

## Evaluation of rotor axial vibrations in a turbo pump unit equipped with an automatic unloading machine

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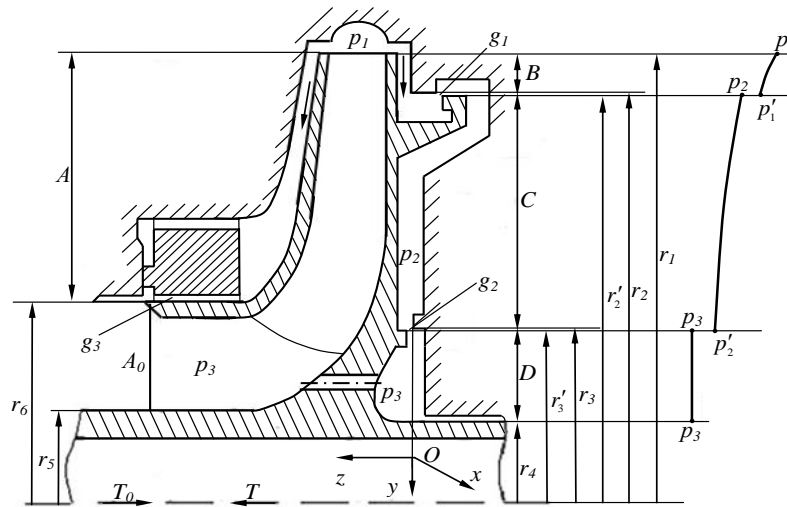
**Abstract.** The article presents forced axial vibrations of the rotor with an automatic unloading machine in an oxidizer pump. A feature of the design is the use in the autoloading system of slotted throttles with mutually inverse throttling. Their conductivity is determined by a numerical experiment in the ANSYS CFX software package.

### 1. Introductory remarks

The article presents (Figure 1) a simplified rotor design of the oxidizer pump equipped with an automatic unloading machine (AUM), in which, unlike in traditional devices [1], inverse slotted throttles are used. Static calculation of the system is given in the article "Static calculation of the rotor unloading automatic machine for a high-pressure centrifugal pump" (V A Martsynkovskyy, A Deineka, A Korczak and G Peczkis), published in this issue of the journal.

Unit rotor together with the automatic unloading machine is a complex dynamic system with distributed parameters that is a subject to periodic external perturbations. Each cross section of the rotor performs interrelated radial, angular and axial oscillations [2]. In regards to reliability, primarily axial vibrations can be dangerous, since their relatively large amplitudes limit the operating life of ball bearings wherein rotor is located. In this paper, as a first approximation, a simplified problem is considered: a rotor is studied as a solid body with an automatic loading system which performs one-dimensional axial oscillations along the support axis. Such a simplified model enables to obtain static and dynamic characteristics in an analytical way and accurately describe the main regularities of the system oscillations.





**Figure 1.** Calculation scheme.

## 2. Derivation of the axial oscillation equation

Expression of the total axial force acting on the impeller, obtained in the article "Static calculation of the rotor unloading automatic machine for a high-pressure centrifugal pump", has the form:

$$F_z = T_0 - Cp_2 + (A - B)p_1 + (A_0 - D)p_3 - K(A^2 - B^2 - C^2)\Omega^2 \quad (1)$$

where  $T_0$  - is the residual axial force to be balanced;  $p_1, p_3$  - supply pressure and pressure at the inlet;  $p_2$  - pressure in the chamber C between the upper and lower throttles, depending on the axial position of the rotor

$$K = \frac{\rho \omega_n^2 \kappa_1^2}{2\pi} \quad (2)$$

$\kappa_1$  - swirl coefficient of the flow in side chambers [3];  $\omega_n$  - nominal rotor speed;  $\Omega = \omega/\omega_n$  - dimensionless rotation speed;  $\rho$  - density of the pumped medium. The last term in expression (1) is the pressure force conditioned by the average angular velocity of liquid in the chambers. Capital letters indicate the flat annular sections shown in Figure 1.

