Investigation of the torsional stiffness of flexible disc coupling

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Abstract. Calculation of flexible coupling torsional stiffness is required when analyzing the torsional vibrations of the reciprocating machinery train. While having the lowest torsional stiffness of all the elements of the train, flexible coupling has a significant influence on the natural frequencies of torsional vibration. However, considering structural complexity of coupling, precise definition of its torsional stiffness is quite a difficult task. The paper presents a method for calculating the torsional stiffness of flexible disc coupling based on the study of its finite element model response under the action of torque. The analysis of the basic parameters that quantitatively and qualitatively affect the coupling torsional stiffness has been also provided. The results of the calculation as well as model adequacy, sufficient for practical application, have been confirmed at the experimental measurement of flexible disc coupling torsional stiffness. The obtained elastic characteristics (dependences of applied torque and torsional stiffness versus twist angle) are nonlinear in the initial stage of loading. This feature should be taken into account when creating reliable mathematical models of torsional vibrations of reciprocating machinery trains containing flexible disc couplings.

1. Introduction
When conducting the torsional vibration analysis of the reciprocating compressor train, there arose need for precise determination of the torsional stiffness of flexible disc coupling that connects engine and compressor rotors.

Flexible coupling often has the lowest torsional stiffness of all the elements of the train that results in a significant effect on the natural frequencies of torsional vibration. However, due to the complex design, it is difficult to accurately determine the torsional stiffness of the coupling [1]. This article is devoted to the description of this problem solution.

Torsional stiffness (c, Nm/rad) is an elastic property that is characterized by the ratio of the applied torque to the twist angle. In general, the coupling torsional stiffness is defined as a derivative of torque with respect to twist angle. The reciprocal of the torsional stiffness is the torsional compliance (e, rad/Nm).

$$c = \frac{1}{e} = \frac{dM}{d\varphi} \tag{1}$$

A distinction is made between linear and nonlinear couplings. For nonlinear couplings, the torsional stiffness is not constant but is a function of the torque magnitude. Typically, a torque vs. twist angle curve is obtained by fixing one end of the coupling and measuring the twist angle of another end with gradual torque increase.

The most accurate coupling torque vs. twist angle curve can be obtained only experimentally. However, this rather expensive process requires special equipment, as well as time for adjustment and measurements.

Flexible disc coupling whose design and operation are well described in [2] consists of different structural elements that deform in complex ways under load. Accordingly, the coupling torsional stiffness depends on a large number of parameters, so analytical calculation is not possible. A more promising approach is a numerical calculation based on the finite element method (FEM). The development of calculation technique and creation of adequate model will enable to obtain elastic characteristics of flexible disc couplings in a very short time.

The relevance of the work is also conditioned by the fact that disc couplings are the most common flexible element couplings due to their excellent performance, high misalignment capacity, compact design, minimal maintenance and relatively low cost. Furthermore, International Standard API 671 specifies metallic flexible-element couplings as the primary type permissible for special-purpose application. Such type of equipment includes high-torque and/or speed machines operating continuously for extended periods, as well as in adverse conditions [3].

2. Literature survey

There is a small number of articles devoted to modeling and numerical analysis of couplings with metallic flexible elements.

Dobre Daniel and colleagues investigated the stress-strain state of flexible membrane coupling under the influence of torque, centrifugal force, axial and angular misalignment [4]. JongHun Kang and his collaborators analyzed the effect of torsional stiffness of flexible disc coupling elements on the natural frequencies of the wind turbine shaft torsional vibrations [5]. In particular, the article gives the calculation results of torsional stiffness of flexible elements packs with different thickness and number of discs. However, both works do not cover the information on the parameters of finite element coupling models used for calculation.

The following two papers are directly dedicated to the modeling of flexible disc couplings and calculation of their torsional stiffness. However, the model proposed by Alex B. Francis in [6] does not take into account two important factors that significantly affect coupling torsional stiffness. They are the separation of flexible elements during coupling deformation from the applied load and bolted joints preload.

