

Study on BTT Coordinated Turn Autopilot Design for Reentry Gliding Vehicle

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Abstract. Two-loop acceleration yaw-autopilot with a PI compensator and the engineering approximation of sideslip angle rate feedback for autopilot inner-loop are put forward accounted for BTT coordinated turn control of reentry gliding vehicle. The effect of turning acceleration to body-frame yaw channel is analyzed deeply, two-loop acceleration yaw-autopilot with a PI compensator is advanced to ensure system rapidity and stability based on minimum output by turning acceleration. The convergence essence of sideslip angle rate feedback is presented, furthermore, the engineering implementation composed by yaw angle rate and feedforward compensator is presented. Finally, the simulation results show that the autopilot design can make sideslip angle to be zero fast, improve the performance of coordinated turn. The design has certain robustness and application value.

1. Introduction

The importance of long-distance, fast and accurate attack is increased rapidly to fulfill requirement of modern war. So reentry gliding vehicle based on glide trajectory is widely concerned by countries[1]. Bank to turn (BTT) control technology must be applied because of high lift-to-drag configuration and maneuverable requirement of flight vehicle, and the strong three-channel coupled effect is produced by high rolling rate[2]. Pitch channel and roll channel are designed to respond command rapidly, and yaw channel must achieve coordinated turn control to eliminate sideslip angle and three-channel coupling, which realizes stable control of missile. So coordinated turn autopilot design of BTT missile is the key technology for control system.

Many experts have pay attention to the coordinated turn technology, and produce many important technical achievements. The autopilot design technology based on classical control theory has three class typical design methods. The first one is to eliminate sideslip by sideslip feedback. The second one is to eliminate sideslip by lateral acceleration feedback, because lateral acceleration of body-frame is produced by sideslip angle. The third one is keeping yaw angular rate unchanged to realize coordinated turn in constant roll angle and flight rate. The lateral acceleration feedback is most common and effective method because of simple system configuration and easy engineering application[3-5].

Based on the design philosophy of yaw acceleration feedback, in this paper the effect of autopilot configuration to coordinated turn performance is analyzed deeply, increasing open-loop gain is the key point to accomplish coordinated turn perfect, so two-loop acceleration yaw-autopilot with a PI compensator is put forward. With the comparison between different feedback information of inner-loop, it comes the conclusion that sideslip angle rate feedback is better than yaw angle rate. To meet engineering requirement, the engineering approximation of sideslip angle rate feedback is also



presented.

2. Yaw channel aerodynamic analysis

During the tuning of flight vehicle, lateral turning acceleration of inertial coordinate system called a_{zd} is produced by the component of lift force in longitudinal-symmetrical plane. Speed vector is deflected by a_{zd} , and sideslip angle is produced. Based on the linear hypothesis, the dynamic equation of yaw channel is as follows[6].

$$\begin{cases} \ddot{\psi} = -a_\beta \beta - a_\omega \dot{\psi} - a_\delta \delta \\ \dot{\psi}_V = b_\beta \beta + b_\delta \delta - \frac{a_{zd}}{V} \\ \beta = \psi - \psi_V \\ a_{zV} = -V \dot{\psi}_V \end{cases} \quad (1)$$

Where $\psi, \dot{\psi}$ and $\ddot{\psi}$ are yaw angle, rate and acceleration, ψ_V and $\dot{\psi}_V$ are heading angle and rate, δ is yaw rudder deflection angle, V is flight speed, β is sideslip angle, $a_\beta, a_\omega, a_\delta, b_\beta, b_\delta$ are the aerodynamic derivatives of yaw channel defined in reference[2]

The overall system dynamics is given by,

$$\begin{bmatrix} \ddot{\psi} \\ \dot{\psi}_V \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -a_\beta & -a_\omega & a_\beta \\ b_\beta & 0 & -b_\beta \end{bmatrix} \begin{bmatrix} \psi \\ \dot{\psi} \\ \psi_V \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -a_\delta & 0 \\ b_\delta & -\frac{1}{V} \end{bmatrix} \begin{bmatrix} \delta \\ a_{zd} \end{bmatrix} \quad (2)$$

