Sustainability of transport structures – some aspects of the nonlinear reliability assessment

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Abstract. Efficient techniques for both nonlinear numerical analysis of concrete structures and advanced stochastic simulation methods have been combined in order to offer an advanced tool for assessment of realistic behaviour, failure and safety assessment of transport structures. The utilized approach is based on randomization of the non-linear finite element analysis of the structural models. Degradation aspects such as carbonation of concrete can be accounted in order predict durability of the investigated structure and its sustainability. Results can serve as a rational basis for the performance and sustainability assessment based on advanced nonlinear computer analysis of the structures of transport infrastructure such as bridges or tunnels. In the stochastic simulation the input material parameters obtained from material tests including their randomness and uncertainty are represented as random variables or fields. Appropriate identification of material parameters is crucial for the virtual failure modelling of structures and structural elements. Inverse analysis using artificial neural networks and virtual stochastic simulations approach is applied to determine the fracture mechanical parameters of the structural material and its numerical model. Structural response, reliability and sustainability have been investigated on different types of transport structures made from various materials using the above mentioned methodology and tools.

1. Introduction

Non-linear analysis is nowadays used by engineers for design of new and assessment of existing structures. This development is supported by the new ***fib*** Model Code 2010 [1] where rational safety assessment approach is presented, which reflects new developments in safety formats based on probabilistic methods. In the Chapter 4 on *Principles of structural design* the probabilistic safety format is introduced as a general and rational basis of safety evaluation. In addition to the partial factor safety concept (which remains as the main safety format for most practical cases) a *global resistance format* is recommended for nonlinear analysis. This concept can be applied in design practice and can offer advanced and rational solutions to modern structural technologies [2].

The fully probabilistic approach represents the most accurate method for the safety assessment of civil engineering structures. The accuracy of this approach is much higher if the non-linear structural analysis is used as a limit state function. The numerical simulation resembles a real testing of

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structures by considering a representative group of samples, which can be statistically analyzed for the assessment of safety.

The probabilistic analysis is performed with software SARA, which integrates program tools ATENA and FReET. The variability of basic properties is described by distribution functions and its parameters (mean, standard deviation, etc.). Probabilistic analysis of the resistance is performed by numerical method such as Latin hypercube sampling method. Resulting array of resistance values is approximated by a distribution function of global resistance, and describes the random properties of the resistance. Finally, for a required reliability index β, or probability of failure P_f, a value of the design resistance R_d shall be calculated.

The fully probabilistic analysis is the ultimate tool for the safety assessment. It is superior to simplified methods because it provides information on the variability of resistance. However, it is computationally demanding and requires good information about random properties of input variables. Therefore, it should be applied in special cases, where consequences of failure substantiate the increased effort.

2. Nonlinear simulation of engineering structures

The nonlinear finite element simulation is recently a well-established approach for analysis of concrete structures. Behavior of the structure under service as well as ultimate conditions can be virtually simulated using computer methods quite realistically. The nonlinear fracture analysis accounting tensile capacity of material enables to exploit reserves, which are usually neglected or diminished in codes or in linear analysis.

The ATENA software [3] was developed for realistic simulation of reinforced concrete structures. It is based on the finite element method with non-linear material models, and utilized for analysis of beams and girders, plates and shells, composite structures, strengthenings, structural details, fastenings, fibre reinforced concrete structures, timber, stonebrick and masonry structures [4], [5], [6], [7], [8], [9]. Considerable part of the analyzed civil engineering constructions belong to the structures of transport infrastructure, such as bridges, tunnels, railway sleepers or load-carrying rail plates etc.

