Dynamic analysis of centrifugal machines rotors supported on ball bearings by combined application of 3D and beam finite element models

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Abstract. This research paper is aimed to investigating rotor dynamics of multistage centrifugal machines with ball bearings by using the computer programs "Critical frequencies of the rotor" and "Forced oscillations of the rotor," which are implemented the mathematical model based on the use of beam finite elements. Free and forces oscillations of the rotor for the multistage centrifugal oil pump NPS 200-700 are observed by taking into account the analytical dependence of bearing stiffness on rotor speed, which is previously defined on the basis of results' approximation for the numerical simulation in ANSYS by applying 3D finite elements. The calculations found that characteristic and constrained oscillations of rotor and corresponded to them forms of vibrations, as well as the form of constrained oscillation on the actual frequency for acceptable residual unbalance are determined.

1. Introduction

The existing numerical methods for calculation of rotor dynamics, which are based on the use of modern programs on the basis of finite element method, use beam elements with taking into account the Bernoulli's Hypothesis or 3D finite elements [1]. In this connection, appliance of 3D models, for example in ANSYS, needs carrying out relatively long operations and computational time [2]. The appliance of beam models [3] is not a long-time process for the initial data preparation, as well as it makes the computational process faster. However, the problem of determination of critical frequencies of rotor systems by using existing programs for the advanced determined dependence of the bearing stiffness and gap seals on rotor speed leads to drawing the Campbell diagram [4] that complicates the computational process with additional graphic constructions. Besides, the determination of rotor oscillations by taking into account the system of imbalances and nonlinear bearing stiffness is the high-complicated scientific problem [5].

In accordance with the abovementioned, the aim of this research is a development of methods for determining eigenfrequencies and critical frequencies, as well as characteristics of rotor oscillations by applying a combined using 3D and beam finite elements on the example multistage centrifugal machines. To achieve this aim the following problems are set:

1) determination of the bearing stiffness with taking into account rotor speed by using ANSYS;

2) development of the mathematical model of rotor dynamics with the predetermined dependence of the bearing stiffness on rotor speed;

3) creation the files of numerical implementation of the purposed mathematical model with the use of beam finite elements;

4) implementation of the developed technique on the example of multistage centrifugal pump NPS 200-700.

2. Pump description and determination of the bearing stiffness

The multistage centrifugal oil pump NPS 200-700 (Figure 1) is designed to pump water, oil, petroleum product, and liquefied carbohydrate gas-oil products at temperature in the range -30… +200°C with particles less than 0,2 μm. The pump's rotor rotates with the operating speed 2950 rpm in two opposite ball bearings KF 7416AC.


(a)