At axial oscillations, the force of viscous resistance  $-c\dot{z}$  ( $c$  is the damping coefficient) acts on the rotor, therefore, on the basis of the 2nd Newton's law, the equation of axial oscillations will take the form:

$$m\ddot{z} + c\dot{z} = -Cp_2 + T_0 + (A - B)p_1 + (A_0 - D)p_3 - K(A^2 - B^2 - C^2)\Omega^2$$

$m$  - indicated rotor weight. Having divided these equation term-by-term on  $Ap_n$ , we obtain the dimensionless form of the equation:

$$T_1^2 \ddot{u} + 2\zeta T_1 \dot{u} = \bar{C}\psi_2 - \bar{T}_0 - (1 - \bar{B})\psi_1 - (\bar{A}_0 - \bar{D})\psi_3 + \bar{K}(A^2 - B^2 - C^2)\Omega^2 \quad (3)$$

$$T_1^2 = \frac{mz_n}{Ap_n}, 2\zeta T_1 = \frac{cz_n}{Ap_n}, \bar{K} = \frac{\rho \omega_n^2 \kappa_1^2}{2\pi Ap_n}, \bar{A}_0 = \frac{A_0}{A}, \bar{B} = \frac{B}{A}, \bar{C} = \frac{C}{A} \quad (4)$$

$$\bar{D} = \frac{D}{A}, \bar{T}_0 = \frac{T_0}{A}; \psi_1 = \frac{p_1}{p_n}, \psi_2 = \frac{p_2}{p_n}, \psi_3 = \frac{p_3}{p_n}; u = \frac{z}{z_n}$$

$z$  - rotor axial displacement equal to the gap in the lower throttle,  $z_n$  - its base value,  $u = z/z_n$  - the dimensionless axial displacement of the rotor.

The equation (3) includes an unknown pressure  $p_2$  in the chamber 2, which depends on the gap size and is determined from the balance equation of the flow rate passing through the slotted throttles of the equilibrium system. For automodeling section of the turbulent flow, the flow rate through the upper and lower throttles is

$$Q_1 = g_1(u)\sqrt{p'_1 - p_2}, \quad Q_2 = g_2(u)\sqrt{p'_2 - p_3} \quad (5)$$

where  $p'_1 = p_1 - KB\Omega^2$ ,  $p'_2 = p_2 - KC\Omega^2$  i.e. decrease in pressure at the lower radius of the corresponding chamber due to the inertial effect is taken into account. In static calculation  $Q_{10} = Q_{20} = Q_0$

According to the results of the numerical experiment, conductance almost linearly depends on the gap:

$$g_1(u) = a_1u + b_1, \quad g_2(u) = a_2u + b_2 \quad (6)$$

And for the unit under consideration

$$a_1 = -1,08 \cdot 10^{-5} \left( m^7/kg \right)^{0,5}, \quad b_1 = 1,43 \cdot 10^{-5} \left( m^7/kg \right)^{0,5}$$

$$a_2 = 0,814 \cdot 10^{-5} \left( m^7/kg \right)^{0,5}, \quad b_2 = 0,249 \cdot 10^{-5} \left( m^7/kg \right)^{0,5}$$

Negative value of the angular coefficient  $a_1$  is due to the fact that when the lower gap  $z$  increases, the upper gap, and hence the conductance, decreases: inverse throttling takes place.

In the dynamics by rotor axial oscillations, the balance equation includes the displacement flow rate  $C\dot{z}$  and compression  $\frac{V}{E} \dot{p}_2$  of liquid in the chamber [4].

$$g_1(u)\sqrt{p'_1 - p_2} = g_2(u)\sqrt{p'_2 - p_3} + C\dot{z} + \frac{V}{E} \dot{p}_2 \quad (7)$$

The obtained first-order differential equation relating to the unknown  $p_2$  is nonlinear. We linearize it in a neighborhood of the equilibrium position, passing to the variational equations. It should be taken into account that inertial forces in the corresponding chambers are:

$$KB\Omega^2 = p_1 - p'_1, \quad KC\Omega^2 = p_2 - p'_2$$

and their variations

$$2KB\Omega_0\delta\Omega = \delta p_1 - \delta p'_1, \quad 2KC\Omega_0\delta\Omega = \delta p_2 - \delta p'_2$$