A group of scientists headed by Bin Zhao conducted the most complex numerical and experimental investigation of flexible disc coupling torsional stiffness [7]. But due to accounting for clearances between bolts and disc pack holes, their model is too complicated for practical application. In addition, clearances in the coupling must be kept to a minimum in order to avoid the increased imbalance and deterioration of the vibrational state of the unit.

The paper presents a method for calculating the torsional stiffness of flexible disc coupling based on the study of its finite element model response under the action of torque. The analysis of the basic parameters that quantitatively and qualitatively affect the coupling torsional stiffness has been also provided. The results of the calculation as well as model adequacy, sufficient for practical application, have been confirmed at the experimental measurement of flexible disc coupling torsional stiffness.

3. Numerical investigation of flexible disc coupling torsional stiffness

ANSYS Mechanical 15.0 simulation software suite was used as a finite element analysis tool.

The investigated flexible disc coupling consists of two hubs, two disc packs (26 circular discs with individual disc thickness of 0.5 mm), a spacer, as well as connecting bolts and nuts. The 3D model of the coupling is shown in Figure 1 and its basic design parameters are shown in Figure 2.

3.1. Finite element model of the coupling

When creating the finite element model, the coupling symmetry is used to reduce computational costs. Since the design is symmetrical to the middle of the spacer, first we define a twist angle of half a

coupling. In addition, the coupling has 3-fold rotational symmetry, so it is enough to model only 1/3 of it (a sector with a 120° central angle) and apply cyclic symmetry boundary condition. The acquired finite element coupling model is shown in Figure 3. The mesh is generated in an automatic mode and additionally improved in contact areas.
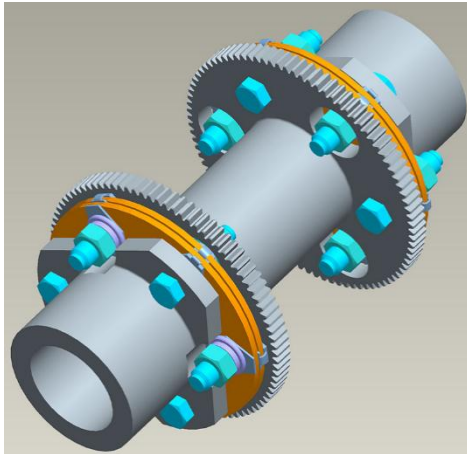

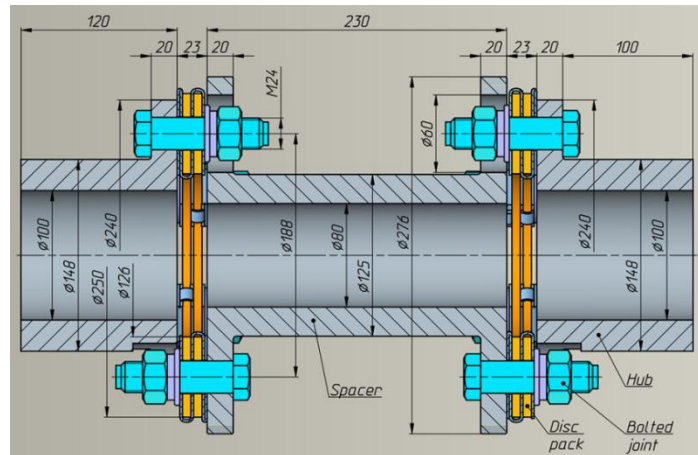**Figure 1.** 3D model of the flexible disc coupling.

Figure 2. Basic design parameters of flexible disc coupling.

3.2. Making contacts, fixing and loading
Three numerical models were created to show the basic parameters that affect flexible disc coupling torsional stiffness. The models differ in contact interaction between the coupling elements and bolted joints preload. Table 1 shows the differences in coupling models.

Table 1. The parameters of coupling numerical models.