(b)
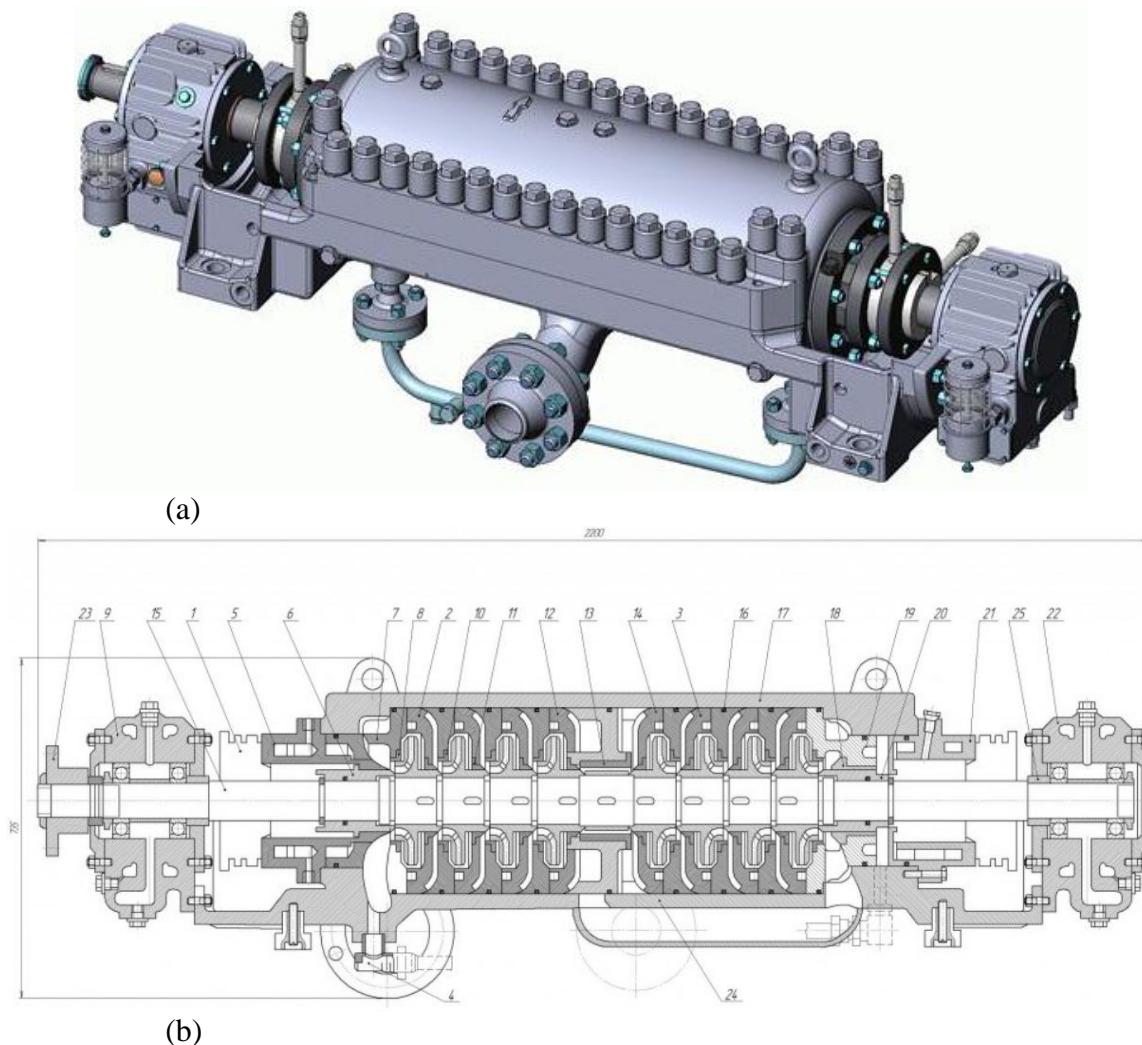
Figure 1. 3D model (a) and frontal section (b) of the centrifugal pump NPS 200-700.

The program ANSYS is used for determining the bearing stiffness with taking into account rotor speed for 2D beam elements. The related scheme of support loading is given on Figure 2 (a).

Considering the rotor speed and the housing stiffness is due to the fact, that the first factor determines the increasing quadratic dependence of bearing stiffness on rotor speed, and the second one leads to decreasing critical frequencies.

The geometry of pump housing and bearing support are chosen in accordance with the pump's engineering drawing. Rotation impact is carried out by applying the inertial forces to the inner bearing

race. Stiffness factor is defined by the analysis of loading of bearing assembly for different rotor speeds (Figure 2 (b)).

(a)   (b)
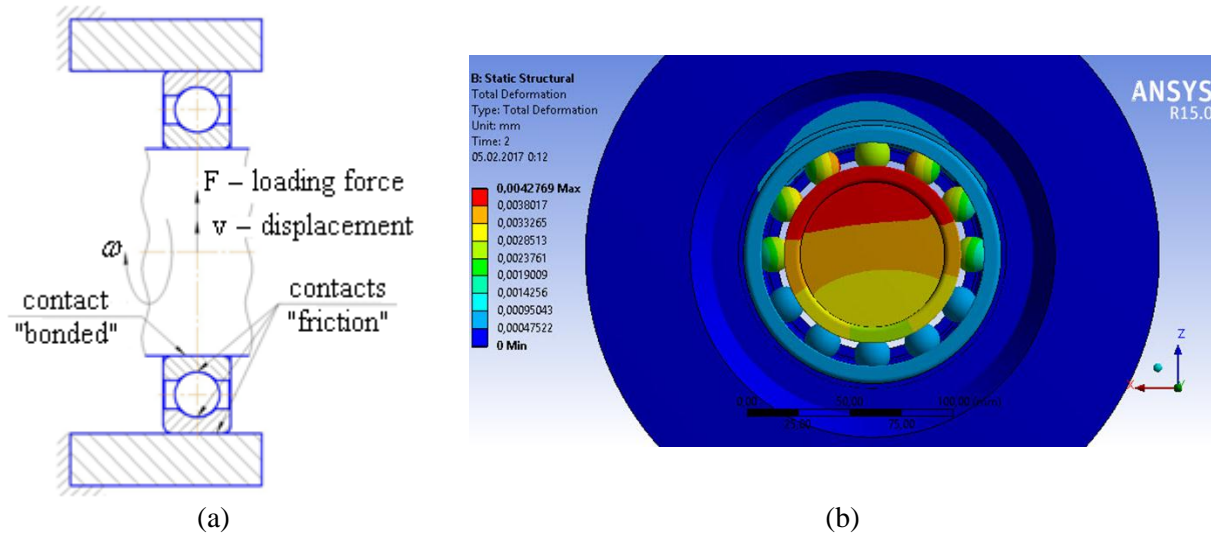
Figure 2. Calculation scheme of support loading (a) and results visualization (b).

