

# Simulation of Vortex Flow in Centrifugal Pump by Method of Hydrodynamic Features

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**Abstract.** A numerical experiment performed by ECM based on different mathematic models and algorithms permits to investigate and evaluate internal and external dynamics of centrifugal pumps being operated in regimes of a flow without separation and separation one around bodies with excessive degree of reliability. This paper researches simulation of the flow within a centrifugal pump impeller by the method of hydrodynamic features. Streamlines and vortex lines were developed. Their relative positions were studied. Friction losses and losses induced by vortex dissipation have been evaluated in dependence to a flow rate.

## 1. Introduction

When simulating of a vortex flow within different channels or confined spaces, vortex distribution is usually considered to be final and distributed one at the internal boundary on which there is a tangential discontinuity of a velocity vector. A magnitude of a tangential velocity jump is defined according to the integral equation which provides the condition of a flow lack through the body surface.

The method of discrete vortices developed to simulate an ideal liquid [1] is the basic vortex method. This method used successfully for numerical calculations of an unsteady flow around bodies with a tail of a vortex wake at sharp edges [2]. The method of discrete vortices in conjunction with a boundary layer model was used to simulate the separation flow at a smooth surface [3, 4]. Schemes that simulate generation of vorticity at the surface of streamlined bodies are shown in [5, 7].

## 2. The research purpose

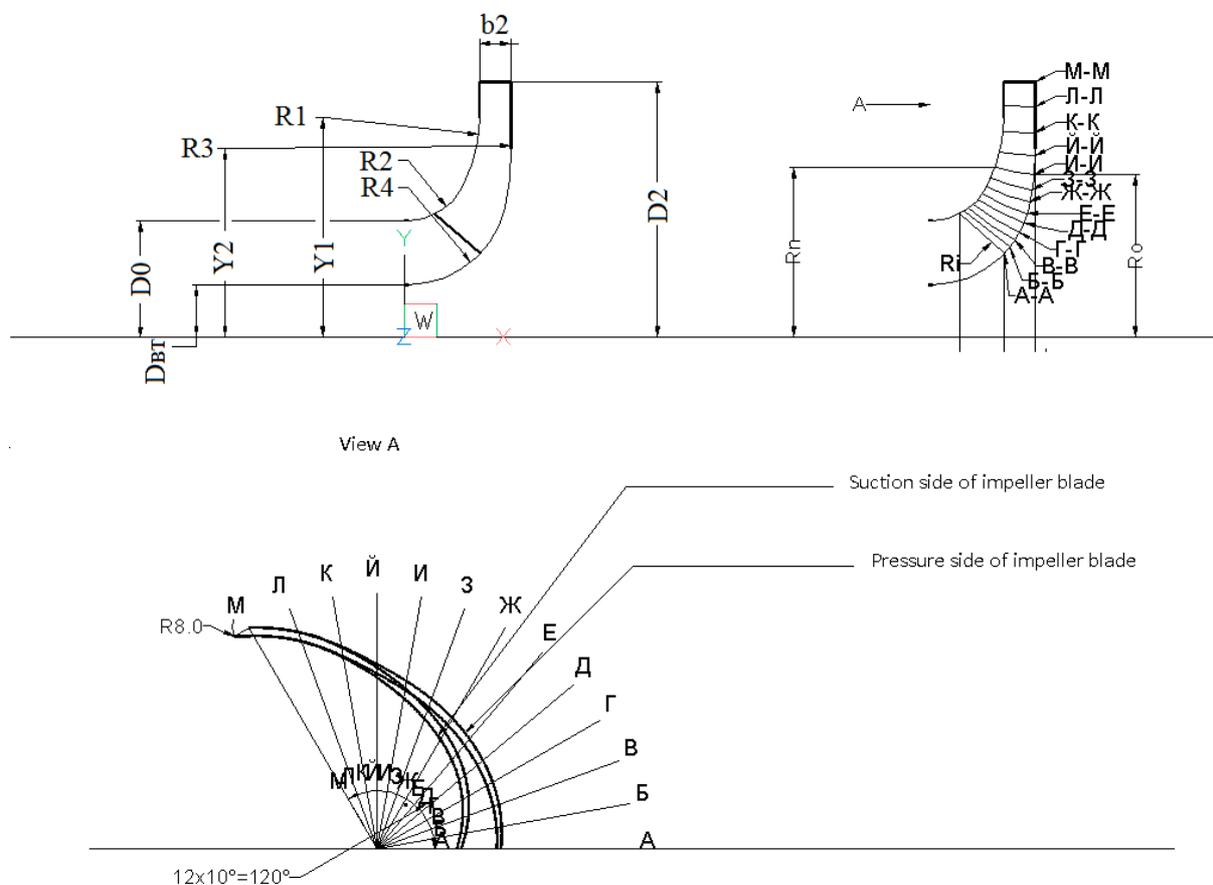
Friction losses and losses induced by dissipation of a vortex in dependence to the flow rate are to be evaluated according to the results of 3D simulation of the vortex flow of an ideal incompressible liquid.

For research purposes the centrifugal pump impeller for parameters of  $Q=130 \text{ m}^3/\text{h}$ ,  $H=80 \text{ m}$ ,  $n=2979 \text{ rpm}$  was simulated by inversion proprietary software.