Further, we go into the quasistatic change in the rotation frequency, therefore  $\delta\Omega = 0$ ,  $\delta p'_1 = \delta p_1$ ,  $\delta p'_2 = \delta p_2$ , and variational equations are (7):

$$\frac{V}{EQ_0} \delta \dot{p}_2 + \frac{1}{2} \left( \frac{1}{p'_{10} - p_{20}} + \frac{1}{p'_{20} - p_{30}} \right) \delta p_2 =$$

$$= - \frac{Cz_n}{Q_0} \delta \dot{u} - \left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right) \delta u + \frac{1}{2} \left( \frac{\delta p_1}{p'_{10} - p_{20}} + \frac{\delta p_3}{p'_{20} - p_{30}} \right)$$

Thereafter, we move to the dimensionless pressures, denoting

$$\Delta\psi_{10} = \psi'_{10} - \psi_{20}, \quad \Delta\psi_{20} = \psi'_{20} - \psi_{30}, \quad \Psi = \left( \frac{1}{\Delta\psi_{10}} + \frac{1}{\Delta\psi_{20}} \right)^{-1} \quad (8)$$

$$\frac{Vp_n}{EQ_0} \delta\dot{\psi}_2 + \frac{1}{2} \left( \frac{1}{\Delta\psi_{10}} + \frac{1}{\Delta\psi_{20}} \right) \delta\psi_2 = -\frac{Cz_n}{Q_0} \delta\ddot{u} - \left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right) \delta u + \frac{1}{2\Delta\psi_{10}} \delta\psi_1 + \frac{1}{2\Delta\psi_{20}} \delta\psi_3$$

To obtain the equation in a short form, it is necessary to divide the last equation term-by-term on a coefficient  $\frac{1}{\Delta\psi_{10}} + \frac{1}{\Delta\psi_{20}} = \frac{1}{\Psi}$  when desired quantity is  $\delta\psi_2$

$$2 \frac{Vp_n}{EQ_0} \Psi \delta\dot{\psi}_2 + \delta\psi_2 = -\frac{2Cz_n\Psi}{Q_0} \delta\ddot{u} - 2\Psi \left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right) \delta u + \frac{\Psi}{\Delta\psi_{10}} \delta\psi_1 + \frac{\Psi}{\Delta\psi_{20}} \delta\psi_3$$

To reduce the recording, variation sign will further be omitted, keeping in mind, however, that this is not about the absolute values of the variables, but about their small deviations from the established values. The latter are marked with an additional zero index and are determined by static calculation. Having denoted the time constants and transmission coefficients as:

$$T_2 = 2 \frac{Vp_n}{EQ_0} \Psi, \quad \kappa_s = 2\Psi \left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right), \quad \tau_2 = \frac{Cz_n}{\left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right) Q_0}, \quad k_1 = \frac{\Psi}{\Delta\psi_{10}}, \quad k_3 = \frac{\Psi}{\Delta\psi_{20}} \quad (9)$$

we obtain the standard form of the linearized flow rate balance:

$$T_2\dot{\psi}_2 + \psi_2 = -\kappa_s(\tau_2\dot{u} + u) + k_1\psi_1 + k_3\psi_3 \quad (10)$$

Pressure in the chamber can be considered as a controlling action, and equation (10) - as the equation of the automatic controller

If we introduce an operator of differentiation in time  $s \equiv d/dt$ , then equation (10) takes the operator form:

$$(T_2s + 1)\psi_2 = -\kappa_s(\tau_2s + 1)u + k_1\psi_1 + k_3\psi_3$$

we can derive the control influence of it:

$$\psi_2 = -\kappa_s \frac{M_2(s)}{D_2(s)} u + \frac{1}{D_2(s)} (k_1\psi_1 + k_3\psi_3) \quad (11)$$

where

$$D_2(s) = T_2s + 1, \quad M_2(s) = \tau_2s + 1 \quad (12)$$

proper operator of the controller and operator of the actions by error, respectively.