Considering the requirement of information measure and coordinated turn research, and defining system output are a_{zb} , $\dot{\psi}$ and β , a_{zb} is yaw acceleration, the system dynamics is

$$\begin{bmatrix} a_{zb} \\ \dot{\psi} \\ \beta \end{bmatrix} = \begin{bmatrix} -b_\beta V & 0 & b_\beta V \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \psi \\ \dot{\psi} \\ \psi_V \end{bmatrix} + \begin{bmatrix} -b_\delta V & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ a_{zd} \end{bmatrix} \quad (3)$$

Apparently a_{zb} is not correlative with a_{zd} , but ψ_V is varied because of a_{zd} , a_{zd} can influence a_{zb} also. The transfer function a_{zb}/a_{zd} and $\dot{\psi}/a_{zd}$ are as follows.

$$\frac{a_{zb}}{a_{zd}} = \frac{-b_\beta(s+a_\omega)}{s^2 + (b_\beta + a_\omega)s + (a_\omega b_\beta + a_\beta)} = \frac{-b_\beta(s+a_\omega)/\omega_m^2}{s^2/\omega_m^2 + 2\mu_m s/\omega_m + 1} \quad (4)$$

$$\frac{\dot{\psi}}{a_{zd}} = \frac{-a_\beta/V}{s^2 + (b_\beta + a_\omega)s + (a_\omega b_\beta + a_\beta)} = \frac{-a_\beta/(V\omega_m^2)}{s^2/\omega_m^2 + 2\mu_m s/\omega_m + 1} \quad (5)$$

Where $\omega_m = \sqrt{a_\beta + a_\omega b_\beta}$ is undamped natural frequency, $\mu_m = (b_\beta + a_\omega)/(2\sqrt{a_\omega b_\beta + a_\beta})$ is damping radio. Steady-state a_{zb} and $\dot{\psi}$ are as follows.

$$a_{zb} = -b_\beta a_\omega a_{zd} / (a_\omega b_\beta + a_\beta) = -b_\beta a_\omega a_{zd} / \omega_m^2 \quad (6)$$

$$\dot{\psi} = (-a_\beta/V) a_{zd} / (a_\omega b_\beta + a_\beta) = (-a_\beta/V) a_{zd} / \omega_m^2 \quad (7)$$

If static stability is lower, ω_m is smaller, a_{zb} and $\dot{\psi}$ during turning are bigger, β is bigger, which is not benefit to coordinated turn.

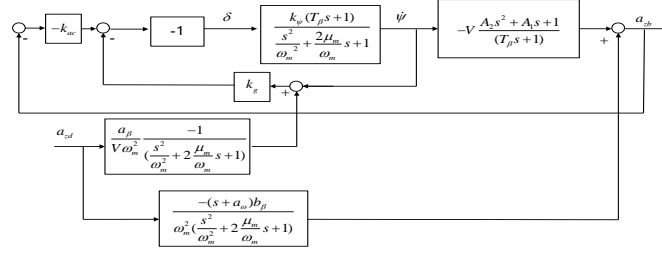


Figure1. A typical block diagram for coordinated turn autopilot design

The command is zero for coordinated yaw autopilot design all the time, a_{zb} and $\dot{\psi}$ produced by a_{zd} are disturbance to yaw autopilot. So the mission of autopilot design can be described that minimizing the output of a_{zb} when there is disturbance input of a_{zd} . The block diagram is shown in Fig.1, where k_{ac} and k_g are design parameters of autopilot, $A_1, A_2, T_\beta, k_\psi$ are the aerodynamic derivatives of yaw channel defined in reference [2].