For developing the mathematical model of free and forced oscillations of the rotor with the impact of the rotor speed on bearing stiffness is considered by the proposed analytical dependence [6, 7]:

$$c(\omega) = c_0 + \alpha\,\omega^2 \tag{1}$$

where c – bearing stiffness, N/m; ω – rotor speed, rad/s; c_0 – bearing stiffness in case of $\omega = 0$; α – coefficient, N·s^2/m, which is determined by regression dependence [8-10]:

$$\alpha = \frac{\sum\limits_{k=2}^{n}(c_k - c_0)\omega_k^2}{\sum\limits_{k=2}^{n}\omega_k^4} \tag{2}$$

k – number of experimental point ($k = 2\dots n$); n – number of points within the numerical simulation by ANSYS; c_k – bearing stiffness, which is determined by the results of numerical simulation in case of frequency $\omega = \omega_k$ (Figure 3).

As a result, the following values were obtained: $c_0 = 2,4\cdot10^8$ N/m, and $\alpha = 46,6$ N·s^2/m. Approximating curve of the dependence (2) is represented on Figure 3.
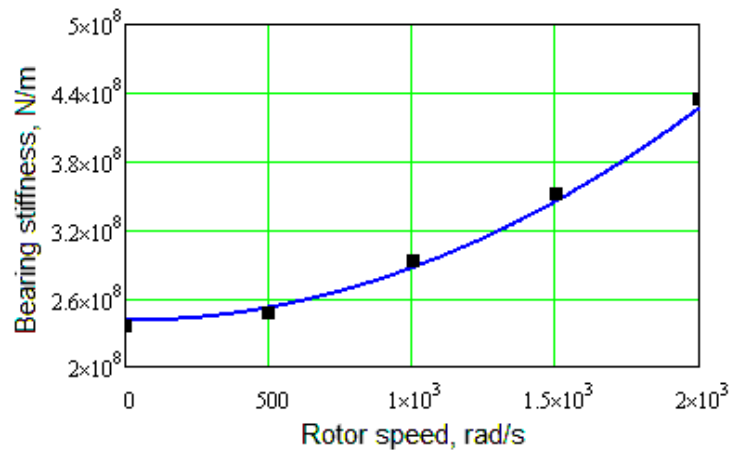
Figure 3. Dependence of the bearing stiffness on the rotor speed.

3. Mathematical model of rotor dynamics

Mathematical model of rotor dynamics system is based on the application of finite element method [3]. Local matrices of the stiffness $[C(\omega)]_e$ and inertia $[M]_e$ binodal finite elements with four degrees of freedom, which correspond to nodes cross motion of and rotation angles are determined by the following formulas:

$$\left[C(\omega)\right]_e = \frac{EI}{l^3}\begin{bmatrix} 12+c(\omega) & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}; \; \left[M\right]_e = \frac{m}{420}\begin{bmatrix} 156+m_0 & 22l & 54 & -13l \\ 22l & 4l^2-k_g & 13l & -3l^2 \\ 54 & 13l & 156 & 22l \\ -13l & -3l^2 & 22l & 4l^2 \end{bmatrix} \tag{3}$$

where E – Young's modulus of material, N/m^2; $I = \pi(D^4 - d^4)/64$– moment of inertia of cross section; D, d – outer and inner diameters of cross section, m; l – element length, m; $c(\omega)$ – bearing stiffness, N/m; $m = \pi \rho l(D^2 - d^2)/4$ – mass of the finite element; m_0 – mass of the bushing part or impeller; ρ – material density, kg/m^3; $k_g = 420\{(D^2 + d^2)/16 + l^2/3 + I_g/m\}$ – parameter, which considers gyroscopic moment of inertia for finite elements and moment of inertia Ig (kg·m^2) of impellers or bushing parts.