Relation of the controller reaction to the stimulus is the transfer function of the controller. If the stimulus is a harmonic function, then an operator of time differentiation is replaced by an imaginary operator  $i\omega$ :

$$\frac{\psi_2}{u} = -\kappa_s \frac{\tau_2 i\omega + 1}{T_2 i\omega + 1} = -\kappa_s [U_2(\omega) + i\omega V_2(\omega)] = W_2(i\omega) \quad (13)$$

$$U_2(\omega) = \frac{1 + T_2\tau_2\omega^2}{1 + T_2^2\omega^2}, \quad V_2(\omega) = \frac{\tau_2 - T_2}{1 + T_2^2\omega^2}$$

and the transfer function  $W_2(i\omega)$  becomes the frequency transfer function or dynamic stiffness of the controller. In the steady-state  $\omega=0$  and  $W_2(0)=-\kappa_s$ , i.e. dynamic rigidity transforms into hydrostatic stiffness of the system. Its negative value  $(-\kappa_s < 0, \kappa_s > 0)$  is a condition of static stability, therefore the system is functional only in the section where  $\kappa_s > 0$ . This condition is met if  $\frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} > 0$ . For

the design considered here  $a_1 < 0, a_2 > 0$ , so this condition of static stability is fulfilled at the whole range of changes in external influences.

The real part  $U_2$  characterizes the controller's stiffness itself, and the imaginary part  $V_2$  - its "contribution" to damping [2]. In the absence of external damping  $c = 0$ , axial oscillations decay, if  $V_2 > 0$  or

$$\tau_2 > T_2 \quad (14)$$

This condition with some margin for the stabilizing effect of external damping can be considered as a condition for rotor axial stability.

Having substituted expression of the regulating effect (11) into the equation of axial oscillations (3), we obtain motion equation of the system "impeller-balancing unit". Preliminary equation (3) must be linearized, i.e. move to variations. Herein the last term on the right-hand part is  $2\bar{K}(A^2 - B^2 - C^2)\Omega_0\delta\Omega = 0$ . After some transformations, we obtain:

$$\begin{aligned} & \left[ (T_1^2 s^2 + 2\zeta T_1 s)(T_2 s + 1) + \kappa_s \bar{C}(\tau_2 s + 1) \right] u = -(T_2 s + 1) \bar{T}_0 - \\ & - \left[ (1 - \bar{B})(T_2 s + 1) - \bar{C} k_1 \right] \mu_1 - \left[ (\bar{A}_0 - \bar{D})(T_2 s + 1) - \bar{C} k_3 \right] \mu_3 \end{aligned}$$

Having grouped the terms in powers of the differentiation operator in time, we obtain final form of the system equation:

$$D_0(s)u = (1 - \bar{B})N_1(s)\mu_1 + (\bar{A}_0 - \bar{D})N_3(s)\mu_3 - N_T(s)\bar{T}_0 \quad (15)$$

In this equation, the system's proper operator  $D_0(s)$  and the action operators are expressed by the equations:

$$D_0(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3 \quad (16)$$

$$N_1(s) = \frac{k_1 \bar{C}}{(1 - \bar{B})} - (T_2 s + 1), \quad N_3(s) = \frac{k_3 \bar{C}}{(\bar{A}_0 - \bar{D})} - (T_2 s + 1), \quad N_T(s) = T_2 s + 1 \quad (17)$$

$$a_0 = T_1^2 T_2, \quad a_1 = T_1^2 + 2\zeta T_1 T_2, \quad a_2 = 2\zeta T_1 + \bar{C} \kappa_s \tau_2, \quad a_3 = \bar{C} \kappa_s \quad (18)$$

