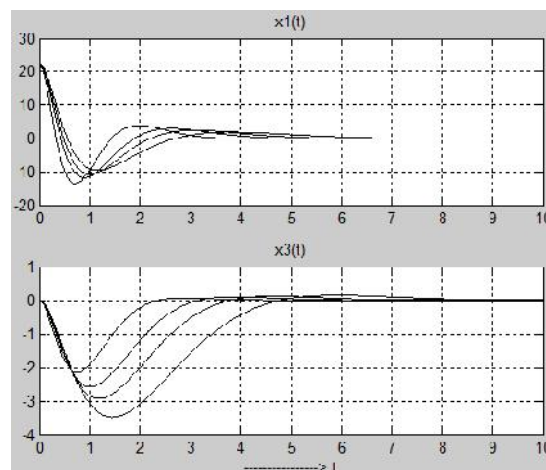

Figure 5. The effect of cart's mass change on position response

When the change of a different load mass appiled to the cart of inverted pendulum, the respon of position was found as shown in Figure 6. All responses show that all load mass changes can be overcome by controller, so the the inverted pendulum always stay stable, and

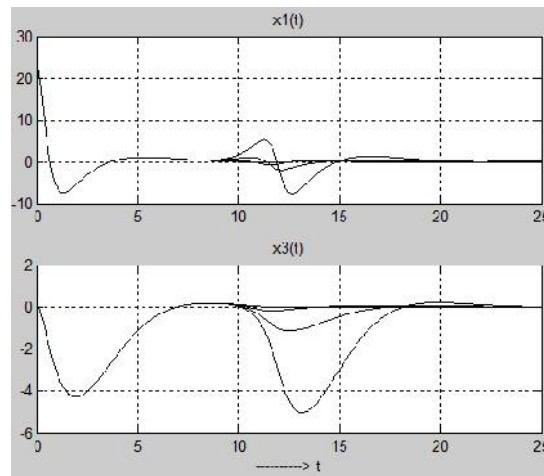the inverted pendulum has already designed to a strong controller.

Figure 6. The effect of mass load disturbance changes on pendulum's position

4. Conclusion

As we have described in the introduction, we need to observe the influences of load mass moving on the stability of pendulum and analyse the limit of mass on inverted pendulum with a large mass, according to the given result on Figure 5 and Figure 6 we can say that we have producted a strong controller with big power and automatic. The limit of load mass which given by the cart has reached 50 kg.

Lastly, the controller has been designed to the strong controller by using the optimal techniques. The weight for optimal conditions to be selected with trial and error several times, so we found the best feedback gain to eliminate all the disturbances.

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