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Dynamic Equilibrium Surfaces for Conical Fluid-Film Bearings

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Abstract. The paper considers the procedure of constructing curves of dynamic equilibrium and problems of applicability of the procedure of linearization of lubricant film reactions around the equilibrium point for conical fluid-film bearings.

1. Introduction

An important role in the study of rotor-bearing units is played by the assumption that it is possible to linearize, generally speaking, essentially nonlinear reactions of the lubricating layer of a fluid-film bearing in the area of a certain equilibrium position. By the position of equilibrium in this case the position of the center of the journal's center is meant, where the external load is completely compensated by the reactions of the lubricating layer, and the set of such points at different rotational speeds of the rotor constitute a spatial curve called the dynamic equilibrium curve. These curves for the case of cylindrical supports constitute a function ϕ in the domain of determining the permissible rotational speeds ω in the plane of the radial cross-section. Such functions are constructed for a given radial load vector F. Equality means $\phi(\omega) = (X, Y)$ that there is equality $\vec{R}(X, Y) = -\vec{F}$ at the point (X, Y), where R is the reaction force of the bearing.

Studies of cylindrical full-coverage and thrust bearings confirm that if there is an asymptotic convergence of the trajectory of the motion of the rotor, then it converges to a point on the dynamic equilibrium curve. In the case when the trajectory of motion has the form of a limit cycle, its geometric center is close to the point of dynamic equilibrium. This gives a formal basis for the linearization of the lubricant layer reactions at these points.

The aim of this paper is to investigate the feasibility of using this method for the case of conical fluid-film bearings. A distinctive feature of this problem is that the dynamic equilibrium curve (if it exists) lies in three-dimensional space, can have discontinuities and other features that are not characteristic of either cylindrical or thrust bearings. At the same time, the solution of this problem is necessary to determine the degree of conformity of the description of the dynamic characteristics of a conical bearing by calculating the stiffness and damping coefficients.

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2. Mathematical model of a 'rotor-conical bearing' system

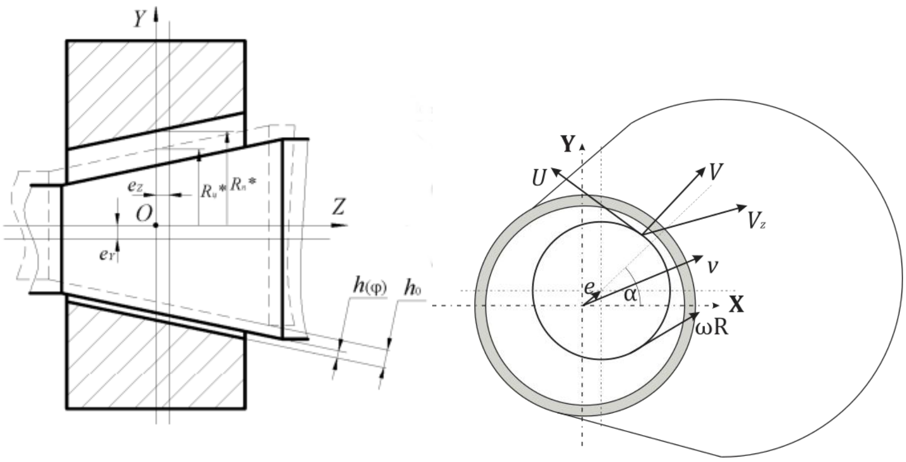


Figure 1. Calculation diagram of a conical fluid-film bearing.

To study the above issues, the mathematical model of the laminar flow of a thin layer of a viscous lubricant in the form of a two-dimensional Reynolds equation [1-3] will be used, which has the following form:

$$div\left(\frac{\rho h^3}{\mu}\,gradp\right) = 6\,div(\rho h V_t) + 12\rho V_n \tag{1}$$

$$V(\alpha) = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} + \begin{pmatrix} 0 \\ \omega R(z) \\ 0 \end{pmatrix}$$

Here V_t, V_n - tangential and normal components of the projection of the velocity $V(\alpha)$ of the lubricant on the bearing surface; $V_x = u, V_z = v$ $V_x = u, V_z = v$ (fig. 1), V_z - speed of the lubricant in the axial direction, $\dot{X}$, $\dot{Y}$ and $\dot{Z}$ - speed of the center of the journal, ω - angular speed, rad/s, h - radial gap function, α - the angle of rotation corresponding to the motion along the axis OX along the supporting surface of the bearing, $\alpha = \dfrac{2\pi x}{R}$.