Global matrix of the stiffness $[C(\omega)]$ and inertia $[M]$ are determined by summarizing the components of local matrices in corresponding nodes.

The numerical calculation of rotor's critical frequencies is to determine the roots of the frequency equation

$$det\left(\left[C(\omega)\right] - \omega^2\left[M\right]\right) = 0 \tag{4}$$

But eigenfrequencies are determined for stiffness matrix $[C(\omega_0)]$, which corresponds to operating rotor speed ω_0.

Column vector $\{Y\}$ of the mode shape for rotor's forced oscillations is determined by the following formula:

$$\{Y\} = \left(\left[C(\omega_0)\right] - \omega_0^2\left[M\right]\right)^{-1}\omega_0^2\{D\} \tag{5}$$

where $\{D\}$ – column vector of imbalances (kg·m), which are applied in nodes of finite elements.

The mathematical model of free and forced oscillations of rotor systems is described and numerically implemented with the use of files "Critical frequencies of the rotor" [11] and "Forced

oscillations of the rotor" [12] within computer program MathCAD. The validation of the purposed methodology is proved in research works [13, 14] for the case of multistage centrifugal machines by comparing the corresponding results obtained in program complex ANSYS, as well as within research works [15-18].

4. Analysis of rotor dynamics for the centrifugal pump NPS 200-700

Figure 4 (a) demonstrates the fragment of the table, which contains the input data for calculation of rotor's critical frequencies for the multistage centrifugal oil pump NPS 200-700. The corresponding design scheme is presented on Figure 4 (b).

	L, m	D, m	d, m	m, kg	I_g, kg·m²	c, N/m	α, N·s²/m	Disb, kg·m
	1	2	3	4	5	6	7	8
1	0.014	0.065	0	0	0	0	0	0
2	0.034	0.065	0	0.64	0	0	0	0
3	0.069	0.065	0	15.75	0	0	0	$1.3 \cdot 10^{-4}$
4	0.025	0.07	0	0.79	0	0	0	0
5	0.026	0.07	0	0.7	0	0	0	0
6	0.057	0.07	0	0	0	$2.4 \cdot 10^8$	46.6	0
7	0.057	0.07	0	1.17	0	0	0	0
...								
46	0.057	0.07	0	0	0	0	0	0
47	0.026	0.07	0	0	0	$2.4 \cdot 10^8$	46.6	0
48	0.012	0.07	0	0	0	0	0	0

(a)

● – local mass;   △ – support;   ◇ – disbalance.
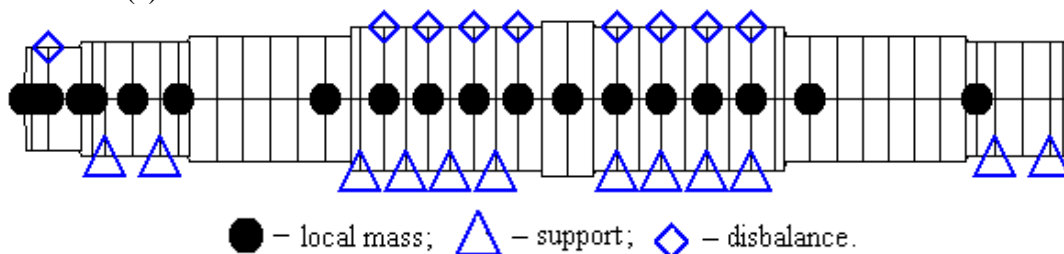
(b)

Figure 4. Fragments of the initial data table (a) and the corresponding design scheme (b) for calculating critical frequencies of the pump rotor.

The calculation results of rotor's eigenfrequencies and critical frequencies are summarized in Table 1, and corresponding forms of free oscillations are represented on Figure 5. Herewith, divergence of the results for eigenfrequencies and critical frequencies by using MathCAD and ANSYS is no higher than 0,7 % and 1,4 % respectively. The resonance safety factor is equal 23 %.

Table 1. Values of eigenfrequencies and critical frequencies, rad/s.