Static rotor displacement relative to the equilibrium position, caused by the deviations of external influences, can be determined using the equation (15), assuming  $s = d/dt = 0$ . Using coefficients of the proper operator one can determine stability of the axial oscillations. According to the Routh-Hurwitz criterion, a third-order system is stable if the following condition is fulfilled:

$$a_1 a_2 > a_0 a_3 \quad (19)$$

for all positive coefficients. It can be seen from the formulas (18) that all the coefficients are positive if  $\kappa_s > 0$ , i.e. under condition that the system is statically stable. If we neglect the effect of external damping ( $\zeta = 0$ ), then the equation (19) after substitution (18) is reduced to the value which was obtained earlier (14):  $\tau_2 > T_2$ . Having substituted (9), we obtain a restriction of the chamber volume:

$$V < \frac{ECz_n}{2p_n \left( \frac{a_2}{g_{20}} - \frac{a_1}{g_{10}} \right) \Psi} \quad (20)$$

### 3. Analysis of rotor axial vibrations

For the model under consideration, axial vibrations are described by the third-order differential equation. Operator  $D_0(s)$  is the system's proper operator, and equation  $D_0(s)u = 0$  is the equation of free axial vibrations of the impeller. Here we confine ourselves to analyzing forced oscillations and stability. A similar problem of rotor axial oscillations of a shaftless motor pump with the traditional design of an automatic unloading machine was considered in the article [5].

We will assume that variations of external influences  $\psi_1, \psi_3, \bar{T}_0$  change according to a harmonic law  $\psi_1 = \psi_{a1} e^{i\omega t}$ ,  $\psi_3 = \psi_{a3} e^{i\omega t}$ ,  $\bar{T}_0 = T_a e^{i\omega t}$ . In this case, linear system response to each of these changes (component of the specific solution) has the form:

$$u_1 = u_{1a} e^{i(\omega t + \phi_1)}, \quad u_3 = u_{3a} e^{i(\omega t + \phi_3)}, \quad u_T = u_{Ta} e^{i(\omega t + \phi_T)}$$

Amplitudes and phases of harmonic reactions, i.e. relations of the reactions to individual harmonic influences, are determined by the frequency transfer functions  $W(i\omega)$  that can be obtained by introducing a substitution  $s = i\omega$  into the equation of axial oscillations (15):

$$\begin{aligned} W_1(i\omega) &= \frac{u_1}{(1 - \bar{B})\psi_1} = \frac{u_{1a}}{(1 - \bar{B})\psi_{1a}} e^{i\phi_1} = \frac{N_1(i\omega)}{D_0(i\omega)} \\ W_3(i\omega) &= \frac{u_{3a}}{(\bar{A}_0 - \bar{D})\psi_{3a}} e^{i\phi_3} = \frac{N_3(i\omega)}{D_0(i\omega)}, \quad W_T(i\omega) = \frac{u_{Ta}}{T_a} e^{i\phi_T} = \frac{N_T(i\omega)}{D_0(i\omega)} \end{aligned} \quad (21)$$

We single out real and imaginary parts in the proper operator (16) and the action operators (17), replacing  $s = i\omega$ :

$$D_0 = U_0 + i\omega V_0, \quad U_0(\omega) = a_3 - \omega^2 a_1, \quad V_0(\omega) = a_2 - \omega^2 a_0 \quad (22)$$

$$N_1 = U_1 + i\omega V_1, \quad U_1 = \left( \frac{k_1 \bar{C}}{1 - \bar{B}} \right) - 1, \quad V_1 = -T_2$$

$$N_3 = U_3 + i\omega V_3, \quad U_3 = \left( \frac{k_3 \bar{C}}{\bar{A}_0 - \bar{D}} \right) - 1, \quad V_3 = -T_2 \quad (23)$$

$$N_T = U_T + i\omega V_T, \quad U_T = 1, \quad V_T = T_2$$

Having substituted these expressions into formulas (21), we obtain frequency transfer functions in the form of complex numbers, for example:

$$W_1(i\omega) = \frac{U_1 + i\omega V_1}{U_0 + i\omega V_0} = \frac{u_{1a}}{(1 - \bar{B})\psi_{1a}} e^{i\phi_1} = B_1(\omega) e^{i\phi_1(\omega)}$$