3. Coordinated turn autopilot structure research

Switching the yaw-autopilot structure based on Fig.1, a block diagram for two-loop acceleration autopilot with input a_{zd} is shown in Fig.2.

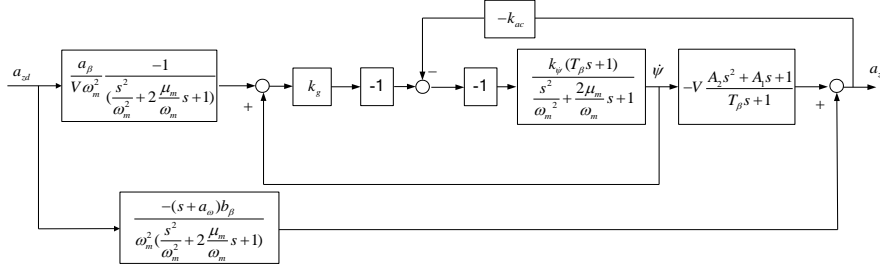


Figure2. The block diagram for two-loop acceleration autopilot with input a_{zd}

The transfer function a_{zb}/a_{zd} is as follows

$$a_{zb}/a_{zd} = k_g k_\psi \left(\frac{s}{a_\beta^2 k_g k_\psi} + 1 \right) / \left(\frac{s^2}{a_\beta} + \frac{b_\beta}{a_\beta} s + 1 \right) (1 - k_{ac} k_\psi V - k_g k_\psi) \quad (8)$$

Defining $K_{a_{zb}/a_{zd}}$ is steady state yaw acceleration of per lateral turning acceleration, so smaller $K_{a_{zb}/a_{zd}}$ represents better coordinated turn performance. $K_{a_{zb}/a_{zd}}$ is given by

$$K_{a_{zb}/a_{zd}} = k_g k_\psi / (1 - k_{ac} k_\psi V - k_g k_\psi) \quad (9)$$

The open-loop gain of autopilot is given by

$$K_G = k_{ac} k_\psi V / (k_g k_\psi - 1) \quad (10)$$

Substitution of expression for K_G into equation (10) yields

$$K_{a_{zb}/a_{zd}} = k_g k_\psi / (K_G + 1)(1 - k_g k_\psi) \quad (11)$$

Where k_ψ is nature ability of flight vehicle, and adjusting k_g to reduce $K_{a_{zb}/a_{zd}}$ is also limited by k_ψ evidently. So the most direct and effective way is increasing K_G , which means that increasing open-loop gain is most benefit to improve coordinated turn ability.

With open-loop gain of 0.49 and 0.98 respectively, the time domain and frequency domain results are given in Fig.3 and Fig.4 for 1g input a_{zd}

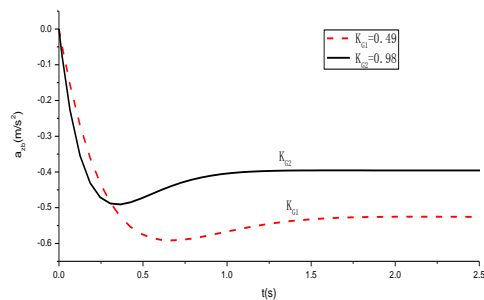


Figure 3. Time domain curves with K_G

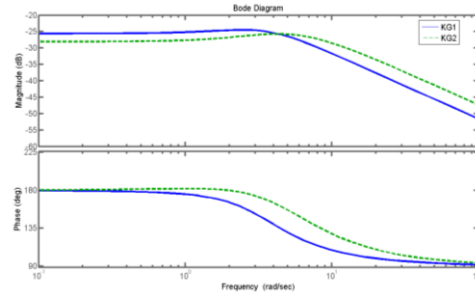


Figure 4. Frequency domain Bode curves

with K_G

Fig.3 displays the steady-state disturbance output can be reduced by increasing K_G . Fig.4 shows that system low-frequency gain degenerated obviously, which promotes the coordinated turn ability, but can not make disturbance output to be zero. The disturbance output is eliminated drastically required too large open-loop gain, which will make the system unstable. So the essential method is to add a proper PI compensator in forward channel, which can make low-frequency gain degenerated and maintain system stable[7-8].