The design technique that was used to simulate an impeller blade according to defined flow pattern is set out in studies [7, 8].

Hereinafter, Figure 1 gives a software-generated line drawing of the impeller being investigated.





**Figure 1.** Software-generated line drawing of impeller being investigated.

### 3. Direct problem

A vortex model of the ideal incompressible liquid based on studies [10-12] has been used for simulation of 3D-flow.

A vortex frame and point dipoles were the main hydrodynamic features being in use, their descriptions are given in [13].

The flow within centrifugal pump impeller with a vaneless diffuser was under consideration.

Boundary conditions.

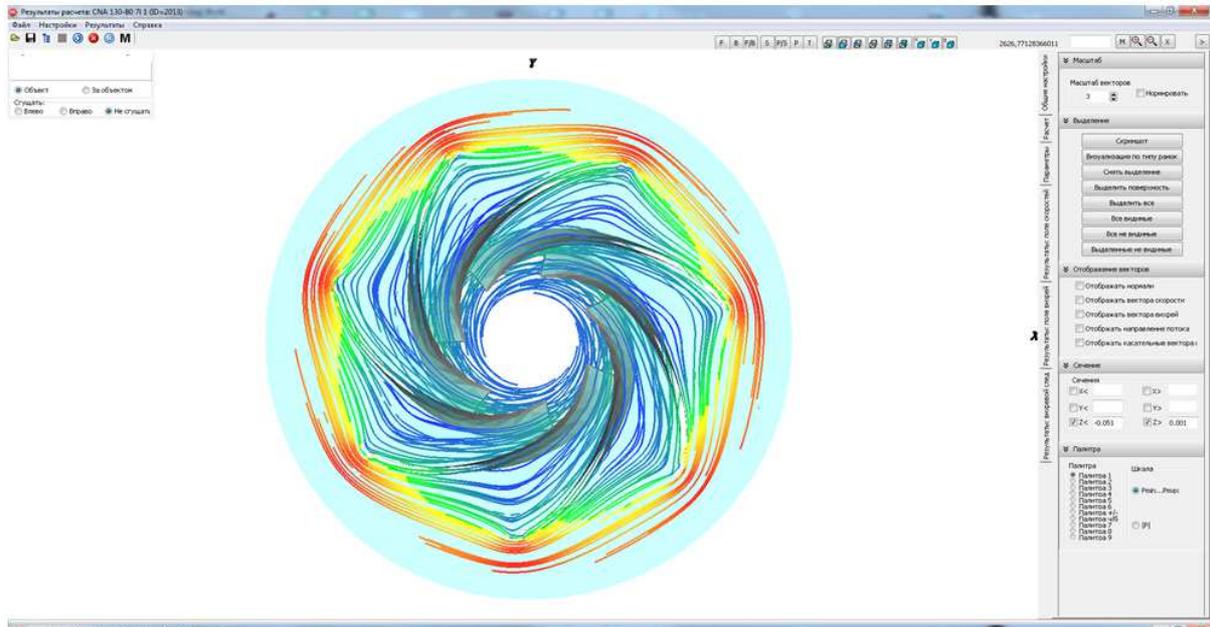
The uniform velocity distribution corresponding to the required flow rate was specified at the impeller inlet.

Since a backflow was supposed a condition of an open flow (opening) was specified at the impeller outlet.

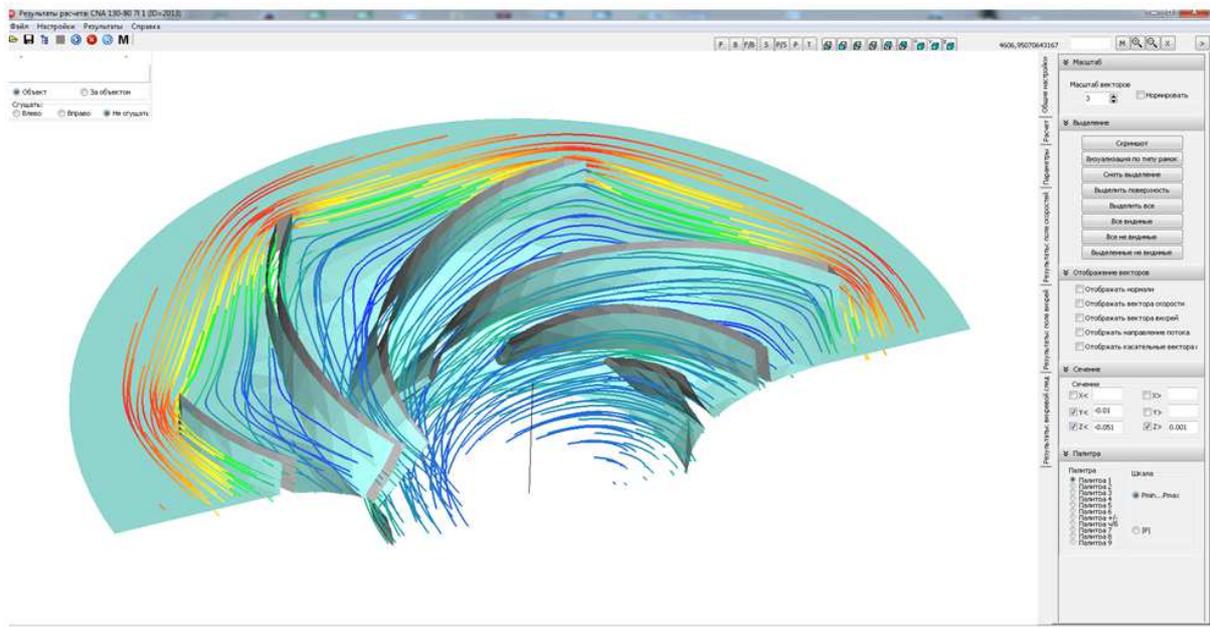
The impermeability boundary condition was specified at solid surface rotating with specified revolutions. The no-slip boundary condition was not specified.