Model number	Primary contact type	Friction coefficient	Bolted joints preload F_p, N
1	"No Separation"	-	-
2	"Frictional"	0.15	10
3	"Frictional"	0.15	120830

Notes:
- in all models a bolt and a nut are rigidly connected (a contact type "Bonded");
- for model No. 2 a minimum bolt preload is specified to improve the calculation convergence;
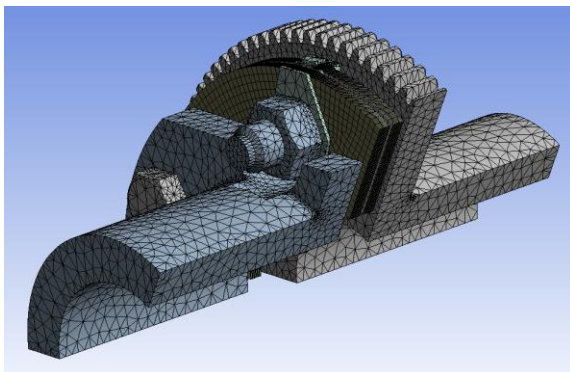- the initial clearances are absent in all frictional contacts.

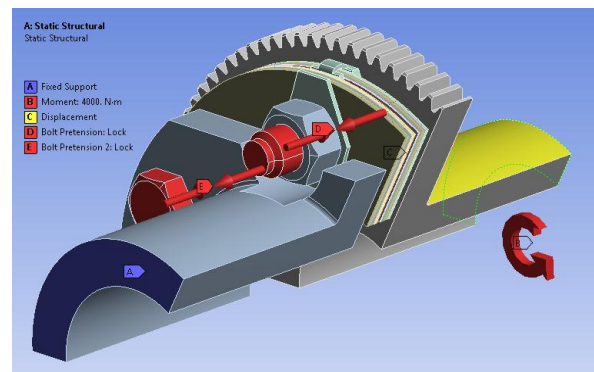**Figure 3.** Finite element coupling model.

Figure 4. Constraints and loads arrangement.

All the models have the same constraints: the end surface of the hub is rigidly fixed, the radial motions of spacer cylindrical surface are ruled out. Figure 4 shows constraints and loads arrangement.

The main load in all models is a 4000 Nm torque (1/3 of the maximum torque) applied to the spacer end surface. The torque loading is divided into 20 substeps. The values of the coupling twist angle are calculated at each substep. The coupling torque vs. twist angle curve is plotted on the basis of the obtained values.

The bolt preload is calculated from the tightening torque on the formula [8]:

$$F_p = T / (Kd) \qquad (2)$$

where

$T = 580$ Nm - tightening torque;

$K = 0.2$ - nut factor (when friction coefficient is 0.15);

$d = 0.024$ m - nominal diameter.

Thus,

$$F_p = 580 / (0.2 \cdot 0.024) = 120830 \text{ (N)}$$

3.3. Obtaining results

Figures 5 and 6 show the calculation results: coupling and disc pack deformation for coupling model No. 3 under maximum torque.
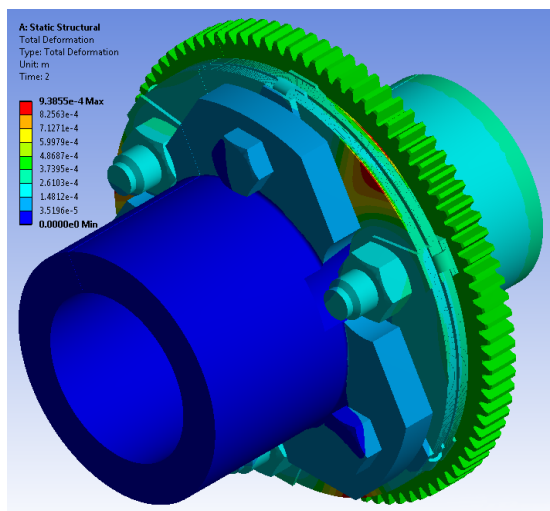