With a PI compensator in forward channel, the block diagram is shown in Fig.5.

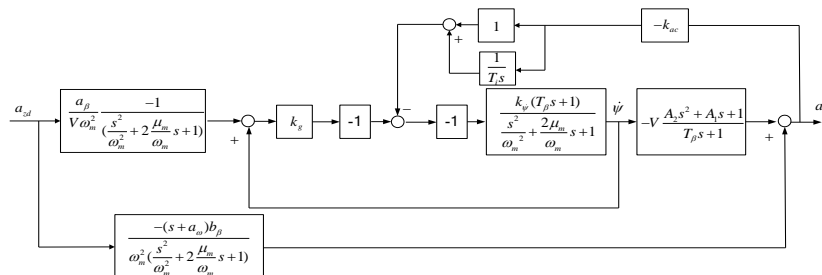


Figure5. The block diagram for two-loop acceleration autopilot with a PI compensator

Where T_i is compensator time constant. With open-loop gain of 0.49 for two-loop acceleration autopilot with a PI compensator, the time domain and frequency domain results are given in Fig.6 and Fig.7 for 1g input a_{zd} .

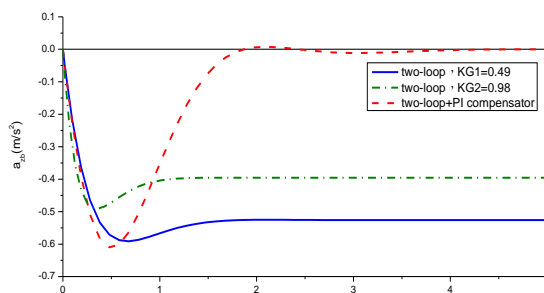


Figure6. Responses with or without PI compensator

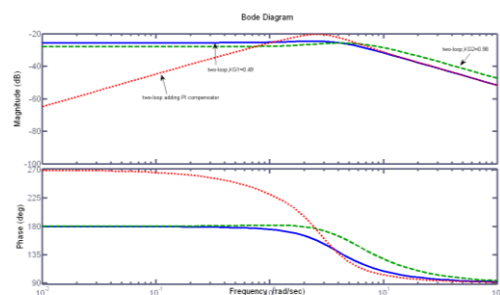


Figure7. Bode curves with different open-loop

Fig.6 and Fig.7 display the proper PI compensator can make low-frequency gain degenerated, which will make yaw-acceleration caused by lateral turning acceleration to be zero quickly and maintain system stable. So the two-loop acceleration autopilot with a PI compensator is much better to accomplish coordinated turn.

4. Inner-loop feedback characteristic

Generally, inner-loop of yaw-autopilot is designed to improve system damp and yaw angle rate is selected as feedback information. However, there is less research on the effect of inner-loop feedback to coordinated turn with input lateral turning acceleration. In steady-state of Eq.(7) $\dot{\psi} = (-a_{\beta}/V)a_{zd}/(a_{\omega}b_{\beta} + a_{\beta}) \approx -a_{zd}/V = \dot{\psi}_v$, the disturbance input of yaw angle rate exist all the time during sustained turning. The sideslip angle can be zero because of $\dot{\psi} \approx \dot{\psi}_v$ approximately. Sideslip angle is present from Eq. (1) as follow.

$$\dot{\beta} = [-b_{\beta} \quad 1 \quad b_{\beta}] [\psi \quad \dot{\psi} \quad \psi_v]^T + [-b_{\delta} \quad \frac{1}{V}] [\delta \quad a_{zd}]^T \quad (12)$$