where  $B_1(\omega) = u_{1a} / ((1 - \bar{B})\psi_{1a}) = |W_1(i\omega)|$ ,  $\phi_1(\omega) = \arg W_1(i\omega)$  - are the amplitude and phase frequency response characteristics. We divide the real and imaginary parts in the expression  $W_1$ :

$$W_1 = \frac{U_1 + i\omega V_1}{U_0 + i\omega V_0} = \frac{U_0 U_1 + \omega^2 V_0 V_1}{U_0^2 + \omega^2 V_0^2} + i\omega \frac{(U_0 V_1 - U_1 V_0)}{U_0^2 + \omega^2 V_0^2}$$

Now the amplitude and phase  $W_1$  have a form:

$$B_1(\omega) = \frac{u_{1a}}{(1-\bar{B})\psi_{1a}} = \sqrt{\frac{U_1^2 + \omega^2 V_1^2}{U_0^2 + \omega^2 V_0^2}}, \phi_1(\omega) = \arctg \omega \frac{U_0 V_1 - U_1 V_0}{U_0 U_1 + \omega^2 V_0 V_1} \quad (24)$$

By analogy, frequency response characteristics according to the influence of  $\psi_3$  and  $\bar{T}_0$  can be determined:

$$B_3(\omega) = \frac{u_{3a}}{(\bar{A}_0 - \bar{D})\psi_{3a}} = \sqrt{\frac{U_3^2 + \omega^2 V_3^2}{U_0^2 + \omega^2 V_0^2}}, \phi_3(\omega) = \arctg \omega \frac{U_0 V_3 - U_3 V_0}{U_0 U_3 + \omega^2 V_0 V_3} \quad (25)$$

$$B_T(\omega) = \frac{u_{Ta}}{T_a} = \sqrt{\frac{U_T^2 + \omega^2 V_T^2}{U_0^2 + \omega^2 V_0^2}}, \phi_T(\omega) = \arctg \omega \frac{U_0 V_T - U_T V_0}{U_0 U_T + \omega^2 V_0 V_T}$$

Calculation formulas can be obtained using the expressions for the real and imaginary parts of the proper operator (16) and the operators of the action (17):

$$B_l(\omega) = \frac{u_{la}}{(1-\bar{B})\psi_{la}} = \sqrt{\frac{\left(\frac{k_l \bar{C}}{1-\bar{B}} - 1\right)^2 + \omega^2 T_2^2}{\left(a_3 - \omega^2 a_l\right)^2 + \omega^2 \left(a_2 - \omega^2 a_0\right)^2}} \quad (26)$$

$$\phi_l(\omega) = -\arctg \omega \frac{\left(a_3 - \omega^2 a_l\right) T_2 + \left(\frac{k_l \bar{C}}{1-\bar{B}} - 1\right) \left(a_2 - \omega^2 a_0\right)}{\left(a_3 - \omega^2 a_l\right) \left(\frac{k_l \bar{C}}{1-\bar{B}} - 1\right) - \omega^2 \left(a_2 - \omega^2 a_0\right) T_2}$$