Figures 2, 3 give reference streamlines in the longitudinal section throughout the height of  $b/2$ . It is properly observed the effect of “no-slipping” of streamlines to the suction side of impeller blade and “repulsing” from the pressure side within interblade channels, that is also simulated by CFD.

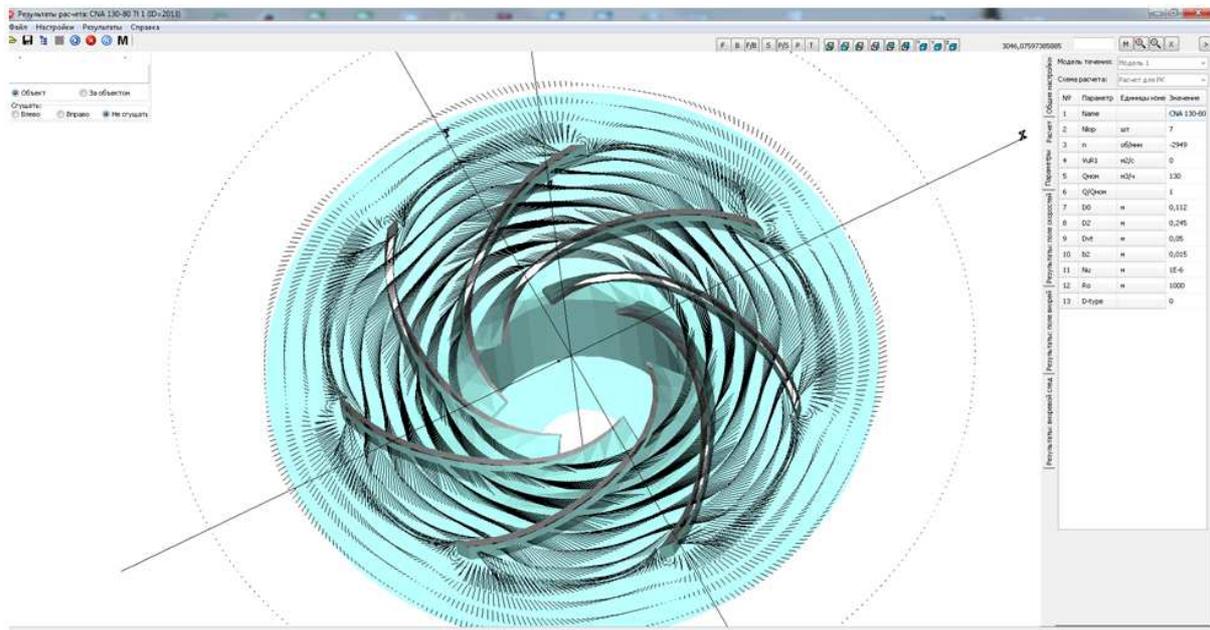
Figures 4, 5 give distribution diagrams of absolute and relative velocities within the interblade channels. It is observed significant areas of the backflow that is not realized as a part of the potential flow only.



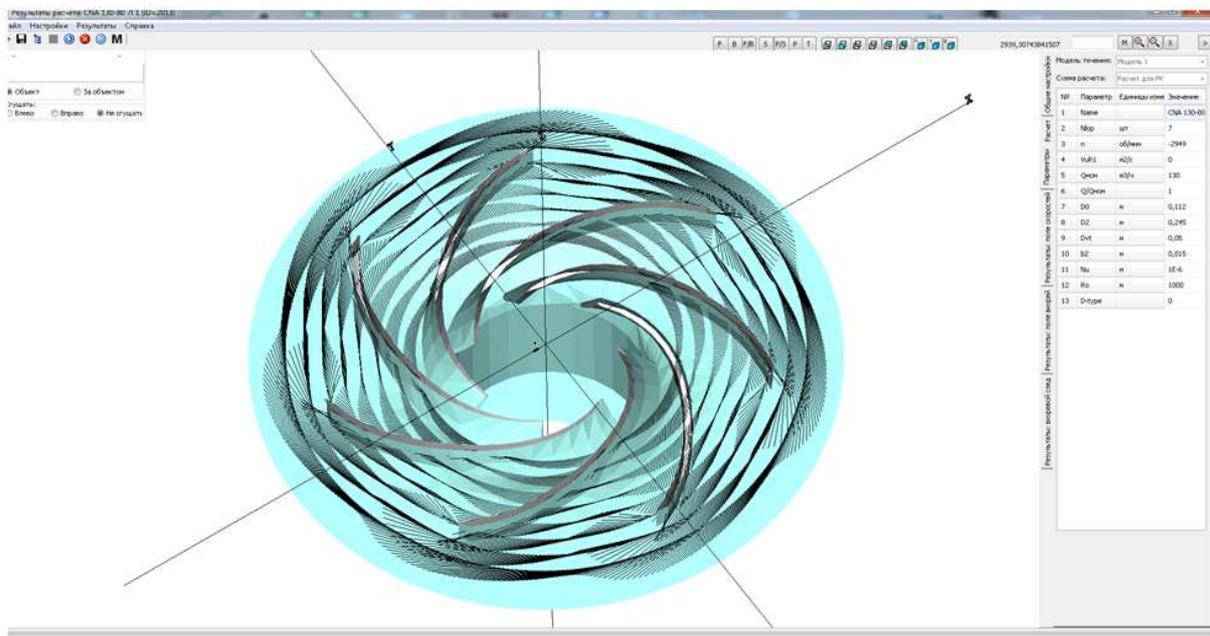
**Figure 2.** Reference streamlines at rated flow regime.