In the framework of this paper, isothermal formulation of the problem is taken, assuming, therefore, that the viscosity and density are constant throughout the lubricating layer.

The calculated area in this case is a "supporting surface", topologically equivalent to the surface of the truncated cone. Equation (1) in this case is conveniently written in polar coordinates, taking as the pole the point of convergence of a given cone [2]:

$$\frac{\partial}{\partial r}\left[\frac{r\rho h^3}{\mu}\frac{\partial p}{\partial r}\right] + \frac{\partial}{\partial \phi}\left[\frac{\rho h^3}{\mu}\frac{\partial p}{r\partial \phi}\right] = 6\frac{\partial}{\partial r}(r\rho h V_r) + 6\frac{\partial}{\partial \phi}(\rho h V_\phi) + 12 r\rho V_n$$

$$\begin{pmatrix} V_\phi \\ V_n \\ V_r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} \tag{2}$$

The boundary conditions to equation (2) are the pressure values p_l and p_r, set on the boundaries and taken as equal to 1 bar within this study.

Determination of the gap function for the general case, which allows the rotation of the local coordinate reference frame of the rotor relative to the fixed bearing's frame, is given in [3]. In the present case of the problem, a simpler formulation of the problem will be considered in which only one conical fluid-film bearing is present, and the coordinate system associated with the rotor can be obtained from the coordinate system associated with the bearing only by the parallel transfer operation. This allows one to write the gap function in a simpler and more convenient form using the expression for the cylindrical bearing clearance function defined by three parameters: the radius of the bearing, the radius of the journal and the displacement vector of its geometric center in the radial plane:

$$h_{conic}(R^b_{\max}, R^b_{\min}, R^r_{\max}, R^r_{\min}, X, Y, Z, \alpha, z) = \tag{3}$$

$$= h_{cylindric}(\tau(0,z)R^b_{\max} + (1-\tau(0,z))R^b_{\min}, \tau(Z,z)R^r_{\max} + (1-\tau(Z,z))R^r_{\min}, X, Y, \alpha)$$

$$h_{cylindric}(R^b, R^r, X, Y, \alpha) =$$

$$= R^b - (X\cos(\alpha) + Y\sin(\alpha) + \frac{\sqrt{(2X\cos(\alpha) + 2Y\sin(\alpha))^2 - 4\cdot(X^2 + Y^2 - R^2)}}{2})$$

$$\tau(Z,z) = \frac{z-Z}{L}$$

Here R^b and R^r - radiuses of the bearing and the rotor accordingly, L - length of the bearing. In practical constructions, the parameters α and z that determine the coordinates of a point on a reference supporting surface are considered free, and all others are considered connected.

One of the characteristics of a fluid-film bearing is the reaction force vector of the lubricating layer, the components of which can be obtained by integrating the projections of the vector $p\cdot\vec{n}$ along the surface, where p - pressure, $\vec{n}$ - the internal normal to the surface of the bearing:

$$R_X = \int_{-\beta}^{\beta} \int_{R^b_{min}/\sin\beta}^{R^b_{max}/\sin\beta} -p(r,\phi)\cdot\cos\left(\frac{X(r,\phi)}{R(Z(r,\phi))}\right)\cos\alpha\, dr\, d\phi$$