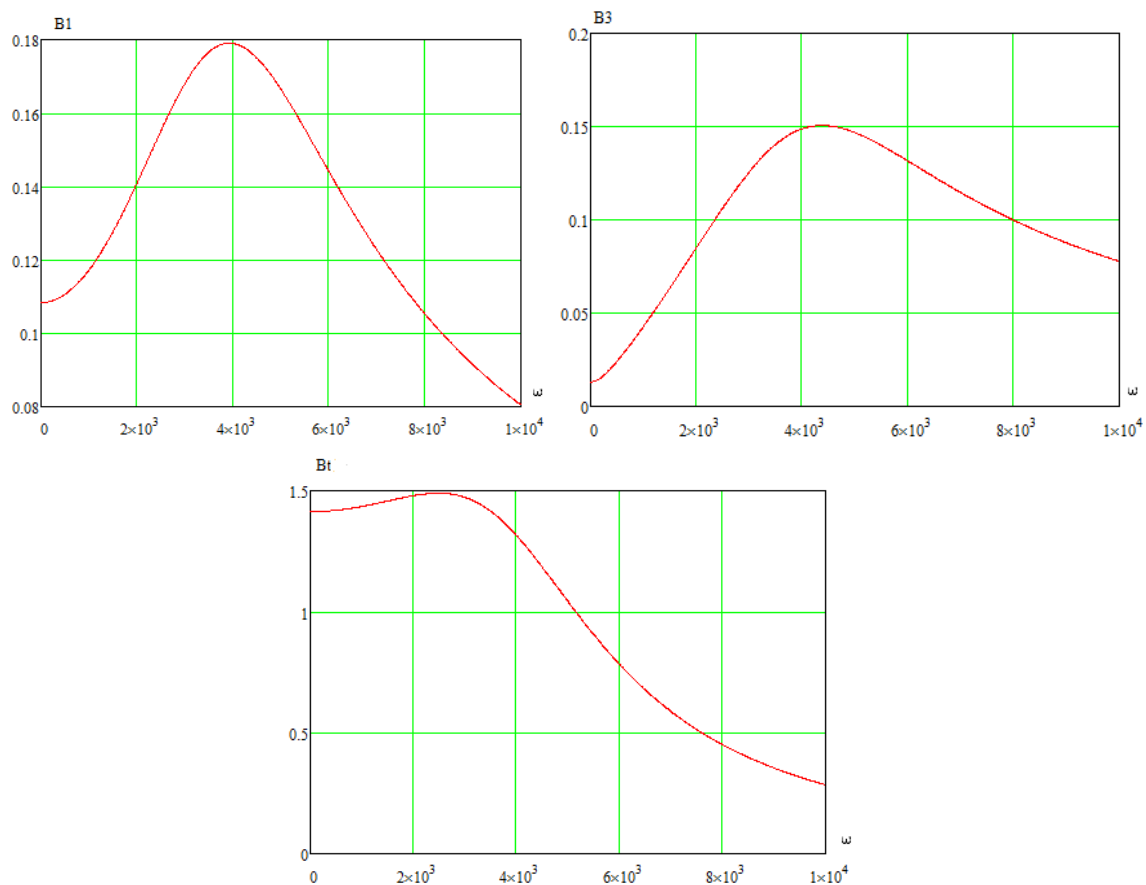
$$B_3(\omega) = \frac{u_{3a}}{(\bar{A}_0 - \bar{D})\psi_{3a}} = \sqrt{\frac{\left(\frac{k_3 \bar{C}}{\bar{A}_0 - \bar{D}} - 1\right)^2 + \omega^2 T_2^2}{\left(a_3 - \omega^2 a_l\right)^2 + \omega^2 \left(a_2 - \omega^2 a_0\right)^2}} \quad (27)$$

$$\phi_3(\omega) = -\arctg \omega \frac{\left(a_3 - \omega^2 a_l\right) T_2 + \left(\frac{k_3 \bar{C}}{\bar{A}_0 - \bar{D}} - 1\right) \left(a_2 - \omega^2 a_0\right)}{\left(a_3 - \omega^2 a_l\right) \left(\frac{k_3 \bar{C}}{\bar{A}_0 - \bar{D}} - 1\right) - \omega^2 \left(a_2 - \omega^2 a_0\right) T_2}$$

$$B_T(\omega) = \frac{u_{Ta}}{T_a} = \sqrt{\frac{1 + \omega^2 T_2^2}{\left(a_3 - \omega^2 a_l\right)^2 + \omega^2 \left(a_2 - \omega^2 a_0\right)^2}} \quad (28)$$

$$\phi_T(\omega) = \arctg \omega \frac{\left(a_3 - \omega^2 a_l\right) T_2 - \left(a_2 - \omega^2 a_0\right)}{\left(a_3 - \omega^2 a_l\right) + \omega^2 \left(a_2 - \omega^2 a_0\right) T_2}$$