Transfer functions $\dot{\beta}/a_{zd}$ and $\dot{\beta}/\delta$ can be written as follows.

$$\dot{\beta}/a_{zd} = s(s + a_{\omega})/V\omega_m^2(s^2/\omega_m^2 + 2\mu_m s/\omega_m + 1) \quad (13)$$

$$\dot{\beta}/\delta = (k_{\beta}B_1s^2 + k_{\beta}s)/(s^2/\omega_m^2 + 2\mu_m s/\omega_m + 1) \quad (14)$$

Where k_{β}, B_1 are the aerodynamic derivatives of yaw channel defined in reference[2]. The conclusion that sideslip angle comes to zero with input turning acceleration in steady-state is clarified again. Comparing with inner-loop yaw angle rate feedback, we can draw the conclusion from Fig.8 that taking sideslip angle rate as feedback information is more propitious to eliminate yaw acceleration.

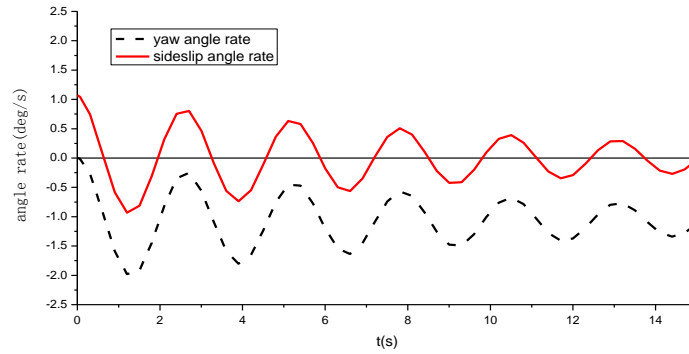


Figure8. Compare of inner-loop yaw and sideslip angle rate

5. Engineering approximation for sideslip angle rate inner-loop

Inner-loop sideslip angle rate feedback is more effective to coordinated turn, but the problem is how to measure sideslip angle rate accurately, the common method sideslip angle differential coefficient can bring serious noise problem. Taking channel-coupling into consideration, in the yaw channel there is [9-10]

$$\dot{V}_{zb} + V_{yb}\omega_{xb} - V_{xb}\omega_{yb} = F_{cz}/m \quad (15)$$

Where V_{xb}, V_{yb}, V_{zb} are the velocity in body coordinate system, ω_{xb}, ω_{yb} are roll and yaw angle rate, F_{cz} is total force of yaw channel, m is mass of flight vehicle.

$$\dot{V}_{zb}/V_{xb} = -V_{yb}\omega_{xb}/V_{xb} + \omega_{yb} + F_{cz}/mV_{xb} \quad (16)$$

There are the equations that $\beta = V_{zb}/V_{xb}$ and $\tan \alpha = -V_{yb}/V_{xb}$ approximately, where $F_{cz} = F_{czb} + mg \cos \vartheta \sin \gamma$ contains aerodynamic force and gravity. Because the requirement of coordinated turn, the aerodynamic force $F_{czb} = 0$. g represents gravitational acceleration, ϑ, γ are pitch and yaw attitude angle, and make a hypothesis on $V_{xb} = V$, so from Eq. (16) we have the inner-loop feedback as

$$\dot{\beta} = \omega_{yb} + \omega_{xb} \tan \alpha + g \cos \vartheta \sin \gamma / V \quad (17)$$

Sideslip angle rate $\dot{\beta}$ is composed of three parts which can be obtained from inertial navigation system all, ω_{yb} is yaw angle rate and can be measured directly, $\omega_{xb} \tan \alpha$ is kinematic compensator and can be obtained by measuring roll angle rate and calculating attack angle, $g \cos \vartheta \sin \gamma / V$ is gravitational compensator and can be obtained by measuring attitude angle and velocity. So with kinematic and gravitational compensator, sideslip angle rate feedback is produced for the engineering implementation, which makes the performance of yaw-autopilot coordinated turn to be best.