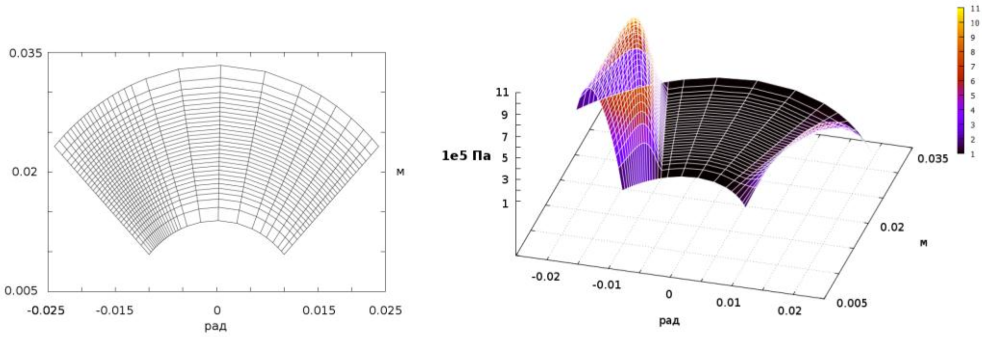
$$R_Y = \int_{-\beta}^{\beta} \int_{R^b_{min}/\sin\beta}^{R^b_{max}/\sin\beta} -p(r,\phi)\cdot\sin\left(\frac{X(r,\phi)}{R(Z(r,\phi))}\right)\cos\alpha\, dr\, d\phi \tag{4}$$

$$R_Z = \int_{-\beta}^{\beta} \int_{R^b_{min}/\sin\beta}^{R^b_{max}/\sin\beta} p(r,\phi)\cdot\sin\alpha\, dr\, d\phi$$

$$\alpha = arctg\left(\frac{R^b_{max} - R^b_{min}}{L}\right), \quad \beta = \pi \sin\alpha, \quad Z(r,\phi) = \left(r - \frac{R^b_{min}}{\sin\beta}\right)\cos\alpha$$

$$X(r,\varphi) = (\tau(0, Z(r,\varphi))R^b_{\max} + (1-\tau(0,Z(r,\varphi)))R^b_{\min}) \cdot \frac{\pi(\varphi+\beta)}{\beta}$$

For the numerical integration of equation (1), the method of finite differences is used with an adaptive grid, described in [4]. This scheme does not imply the uniformity of the grid along the coordinate lines and has the $o(h^2)$ order of accuracy, where h - the average step of the grid. A typical pressure distribution in a fluid-film conical bearing in the steady-state motion is shown in the Figure 2



(a) (b)

Figure 2. To the solution of (2): (a) adaptive calculation grid; (b) pressure distribution on a conical bearing.

3. Dynamic equilibrium curves formation

Using the presented mathematical model, the question of determination of points of a dynamic equilibrium curve could be addressed, given the geometric parameters of the bearing, thermophysical properties of the lubricant and the given external force [2]. It is easy to show, that if for some ω such point exists, then it is the only one. To do this, it suffices to note that the norm of the bearing's reaction force vector is a monotonically increasing eccentricity function, that is, the amount of displacement of the rotor relative to its central position. Then, because of the symmetry of the bearing, the vector equation $\vec{R}(X,Y) = -\vec{F}$ can be divided into two equations:

$$\| R(X,0) \| = \| F \| \tag{5}$$

$$\frac{\langle A(\alpha)R(X,0), F\rangle}{\| F \|^2} = 1 \tag{6}$$

where $A(\alpha)$ is the matrix of the left-hand rotation by the angle α, $\langle\cdot,\cdot\rangle$ is the Euclidean scalar multiplication. Obviously, these equations can be solved in this order, that is, first to find the reaction force vector with a predetermined displacement direction averaged to the given load, and then determine the angle to which the given vector should be rotated in order to completely balance the applied force. Because of the monotonicity of the reaction, simple one-dimensional optimization methods can be used to solve the first equation, in particular, the half-division method.