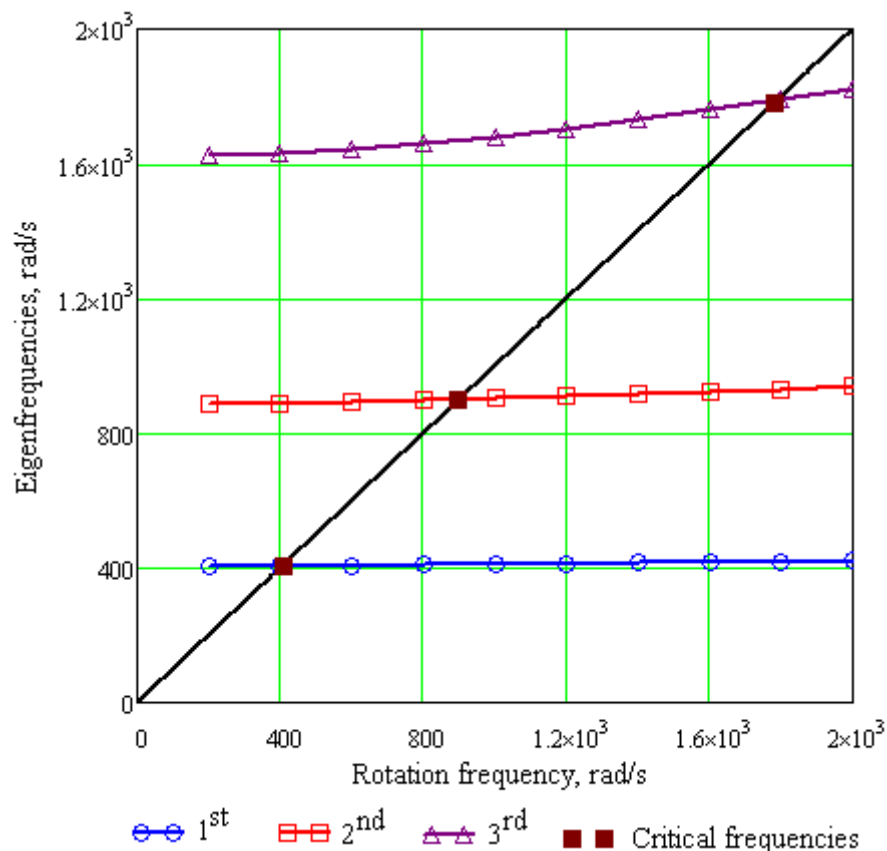| Frequency	Frequency number		
	1	2	3
Eigenfrequency	406 (406)*	890 (887)	1640 (1629)
Critical frequency	407 (407)	901 (896)	1805 (1780)

* The values in parentheses are defined by using ANSYS.
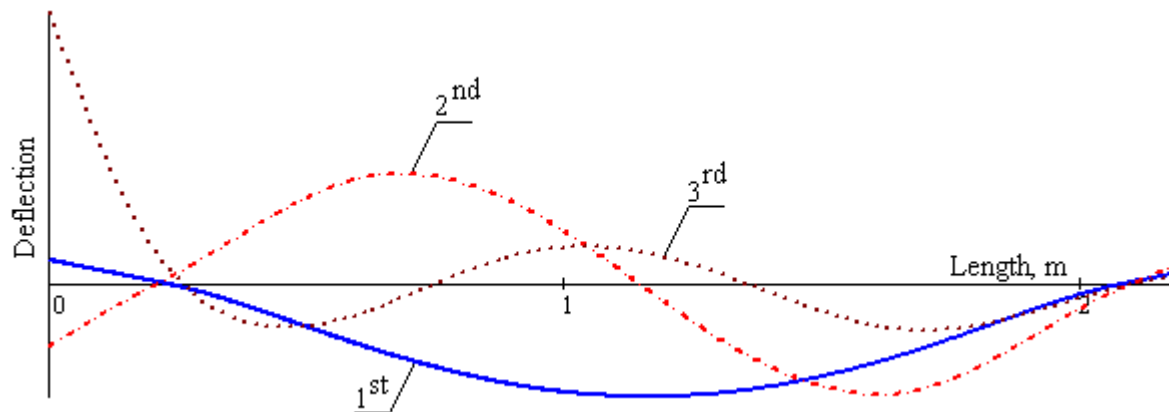
Figure 5. Forms of free oscillations of the rotor.