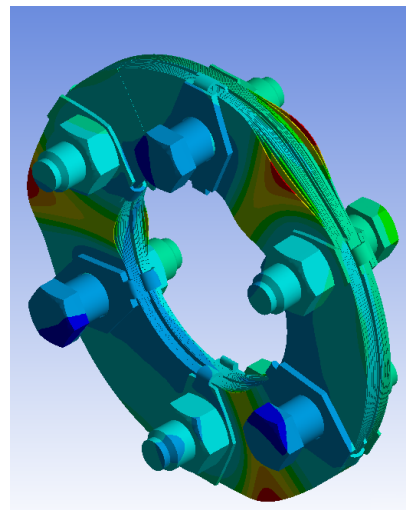**Figure 5.** Deformation of the coupling (true scale).

Figure 6. Deformation of the disc pack (10x deformation scale).

The tool "Deformation Probe" is used to obtain the values of the coupling twist angle. It determines the total displacement of the point located on the outer circumference of the spacer end face at each load substep (see Figure 7).

The twist angle of half a coupling is calculated by the total point displacement at each load substep:

$$\varphi_i^* = \frac{2\Delta_i}{D_s} \qquad (3)$$

where

φ_i^* - the twist angle of half a coupling at the i-th load substep;

Δ_i - the total point displacement at the i-th load substep;

$D_s = 0.125$ m - outer diameter of the spacer cylindrical area.
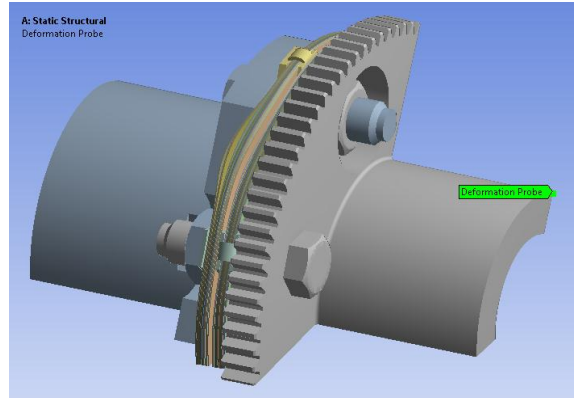
Figure 7. "Deformation Probe" point position.

To make the further use convenient and to compare with the experimental data, the total coupling torsional stiffness is determined excluding the torsional stiffness of hubs cylindrical areas.

The total coupling twist angle at each load substep is as follows:

$$\varphi_i = 2(\varphi_i^* - \varphi_c) = 2(\varphi_i^* - M_i \cdot e_c) \qquad (4)$$

where

M_i - applied torque at the *i*-th load substep;

e_c - torsional compliance of the hub cylindrical area.

$$e_c = \frac{32}{\pi G} \cdot \frac{l_c}{D_c^{\,4} - d_c^{\,4}} \qquad (5)$$

where

$l_c = 0.1\,\mathrm{m}$, $D_c = 0.148\,\mathrm{m}$, $d_c = 0.1\,\mathrm{m}$ - length, outer and inner diameters of the hub cylindrical area; $G = 7.8 \cdot 10^{10}\,\mathrm{Pa}$ - shear modulus.

$$e_c = \frac{32}{3.14 \cdot 7.8 \cdot 10^{10}} \cdot \frac{l_c}{0.148^4 - 0.1^4} = 3.438 \cdot 10^{-8}\ (\mathrm{rad/Nm})$$

In order to find the analytical expression for the coupling torque vs. twist angle curve, the values calculated by the equation (4) are approximated by a polynomial function. Then the coupling torsional stiffness vs. twist angle curve is calculated on the formula (1). The obtained curves for each numerical model will be presented in section 5.

4. Experimental determination of flexible disc coupling torsional stiffness
To determine which of the three numerical models is the most appropriate for torsional stiffness analysis of flexible disc coupling, experimental twist angle measurement of a real coupling was carried out. For this purpose, we have developed methodology that does not require coupling dismantling and allows to take measurements directly on the compressor unit.