**Figure 3.** Reference streamlines at rated flow regime (a fragment).



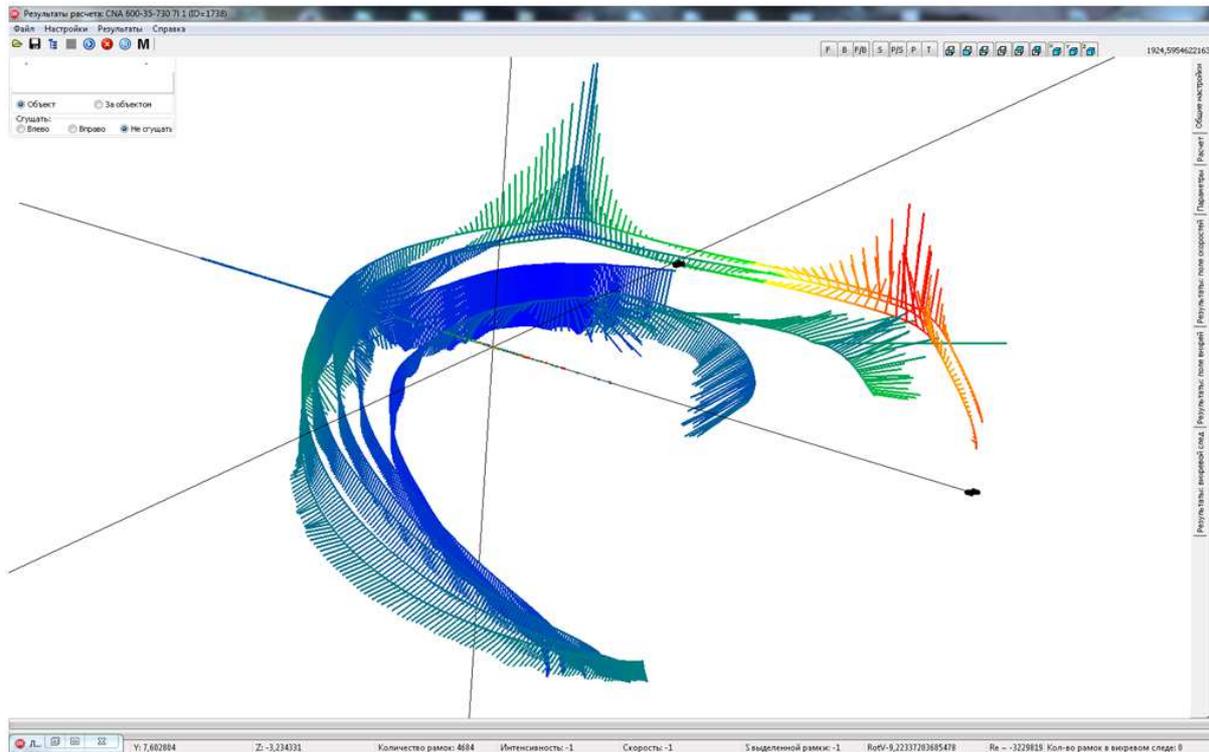
**Figure 4.** Distribution diagram of absolute velocity vectors within interblade channels.



**Figure 5.** Distribution diagram of relative velocity vectors within interblade channels.

Hereinafter, Figure 6 gives some separately depicted streamlines which run from the impeller inlet to outlet to be viewed more conveniently without impeller geometry. At every computational point of streamlines a vortex vector at this point was calculated and shown by a segment being oriented in direction of vortex vector. The segment length is proportional to a modulus of the vortex.