For the case of a laminar isothermal flow of the lubricant in the bearing, it can also be shown that there is a single curve of dynamic equilibrium, determined to within an arbitrary factor in terms of

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accuracy having the dimension of dynamic viscosity. Indeed, for the case of constant viscosity, one can make a $p \to p_0\mu$ substitution of variables and solve the Reynolds equation (1) in the form:

$$\operatorname{div}\left(\rho h^3 \operatorname{grad} p_0\right) = 6\operatorname{div}(\rho h V_t) + 12\rho V_n$$

Then the reaction forces will be determined as

$$R_X = \int_\Omega -\mu p_0(\vec{x}) \cdot \cos\alpha(\vec{x}) dS = \mu \int_\Omega -p_0(\vec{x}) \cdot \cos\alpha(\vec{x}) dS = \mu R_X^0$$

$$R_Y = \int_\Omega -\mu p_0(\vec{x}) \cdot \sin\alpha(\vec{x}) dS = \mu \int_\Omega -p_0(\vec{x}) \cdot \sin\alpha(\vec{x}) dS = \mu R_Y^0$$

So, for a laminar isothermal flow, the shape of the dynamic equilibrium curves is determined only by the gap function and, therefore, is a only geometric characteristic of the bearing.

For the case of a conical bearing, it becomes necessary to consider a three-dimensional reaction vector, which makes it impossible to directly use the above-described decomposition technique. One can, however, get rid of this difficulty, if one observes that for each permissible axial displacement of the rotor there is a corresponding curve of mobile equilibrium in the cutting plane for a given projection of the external counterbalanced force F on it. So, for conical bearings, it is possible to define a surface that can reasonably be called a dynamic equilibrium surface, such that all radial curves of dynamic equilibrium for all permissible axial displacements of the rotor lie on it. On this surface, it is possible to determine the field of forces directed in the axial direction and which are the projection of reaction forces at the corresponding "equilibrium" points of reaction forces to the axis of the bearing. In this case, the problem of finding equilibrium points for a conical bearing can also be decomposed into the following steps:

1) form the surface of dynamic equilibrium Ω;

2) solve the equation $R(X,Y,Z) = -F$ with limitation $(X,Y,Z) \in \Omega$.

It is clear that every equilibrium point lies on the surface of mobile equilibrium. We note, however, that already at the stage of setting the problem, it becomes clear that it is impossible to speak in the general case of any continuous three-dimensional mobile equilibrium curve for a conical bearing. Moreover, the computational experiments carried out show that, in general, there is no reason for a conical bearing to assume the presence of *at least one* point in which both the radial and axial components of the applied force can be simultaneously balanced. A typical example of a mobile equilibrium surface and its corresponding field of axial reactions are shown in Figures 3-6.

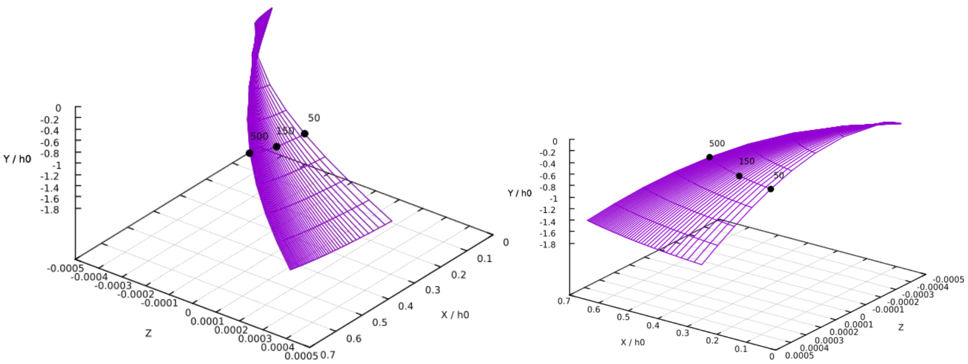


Figure 3. Surfaces of the dynamic equilibrium of the conical bearing; Lubricant: Water.