The ATENA software consists of calculating core ensuring the non-linear numerical analysis, and a user-friendly graphical interface for an efficient communication between end-user and program core. The numerical core covers the finite element technology, non-linear material models and non-linear solution. Since concrete is a complex material with strongly nonlinear response, special constitutive models for the finite element analysis of structures made of concrete and similar materials are employed. The non-linear material models are based on the orthotropic damage theory and special concrete-related theory of plasticity. As one of the main features the non-linear fracture mechanics is employed for concrete cracking in tension. Based on the fracture energy approach the tensile cracks are modeled as smeared material damage which enables utilization of the continuum mechanics even for the damaged material. For the shape of stress-crack opening curve an exponential law derived by Hordijk [10] is used in case of normal concrete. Objectivity of the solution (independency on the finite element mesh) is ensured using crack band method [11], [12]. The material law for concrete exhibits softening after reaching the tensile strength. The behavior of concrete in compression is defined by special theory of plasticity (three-parameter model [13], [14]), with non-associated plastic flow rule and softening. This material model for concrete can successfully reproduce also other important effect, such as volume change under plastic compression or compressive confinement. The native graphical user-interface in ATENA Engineering supports all the specifics of reinforced concrete, e.g. input of discrete reinforcing bars, or evaluation of crack patterns in the damaged structural model.

ATENA Engineering is perfectly suitable for static analysis of concrete structures, obtaining their load-displacement response and resistance, crack pattern including crack widths and identification of the failure mode. It can be used for structure optimization, assessment of retrofitting or reinforcement detailing.

In order to extend ATENA potential and features, the recent development combines the calculating core with AtenaStudio runtime and post-processing environment, and a powerful third-party program

GiD for pre-processing. The resulting product ATENA Science covers broad range of structural and material behavior in time. It enables to model geometrically complicated shapes and it is suitable for analysis of complex structural problems, such as dynamic implicit analysis, dynamic eigenvalue analysis, static stress analysis, creep analysis, transport of heat and fluids or fire analysis. The coupling of the above effects can be often achieved through simultaneous solution of various constitutive models. Thus, dynamic analysis can capture non-linear material response due to cracking, etc. In the eigenvalue analysis vibration frequencies reflect the stiffness changes due to material damage. In creep analysis the cracking of concrete and redistribution of stress due to plastic deformations is reflected. In fire analysis material response is strongly dependent on changing temperature fields.

However, the advanced non-linear analysis is performed in the ATENA software using purely deterministic way, i.e. it is assumed, that all the input parameters (material properties, geometry of the structure, etc.) are fixed, well known values. This is in contradiction to reality, where these values are usually uncertain or random, evtl. variable in space and time. Therefore, stochastic analysis would be much more suitable for obtaining really realistic results.

3. Probabilistic assessment of structural reliability

The probabilistic statistical, sensitivity and reliability analysis of engineering problems is based on the efficient reliability techniques with emphasize on small/sample simulation techniques, in order to be suitable for time demanding nonlinear finite element modeling. The main purpose of this kind of analysis is to account for randomness and epistemic uncertainties of material input parameters and other structural properties in the nonlinear structural analysis.

The probabilistic software FReET [15] allows simulations of uncertainties of the analyzed problem basically at random variables level (typically in civil/mechanical engineering – material properties, loading, geometrical imperfections). The attention is given to those techniques that are developed for analyses of computationally intensive problems; nonlinear FEM analysis is a typical example. Stratified simulation technique Latin hypercube sampling (LHS) is used in order to keep the number of required simulations at an acceptable level. This technique can be used for both random variables and random fields levels. Statistical correlation is imposed by the stochastic optimization technique called simulated annealing. Sensitivity analysis of the input parameters to resulting values is based on nonparametric rank-order correlation coefficients.

The procedure can be briefly outlined: Random input parameters are generated according to their PDF using LHS sampling. Samples are reordered by the Simulated Annealing approach in order to match the required correlation matrix as closely as possible. Generated realizations of random parameters are used as inputs for the analyzed function (computational model). The solution is performed many times and the results (structural response) are saved. At the end of the whole simulation process the resulting set of structural responses is statistically evaluated. The results are: estimates of the mean value, variance, coefficient of skewness and kurtosis, and the empirical cumulative probability density function estimated by an empirical histogram of structural response.

This basic statistical assessment is visualized through the “Histograms” window. It is followed by reliability analysis based on several approximation techniques: (i) the basic estimate of reliability by the Cornell safety index, (ii) the curve fitting approach applied to the computed empirical histogram of response variables and (iii) the simple estimate of probability of failure based on the ratio of failed trials to the total number of simulations.