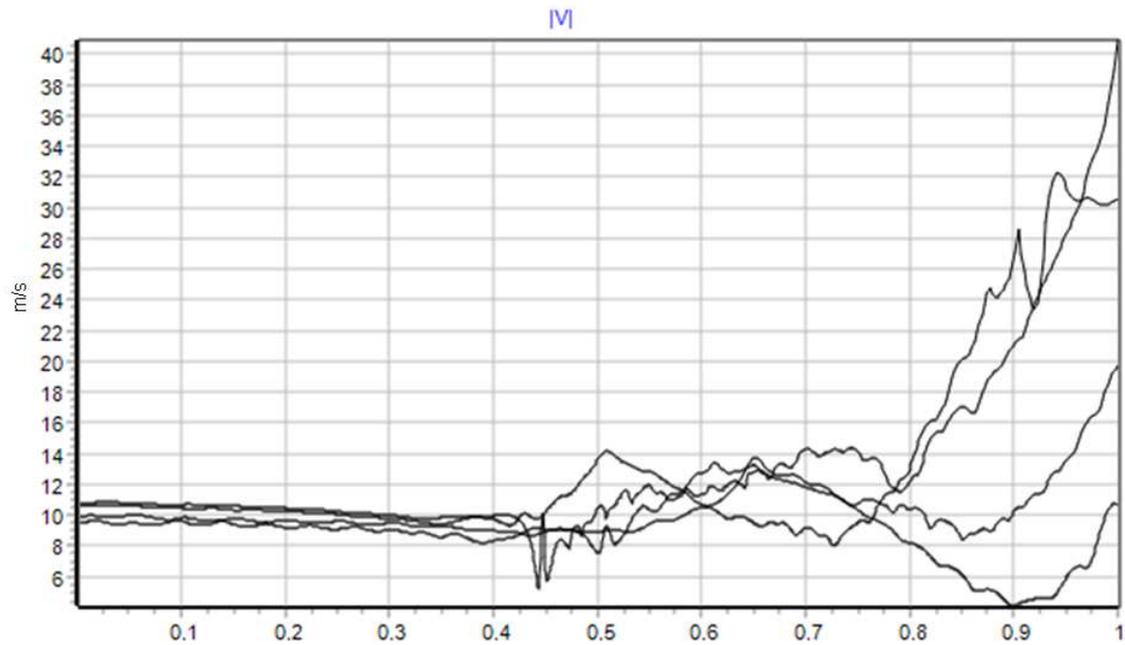
Surges of vortices at the impeller outlet correspond to positions of impeller blade trailing edges (at the design moment of time), where strong trailing vortices form. It is also observed that the particle rotation direction reverses the direction of motion.



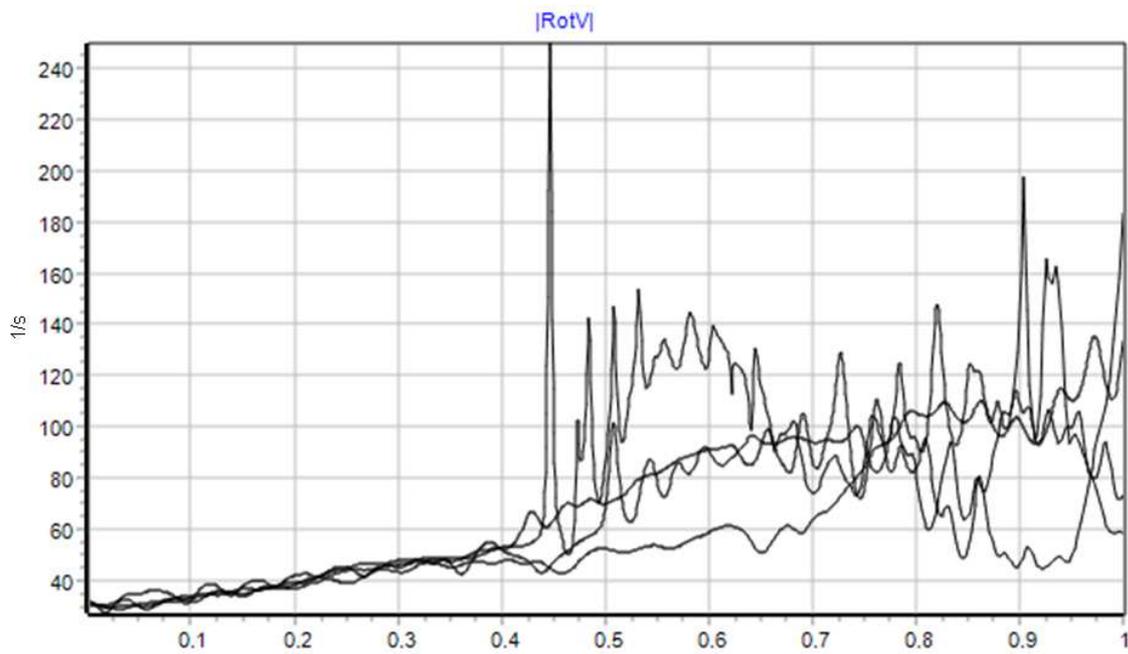
**Figure 6.** Streamlines directed from inlet section to impeller outlet with relative positions of vortex lines on them.

Figure 7 gives the graph of velocity modulus variance along the streamlines, Figure 8 - the graph of vortex modulus variance along the streamlines.