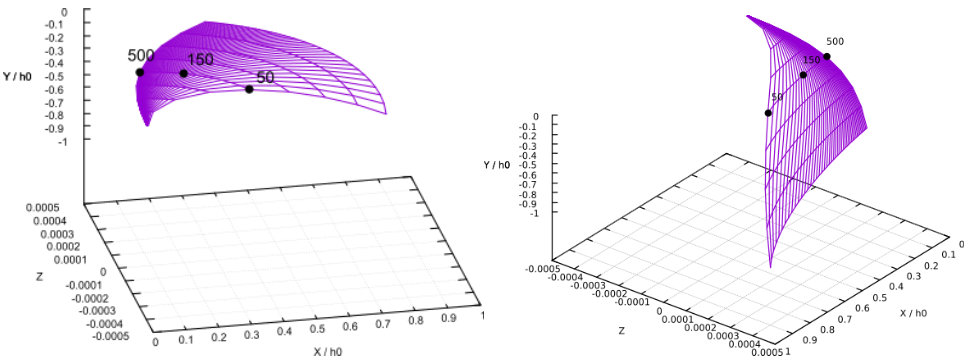
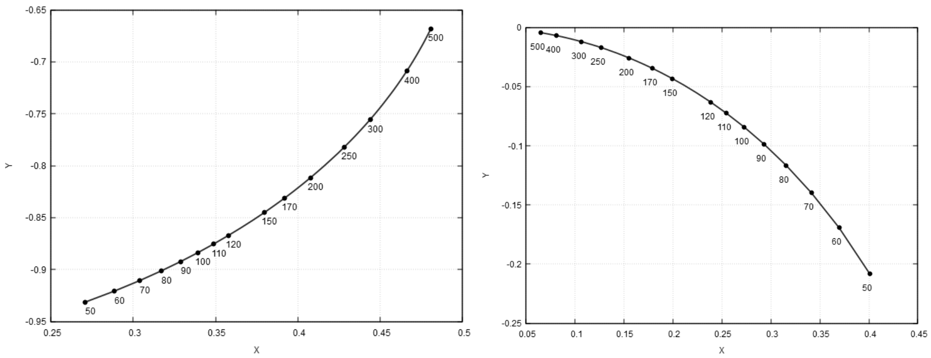
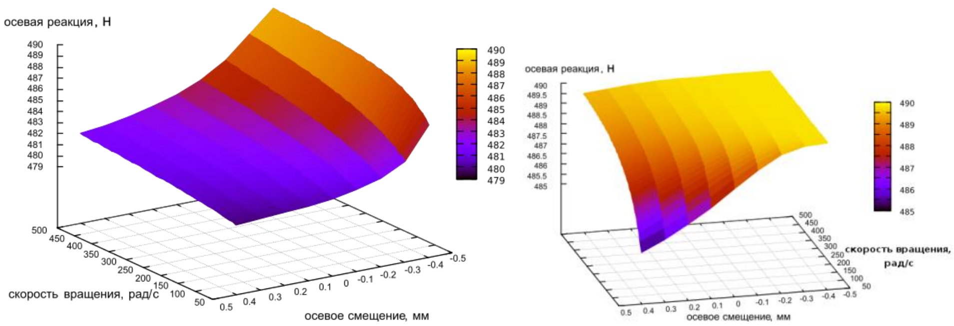


Figure 4. Surfaces of the dynamic equilibrium of the conical bearing; Lubricant: Oil TP-22.



(a) (b)

Figure 5. The curves of dynamic equilibrium in the absence of axial displacement: (a) water; (b) turbine oil TP-22.



(a) (b)

Figure 6. Distributions of axial reactions forces of a conical bearing: (a) water; (b) turbine oil TP-22.

The figure shows that the field of axial reaction forces is practically uniform. From this the following conclusion could be drawn: The conical bearing, by its design at a given speed of rotation ω, is able to stably balance only a certain set of external forces, for which F_Z lies in some sufficiently close proximity $\phi(F_X, F_Y, \omega, 0)$.

First, for a conical bearing, to a lesser extent than for the rest, a procedure for linearizing the reaction in the area of a certain point is suitable, and a description of the effect of the lubricating layer on the rotor through the stiffness and damping coefficients. Secondly, a small dispersion of the axial component of the reaction of the lubricating layer at points lying on the surface of mobile equilibrium indicates that the application of a constant axial force in the general case must lead either to a displacement of the rotor in the direction of increasing gap and loss of bearing's load capacity or to the contact of the surfaces of the rotor-bearing unit due to the inability to compensate the applied axial load. The study of these questions and the determination of parameters that ensure the operability and stability of the motion of the rotor in conical supports require the solution of the dynamic problem.

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Acknowledgments

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