The results of numerical simulation (Table 1) correspond to the theorem about the spectrums of eigenfrequencies and critical frequencies [19]: since all critical frequencies are higher than operating rotor speed, they exceed the corresponding eigenfrequencies.

The additional benefit of applying the proposed methodology for determining eigenfrequencies and critical frequencies with the use of files [11, 12] of the computer program MathCAD is the lack of drawing Campbell diagram [20], which is represented on Figure 6 on the example of the rotor for the pump NPS 200-700 with the application of the program complex ANSYS.

Figure 6. Campbell diagram obtained using ANSYS.

The form of the rotor forced oscillations on the operating frequency for the system of imbalances, which is determined in accordance with the standard [19], is presented on Figure 7. Maximum

deflection in the section of the central gap seal is equal 0,17 mm, which does not exceed the corresponding radial gap.
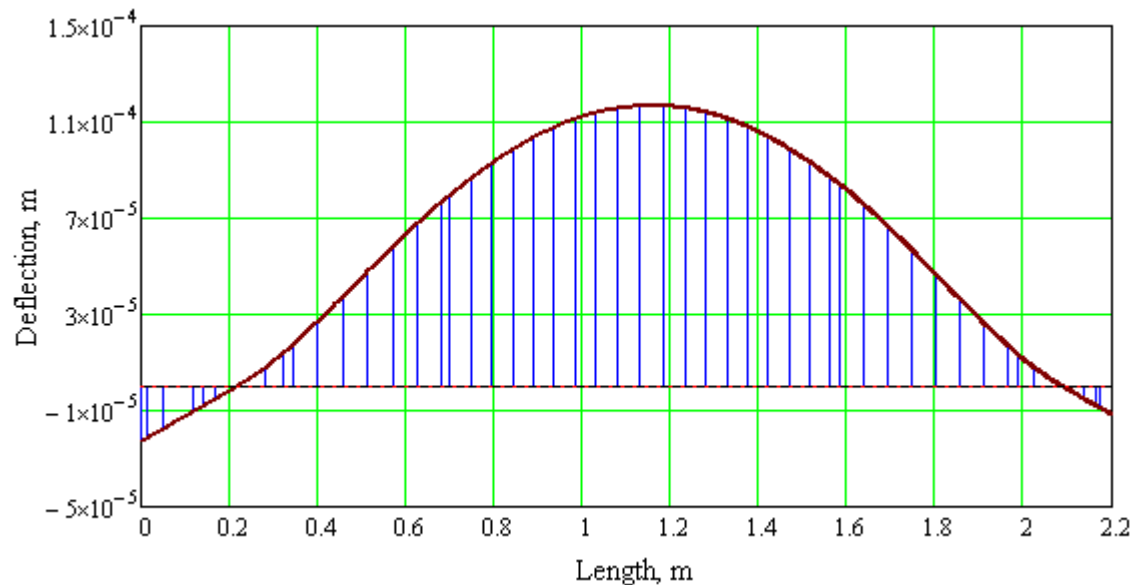
Figure 7. Form of forced oscillations of the rotor.

5. Conclusions

Thus, the work is aimed to develop the mathematical model of rotor dynamics systems and the corresponding methodology for determining eigenfrequencies and critical frequencies, as well as forms of free and forced oscillations by combined implementation of 3D and beam finite element models by using ANSYS and the computer algebra system MathCAD.

Bearing stiffness properties for supports with ball bearings KF 7416AC with taking into account rotor speed is determined by the example of multistage centrifugal oil pump NPS 200-700 by using ANSYS. The bearing stiffness coefficients and determined by the proposed regression dependence.

The represented mathematical model of rotor dynamics systems is based on the application of finite element method, which considers predefined analytical dependence of bearing stiffness on rotor speed, as well as gyroscopic moments of inertia for impellers and other bushing parts. The model is implemented in files "Critical frequencies of the rotor" and "Forced oscillations of the rotor" of the computer program MathCAD by the example of multistage centrifugal oil pump NPS 200-700.

The main advantages of the proposed approach in comparison on using ANSYS are the opportunity to take into account the advanced analytical dependence of bearing stiffness on rotor speed, and lack of drawing Campbell diagram in order to determine critical frequencies, as well as relatively fast preparation of input data and short computational time.

The further research will be aimed at implementation of the proposed methodology fir the case of nonlinear bearing stiffness, as well as for the development and numerical simulation of balancing methodologies.

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