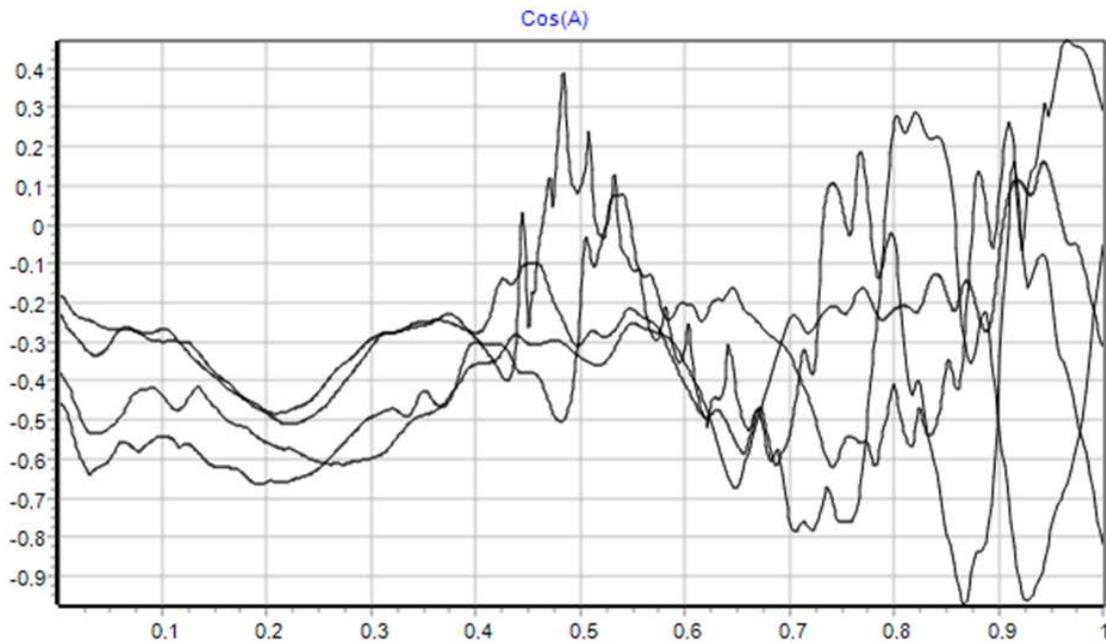
Figure 9 shows the graph of variance of angle cosine between the streamlines and corresponding vortex lines along the streamlines. Energy transfer in the centrifugal pump impeller occurs proportionally to the value of the dot product  $V \cdot \text{Rot}V$ . Therefore at the sections, where an angle between the direction of the vortex vector (streamline) and relative vortex vector is the smaller, the smaller amount of energy could be taken by liquid from the impeller. A swirling flow (a vortex extends in velocity vector direction) doesn't give and take energy from outside. The swirling flow only transfers energy.



**Figure 7.** Graph of velocity modulus variance along streamlines.



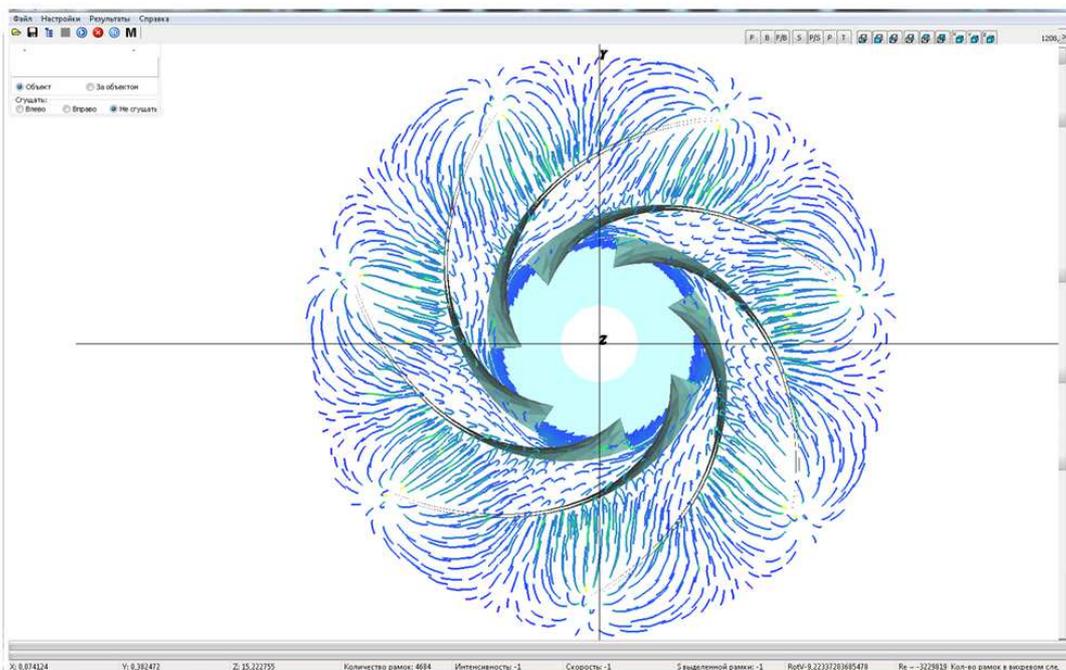
**Figure 8.** Graph of vortex modulus variance along streamlines.



**Figure 9.-** Graph of variance of angle cosine between streamlines and corresponding vortex lines along streamlines.

Figure 10 shows a pattern of vortex lines in the longitudinal section throughout the height of  $b_2/2$  at rated flow regime.