State-of-the-art probabilistic algorithms are implemented to compute the probabilistic response and reliability. The main features of the FReET software are:

1) Stochastic modeling (inputs)

- Direct connectivity to the nonlinear analysis input data
- Friendly Graphical User Environment (GUE)

- 30 probability distribution functions (PDF), mostly 2-parametric, some 3-parametric, two 4-parametric (Beta PDF and normal PDF with Weibullian left tail)
- Unified description of random variables optionally by statistical moments or parameters or a combination of moments and parameters
- PDF calculator
- Statistical correlation (also weighting option)
- Categories and comparative values for PDFs
- Basic random variables visualization, including statistical correlation in both Cartesian and parallel coordinates

2) Probabilistic techniques (solution)

- Crude Monte Carlo simulation
- Latin Hypercube Sampling (3 alternatives)
- First Order Reliability Method (FORM)
- Curve fitting
- Simulated Annealing
- Bayesian updating

3) Response/limit state function (evaluation)

- Numerical form directly connected to the results of nonlinear FE analysis
- Multiple response functions are assessed in same simulation run

4. Identification of material parameters by inverse analysis

The key parameters for the nonlinear fracture mechanics modeling are fracture energy [16], tensile and compressive strength, and modulus of elasticity of the structural material, which are generally not well known, but crucial for a successful computer simulation of the structural response. For identification of fracture mechanical parameters of concrete from experimental results of three-point bending tests on notched-beam specimens an inverse method based on artificial neural networks has been introduced by Lehký et al. [17], [18].

Determination of parameters values is performed by inverse analysis using artificial neural network based method. The background of the inverse analysis is finite element method model which is used for numerical simulation of three-point bending fracture test; the model was created in ATENA software. Subject of identification are the basic three fracture mechanical parameters of the concrete model: modulus of elasticity, tensile strength, fracture energy. These material model parameters are considered as random variables described by probability distribution. The rectangular distribution was chosen as the lower and upper limits represent the bounded range of physical existence. The variables are then simulated randomly based on the small-sample simulation Latin Hypercube Sampling. Multiple calculation of deterministic computational model using random realizations of material model parameters is performed, and statistical set of the virtual structural response is obtained. Random realizations and the corresponding responses from the computational model serve as the basis for the training of the neural network [19]. After the training the neural network is ready to solve the main task: To provide the material parameters for which the numerical simulation will result in the best agreement with the provided experimental data. This task is performed by means of the simulation of the neural network using measured response as an input. It results in a set of identified optimal material input parameters. The last step of the procedure is results verification – calculation of computational model using identified parameters and comparison with the measured data. To obtain statistical characteristics of material parameters inverse analysis is performed for each specimen (L–D diagram) individually. The set of identified values is obtained as the result of individual identification and can be assessed statistically as it is usually done for experiments.

5. Sustainability of reinforced concrete engineering structures

Sustainability of civil engineering structures regards reliability and durability issues, both of which are among the decisive structural performance characteristics [20]. Assessment, together with inspection, maintenance and/or replacement plans, should guarantee the correct performance of a structure (e.g. a bridge) during its intended or residual service life. This time t_S is defined as the assumed period of time, obtained after an assessment process, for which the structure will continue to serve its purposes. When t_S elapses, a new assessment is requested. The intended service life, agreed by the client and justified on the basis of socio-economic criteria, is the design life t_D. This is decided on a case by case basis. An assessment process must always define the reliability (or safety) level of the particular limit state, the residual service life inherent to the final results and the conclusions, decisions and optimization to be carried out for the bridge under consideration

The program FReET-D (FReET Deterioration/Degradation Module [21]) was developed in order to support the life-cycle, sustainability and performance-based assessment and design of reinforced concrete structures. It is an extension to the probabilistic software FReET, and contains a deterioration module for statistical, sensitivity and reliability assessment of degradation effects of reinforced and pre-stressed concrete structures. Number of recognized models for concrete carbonation and deterioration as well as reinforcement corrosion are included. This deterioration module extends abilities of the basic program FReET to degradation effects assessment. It provides modeling of degradation phenomena in concrete structures, assessment of service life and assessment of reliability measure.