As an example, we calculate the amplitude and phase frequency characteristics of the pump rotor, described in the article "Static calculation of the rotor unloading automatic machine for a high-pressure centrifugal pump". We will also use the results of static calculation presented in this paper. To analyze the rotor dynamics, additional parameters are taken: rotor weight  $m = 49,6 \text{ kg}$  damping index of the rotor free axial vibrations without AUM  $\zeta = 0,05$ , elasticity modulus of the pumped medium  $E = 2 \cdot 10^9 \text{ Pa}$ . Steady-state values are used from the static calculation results: pressure in the chamber  $p_{20} = 21,6 \text{ MPa}$ , the dimensionless end clearance in the lower throttle  $u_0 \approx 0,53$  and the volume flow  $Q_0 \approx 23 \cdot 10^{-3} \text{ m}^3/\text{s}$ .



**Figure 2.** Amplitude frequency characteristics of axial oscillations.

Studying the frequency characteristics (Figure 2) one can see that the first resonance occurs at a frequency of  $4000 \text{ s}^{-1}$ . The second proper frequency of axial oscillations is far beyond the achievable rotation frequency, so it is of no practical interest. Using formulas (25) - (27), amplitudes of the resonance oscillations can be determined. If the relative amplitudes of the external perturbations are taken as  $\psi_{1a} = \psi_{3a} = 0,1$ , we obtain the following values  $z_{1a} = 0,014 \text{ mm}$ ,  $z_n = 0,009 \text{ mm}$ ,  $z_{3a} = 0,0083 \text{ mm}$ ,  $z_n = 0,0054 \text{ mm}$ .

#### 4. Conclusions

The article presents a calculation of rotor axial vibrations of the oxidizer pump. To obtain static characteristics of a system there was performed a numerical experiment in the ANSYS CFX software package and there were obtained mass flow rates and pressure values in the discharge chamber depending on the rotor axial displacement (axial gap of the lower throttle). Based on the results of numerical calculations, turbulent conductance of the upper and lower slotted throttles were calculated.



This made it possible to obtain a dependence of the axial displacement on the discharge pressure and, as a result, determine axial gaps sizes of the slotted throttles, as well as the mass flow rate at the nominal operating mode of the pump. Therefore, at a nominal frequency of 18750 rpm mass flow rate is 25.4 kg / s, axial gap of the lower throttle is 0.34 mm, the upper one - 0.31 mm. At a minimum frequency of 10500 rpm mass flow rate is 12.1 kg / s, axial gap of the lower throttle is 0.34 mm, the upper one - 0.31 mm. At a maximum frequency of 21150 rpm, mass flow rate is 27.55 kg / s, axial gap of the lower throttle is 0.33 mm, the upper one - 0.32 mm. Thus, it can be stated that a face gap has almost constant value in the entire range of operating frequencies of the TPU rotor. As it can be seen from the static characteristics (Figure 2), if discharge pressure increases, the face gap value increases, and when rotation speed goes up then the face gap value goes down. Analysis of hydrostatic stiffness (Figure 2) makes it possible to conclude that equilibrium position of the rotor is stable.

To estimate the critical frequencies of the oxidizer pump rotor oscillations, amplitude and phase frequency characteristics were built. The analysis of these characteristics showed that the first proper frequency of axial oscillations is equal to 38200 rpm, the second proper frequency is far beyond the attainable rotational frequencies. The obtained estimates indicate that the critical frequency is more than twice higher than the nominal rotation frequency and 1.8 times higher than the maximum operating frequency. Thus rotor is rigid relating to the axial oscillations. It should be noted that the stability condition of the rotor axial oscillations is also fulfilled, which is proved by the fact that all coefficients of the system characteristic equation are positive and the condition (20) is fulfilled.

Radial, angular and axial oscillations of the TPU oxidizer pump rotor should be a subject of the detailed study, wherein a rotor is studied as a free solid body influenced by a complex system of hydrodynamic forces. It should be born in mind that it is necessity to take into account deformed state of the automatic unloading machine affected by the medium pressure on the distribution of this pressure and vice versa. In fact, it is a related task of hydroelasticity.

## References

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