The torque loading and measurement was conducted by using two specially manufactured levers with a half-flange mount (see Figure 8). The first lever, mounted on the coupling flange from engine side, was being gradually loaded by means of a jack-screw. Thus, the torque was created. The second lever was attached to the coupling hub flange on the compressor side and was rigidly connected to the 'ground' through the electronic dynamometer sensor. Thus, the second end of the coupling was fixed, while the sensor was measuring the applied torque.

The measurement of the coupling twist angle was taken by using two laser indicators mounted on the coupling hubs (see Figure 9). Laser beams were projected onto a screen with millimeter marking.

The screen was installed at a distance of 6 meters from the coupling axis. With increasing torque, there was a vertical movement of laser beam points on the screen. Movement of indicator points was recorded for each load step.

Before starting the measurements, the tentative loading (up to the value of the expected maximum torque) and unloading of the coupling were carried out to remove possible clearances between its elements.

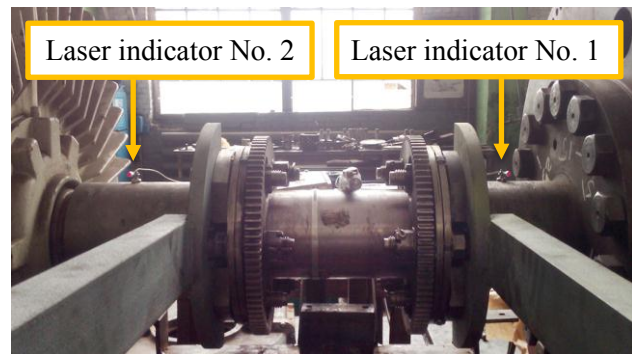**Figure 8.** General view of the experimental facility.

Figure 9. Laser indicators arrangement.

Obviously, laser beam No. 2 displacement is greater than laser beam No. 1 displacement by the coupling twist angle value (see Figure 10).
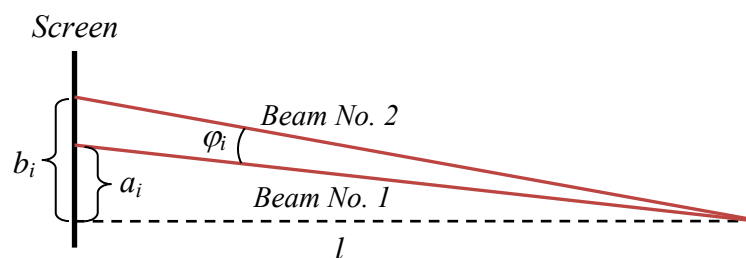

Figure 10. The scheme for defining the coupling twist angle.

The coupling twist angle on each load step is calculated by the formula:

$$\varphi_i = arctg(b_i\,/\,l) - arctg(a_i\,/\,l) \tag{6}$$

where

φ_i - the coupling twist angle at the i-th load step;

a_i, b_i - laser beam No. 1 and laser beam No. 2 displacements at the i-th load step;

l - the distance from the screen to coupling axis.

To find the analytical expression for the experimental coupling torque vs. twist angle curve, the values calculated by equation (6) are approximated by a polynomial function. Then the coupling torsional stiffness vs. twist angle curve is calculated according to formula (1).

5. A comparative analysis of the findings
The curves of the dependence of applied torque versus twist angle obtained for the numerical models, as well as experimentally, are presented in Figure 11. The curves of the dependence of coupling torsional stiffness versus twist angle obtained for the numerical models, as well as on the experimental data, are shown in Figure 12.

As seen from these graphs, the elastic characteristics of the coupling model No. 3 almost correspond to the real coupling characteristics. This proves to adequacy of this model and its acceptability for calculating torsional stiffness of flexible disc couplings with the other design parameters.