Awareness of a vortex field within a domain under consideration gives the following application results – evaluation of losses due to friction on a solid surface (including a smooth surface) and evaluation of losses with dissipation of vortices within the internal region of the flow emerging due to the fact that the flow is the vortex flow rather than the potential one.



**Figure 10.** Reference vortex lines in longitudinal section at rated flow regime.

#### 4. Friction stress analysis

Studies [14, 15] show that force  $F$  acting on a streamlined surface without other liquid-acted forces is related with hydrodynamic momentum  $I$  by the formula

$$F = -\frac{dI}{dt} - \rho \frac{d}{dt} \int r \times (V \times n) dl \tag{1}$$

where hydrodynamic momentum

$$I = \rho \int_s r \times \Omega ds \tag{2}$$

If we differentiate (1) when body moves uniform under the no-slip boundary conditions, we should deduce

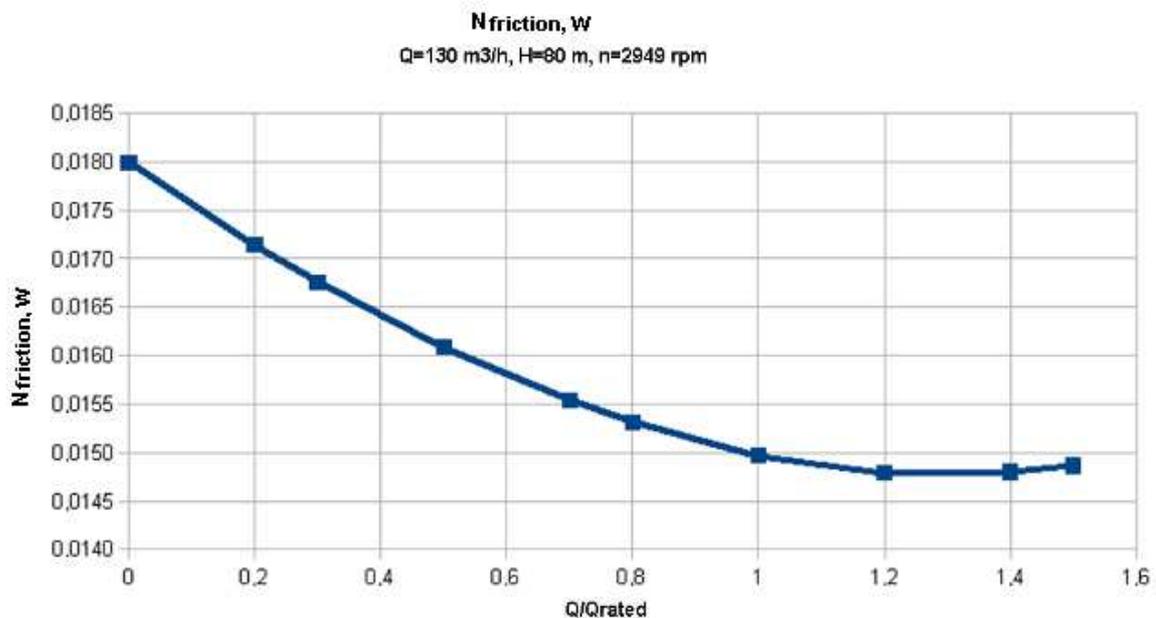
$$F = -\rho \int_l r \times J dl - \rho \int_\Gamma V_d \times dG \tag{3}$$

The first item represents a resultant of a pressure force, the addend represents a resultant of friction forces. Friction stress  $\tau_w$  is expressed by the formula:

The first item represents a resultant of a pressure force, the addend represents a resultant of friction forces. Friction stress  $\tau_w$  is expressed by the formula:

$$\tau_w(l) = -\rho \int_\Gamma I_w r(l), r(G) \times dG \tag{4}$$

Figure 11 shows a change of friction force total energy (resistance forces) per a time unit, that acts on an impeller depending on the flow rate.



**Figure 11.** Change of friction force total energy per a time unit, that acts on an impeller in dependence to the flow rate.

#### 5. Energy Dissipation

The work of all forces acting on mass the liquid in finite volume being defined by dot product of the stress tensor and strain velocity tensor for incompressible liquid is determined by the equation [16]

$$N_{\partial uc} = \mu \left[ \begin{aligned} &2 \left( \frac{\partial V_x}{\partial x} \right)^2 + 2 \left( \frac{\partial V_y}{\partial y} \right)^2 + 2 \left( \frac{\partial V_z}{\partial z} \right)^2 + \\ &\left( \frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y} \right)^2 + \left( \frac{\partial V_z}{\partial y} + \frac{\partial V_y}{\partial z} \right)^2 + \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right)^2 \end{aligned} \right] \quad (5)$$