The (stochastic or deterministic) results can be directly used as inputs in ATENA or SARA programs in order to account for material deterioration and degradation and to perform life-cycle analysis or performance-based design or assessment of concrete engineering structures.

6. Nonlinear reliability assessment of transport structures

Probabilistic analysis is a general tool for safety assessment of civil engineering structures, in particular of concrete structures. In the fully probabilistic non-linear approach the structural resistance R_d is calculated by means of the probabilistic non-linear analysis. The classical statistical and reliability approach is to model material parameters as random variables with prescribed distribution function. The stochastic response requires repeated analyses of the structure with these random input parameters, which reflects randomness and uncertainties in the input values [22].

In this approach the resistance function *r(r)* is represented by nonlinear structural analysis and loading function *s(s)* is represented by action model. Safety can be evaluated by the reliability index β, or alternatively by failure probability P_f taking into account all uncertainties due to random variation of material properties, dimensions, loading, and other.

Probabilistic analysis based on the non-linear numerical simulation includes following steps:

1) Numerical model based on non-linear finite element analysis. This model describes the resistance function *r(r)* and can perform deterministic analysis of resistance for a given set of input variables.
2) Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function *s(s)* (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.
3) Probabilistic analysis of resistance and action. This can be performed by stratified method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide

random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.

4) Evaluation of safety using reliability index β or probability of failure.

Probabilistic analysis can be also used for determination of design value of resistance function *r(r)* expressed as R_d. Such analysis involves the steps 1) to 3) above and R_d is determined for required reliability β or failure probability P_f.

In order to make the application of the probabilistic non-linear analysis user-friendly, special software tool has been developed by the authors and their co-workers The resulting software SARA (Structural Analysis and Reliability Assessment) integrates the above mentioned software tools ATENA and FReET. It is equipped with a user friendly shell SARA Studio, which leads the user interactively through the modelling and randomization process of the solved problem as described above. All the above mentioned features (or selected) of the involved programs including deterioration/degradation phenomena are utilized in the reliability analysis and sustainability and performance-based assessment of concrete structures [23], [24], [25], [26], [27], [28].

7. Examples of sustainability and nonlinear reliability assessment of transport structures

7.1. Bridge Kristienberg, Stockholm

The newly constructed intersection of highway No. E4 is the main traffic node for Kungsholmen Island in Stockholm, Sweden. A reinforced concrete bridge with two-span frame structure is a part of the intersection. Total bridge length is 26 m, bridge deck has width of 7 m. There are two lateral abutments and one intermediate support. The abutments have a significant inclination with respect to road axis and they have a different shape and size. Bridge has been subjected to several loading tests under the loads as two or four heavy trucks (weight of one truck is between 24 and 29 tons). Reliability assessment of this bridge has been performed and presented in [29].

Figure 1. Bridge Kristienberg.

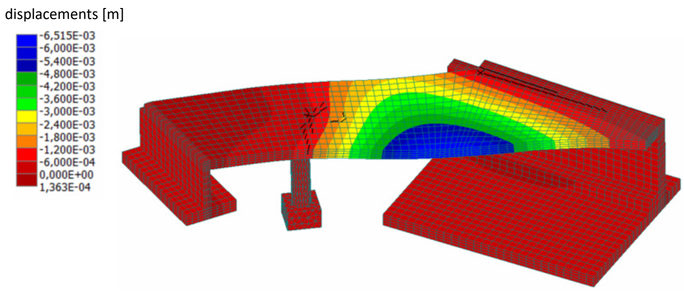

Figure 2. Simulated vertical displacement (colour scale) and cracks (short black lines) in the structural model loaded by four heavy trucks.

The numerical model of the bridge was created in ATENA 3D. Parameters of concrete are based on laboratory experiments, e.g. compressive strength 41.2 MPa. Steel reinforcement (class B500B) is modelled according to the project documentation, shear reinforcement is modelled as smeared reinforcement. For the model consisting of 11351 finite elements both deterministic and probabilistic analyses were performed. For the probabilistic approach, basic material parameters of concrete and reinforcement were randomized, total number of 20 samples-simulations has been calculated. Displacements were compared between experimental and numerical results.