Coupling model No. 1 reproduces the model introduced in [6]. When specifying the contact interaction between the elements of this model, a linear contact type "No Separation" was employed. It assumes mutual sliding of model elements (without friction), but prohibits their separation. Thus, this model does not allow for separation of flexible elements when the coupling deforms from the applied load. This entails an artificial increase in torsional stiffness. Also, this model gives a linear torque vs. twist angle curve in view of the linearity of the contacts used.

When defining the contact interaction between the elements of coupling models No. 2 and No. 3, a more complex nonlinear contact type "Frictional" was used. It allows for friction between the model elements and, accordingly, assumes their separation. However, the second model does not consider the bolted joints preload. Comparing the curves obtained for these numerical models, we can conclude that it is very important to take into account the bolted joints preload. This option increases the torsional stiffness of flexible disc coupling, as well as gives the non-linear nature of the elastic characteristics at the initial stage of loading.
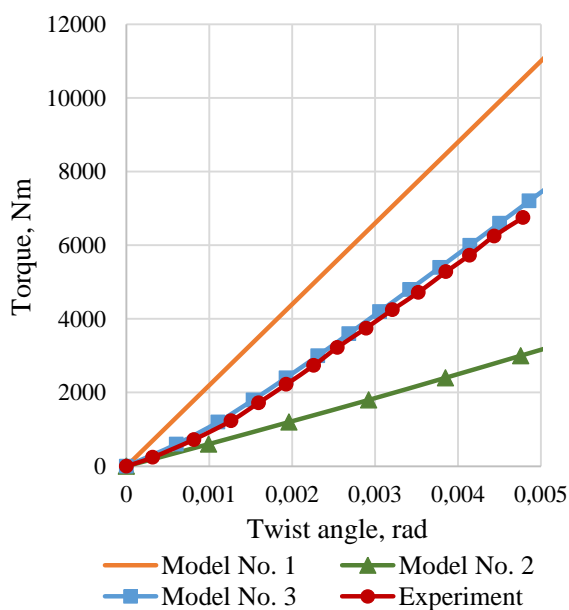

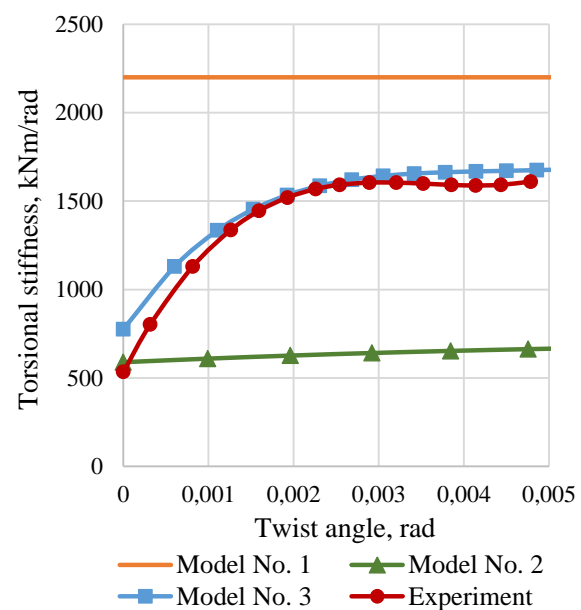**Figure 11.** Coupling torque vs. twist angle curves.

Figure 12. Coupling torsional stiffness vs. twist angle curves.

6. Conclusions

1. A method for calculating torsional stiffness of flexible disc coupling based on the study of its finite element model response under the action of torque has been developed.

2. When creating reliable numerical model of flexible disc coupling it is necessary to allow for friction between the coupling elements as well as the bolted joints preload.

3. The calculation results and sufficient for practical application adequacy of the numerical model are confirmed by the experimental measurement of flexible disc coupling torsional stiffness.

4. The obtained elastic characteristics (dependences of applied torque and torsional stiffness versus twist angle) are nonlinear in the initial loading stage. This feature should be taken into account when creating reliable mathematical models of torsional vibrations of reciprocating machinery trains that involve flexible disc couplings

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