The right-hand side of the equation (5) is always positive except for the case when all derivatives of velocities become zero in the coordinates. Therefore, a viscous incompressible liquid flows without dissipation of mechanical energy only in the case of absence of particle deformations, i.e. when a liquid moves like a solid body. At all other reasons the losses of mechanical energy occur when the liquid is flowing. When subtracted the term

$$2\mu \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)^2 = 0$$

from the right-side and left-side of equation (5) and added components of the vortex we should deduce:

$$N_{\partial uc} = 4\mu \left( \omega_x^2 + \omega_y^2 + \omega_z^2 \right) - 4\mu \left( \begin{aligned} &\frac{\partial V_y}{\partial y} \frac{\partial V_z}{\partial z} - \frac{\partial V_y}{\partial z} \frac{\partial V_z}{\partial y} + \frac{\partial V_z}{\partial z} \frac{\partial V_x}{\partial x} - \\ &\frac{\partial V_z}{\partial x} \frac{\partial V_x}{\partial z} + \frac{\partial V_x}{\partial x} \frac{\partial V_y}{\partial y} - \frac{\partial V_x}{\partial y} \frac{\partial V_y}{\partial x} \end{aligned} \right) \quad (6)$$

When multiplied the right-side and left-side by volume element  $d\tau$  and performed the integration of the whole volume we should obtain an amount of mechanical energy being dissipated per a time unit in finite volume  $\tau$ .

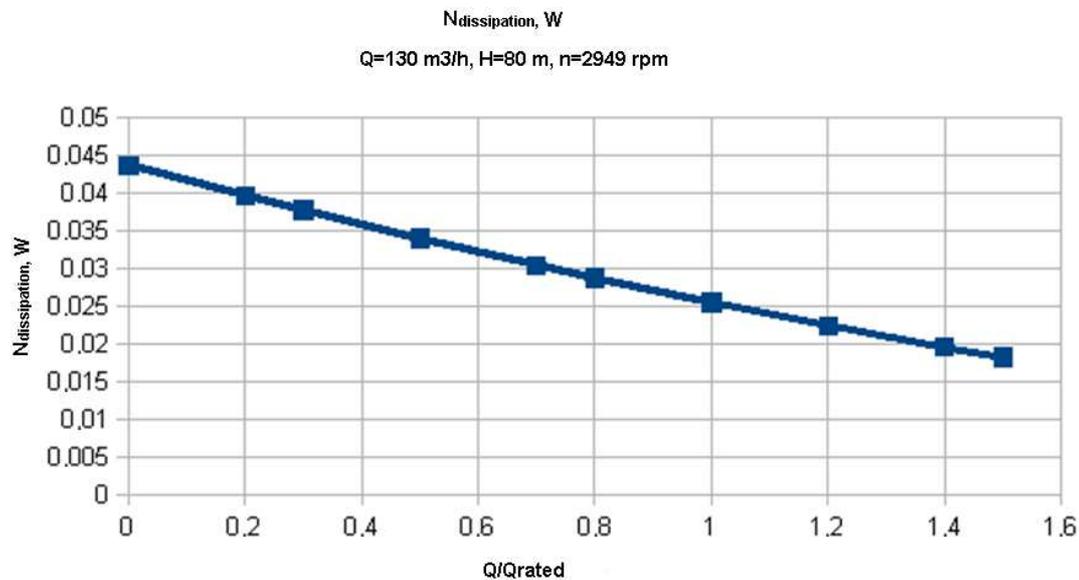
$$\begin{aligned} \iiint_{\tau} N_{\partial uc} d\tau &= 4\mu \iiint_{\tau} \left( \omega_x^2 + \omega_y^2 + \omega_z^2 \right) d\tau - \\ &- 4\mu \iiint_{\tau} \left( \begin{aligned} &\frac{\partial V_y}{\partial y} \frac{\partial V_z}{\partial z} - \frac{\partial V_y}{\partial z} \frac{\partial V_z}{\partial y} + \frac{\partial V_z}{\partial z} \frac{\partial V_x}{\partial x} - \\ &\frac{\partial V_z}{\partial x} \frac{\partial V_x}{\partial z} + \frac{\partial V_x}{\partial x} \frac{\partial V_y}{\partial y} - \frac{\partial V_x}{\partial y} \frac{\partial V_y}{\partial x} \end{aligned} \right) d\tau \end{aligned} \quad (7)$$

If boundaries of the volume  $\tau$  are fixed, solid boundaries whereon the velocities will become zero due to no-slip condition for a vector projection upon integration by parts we deduce:

$$\iiint_{\tau} N_{\partial uc} d\tau = 4\mu \iiint_{L} \left( \omega_x^2 + \omega_y^2 + \omega_z^2 \right) d\tau \quad (8)$$

Therefore, with the motion of the incompressible liquid enclosed in a fixed volume, the total amount of mechanical energy being dissipated per second will depend only on vortex intensity within the volume. This total amount of mechanical energy will represent in the form (8).

Figure 12 shows the graph of variance of energy losses per a time unit within an impeller under consideration depending on the flow rate due to dissipation of vortices. Energy losses decrease with increasing of the flow rate.



**Figure 12.** Change of energy losses per a time unit in the impeller under consideration in dependence to the flow rate due to vortex dissipation.

## 6. Conclusions

- 3D-simulation of the vortex flow of incompressible ideal fluid was performed within centrifugal pump impeller designed for specified flow pattern.
- The velocity fields, streamlines, vortex fields have been calculated.
- Losses induced by friction and vortex dissipation have been evaluated versus the flow rate.
- Obtained results and procedure could be used for target retrofit of hydrodynamic qualities of various hydraulic machines.

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