The simulations were in very good accordance with the experimental results. Moreover, it was possible to predict and asses crack width in critical parts of the bridge, i.e. above the central pillar in this case (see black lines in Figure 2).

7.2. Rohrbach arch bridge, Mattersburg

Simulation of an old masonry railway bridge was presented in [30]. The bridge consists of five arch elements and was built between 1845 and 1847 in Rohrbach bei Mattersburg in Austria. To investigate the sustainability and load bearing capacity of the bridge, model of one typical arch was build in laboratory and tested [31]. The experimental arch has a clear span of 2.96 m, a rise of 1.00 m and for technical reasons a thickness of 0.60 m. In order to replicate the abutment portion of the modelled arch and the associated stiffness springing, these were attached to the steel abutments.

The experiments were reproduced by ATENA 2D model. The individual bricks and the mortar were modelled by a simplified micro modelling. The mortar joints were modelled mostly with so called “Interface Material”. The monitoring points in the ATENA 2D model were placed at the same positions as the measuring points of the laboratory arch.

To adjust the material parameters as close as possible to the laboratory arch, probabilistic simulation was performed and parameters of bricks and mortar were dispersed. The initial material parameters were taken from the PhD thesis [31]. By comparing load-displacement diagrams optimal material parameters were identified.

According to the laboratory test, the arch in the model was loaded to four different damage levels. Model shown in Figure 4 is at damage level 1 where bricks on the left side of the arch are already destroyed and model does not account them in the analysis.

Figure 3. Rohrbach arch masonry bridge.

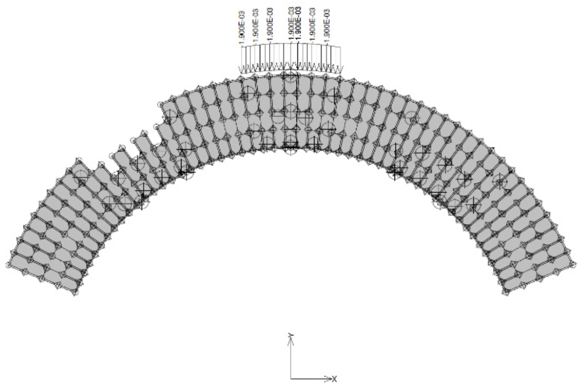

Figure 4. ATENA 2D model at damage level 1.

Comparison of numerical and experimental results on the small scale model documented that the computer model can reproduce real structural behaviour. The material parameters for all structural components are of crucial importance. The loading history and damage level of the structure should be accounted as well. If such preconditions are fulfilled, behaviour and response of the real (full scale) bridge under various actions and its damage under limit conditions can be successfully reproduced by the numerical modelling. Corresponding tests are hardly possible at the real bridge or full scale laboratory specimen.

8. Conclusions

- Assessment of performance and sustainability of civil engineering structures based on randomization of advanced nonlinear computer analysis is presented.
- Structural response, reliability and sustainability have been investigated on different types of transport structures made from various materials using the above mentioned methodology and tools.
- Appropriate identification of material and structural parameters of the numerical model is crucial for the virtual response and failure modelling of structures and structural elements.

- The probabilistic analysis could predict and assess material damage (crack width) in critical parts of the structure.
- Two examples of the numerical simulation for different types of bridge structures are shown in order to document feasibility of the presented approach.
- Very good agreement of simulation with the measured data and experimental results has been obtained.
- The examples demonstrate that the presented techniques can be applied for complex transport structures.
- The results from the nonlinear probabilistic analysis can serve as a rational basis for design, assessment and maintenance of concrete structures.

Acknowledgements
Research reported in this paper was partially performed within the Czech Science Foundation (GA CR) project 16-04132S "Epistemic uncertainty of crack models in reinforced concrete structures" and supported by Competence Centres program of Technology Agency of the Czech Republic (TA CR), project Centre for Effective and Sustainable Transport Infrastructure (No. TE01020168).

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