With the consider of channel coupling, the block diagram of acceleration yaw-autopilot is described in Fig.9, the autopilot adopts two-loop system structure with a PI compensator, inner-loop sideslip angle rate feedback is produced from yaw angle rate adding kinematic and gravitational compensator.

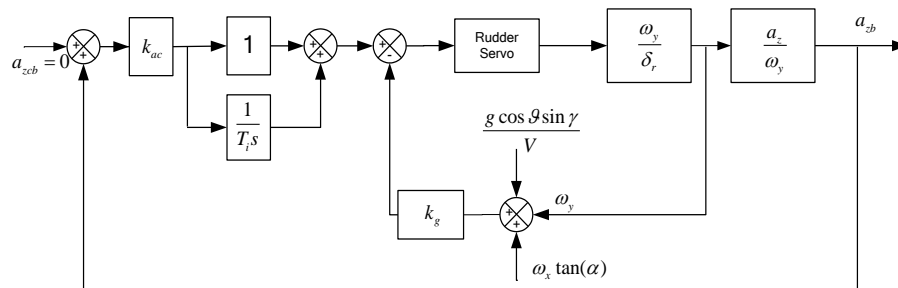


Figure9. The block diagram of acceleration yaw-autopilot

The yaw channel command is $a_{zbc} = 0$, the pitch channel command is $\alpha_c = 10^\circ$, and the roll channel command is $\gamma_c = 45^\circ$. The yaw acceleration and rudder deflection angle with different inner-loop feedback are shown in Fig.10 and Fig.11.

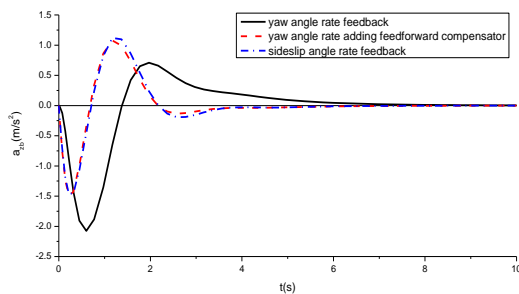


Figure10. The yaw acceleration

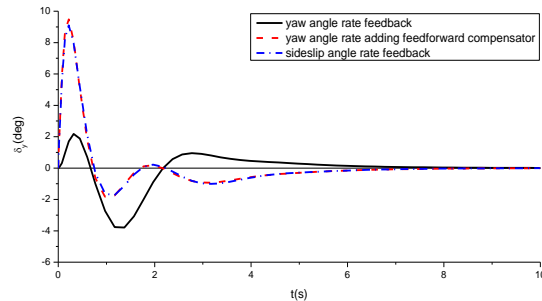


Figure11. The yaw rudder deflection angle

Based on Fig.10 and Fig.11, the feedback of yaw angle rate adding kinematic and gravitational compensator can decrease the maximum of yaw acceleration and make yaw acceleration to zero fast, which is much better for coordinated turn to eliminate sideslip angle, and the effect is equal to sideslip angle rate feedback. Comparing with sideslip angle rate feedback, the feedback of yaw angle rate adding kinematic and gravitational compensator need more rudder resource, which is used to eliminate sideslip angle quickly, so the sufficient rudder resource is the key point to coordinated turn.

6 .Conclusions

Based on the detailed performance analysis on yaw acceleration output by autopilot structure, two-loop acceleration yaw-autopilot with a PI compensator has been presented. The convergence essence of yaw acceleration by inner-loop sideslip angle rate feedback has been validated, and the engineering approximation of sideslip angle rate feedback has been confirmed, the coordinated turn yaw-autopilot structure has been presented. The simulation results show that the yaw-autopilot has been presented in this paper can eliminate the sideslip and channel coupling effectively, improve the coordinated turn ability clearly. The research in this paper has certain application value for control

system design of reentry gliding vehicle.

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