IOP Conf. Series: Materials Science and Engineering **233** (2017) 012045 doi:10.1088/1757-899X/233/1/012045

Complex double-mass dynamic model of rotor on thrust foil gas dynamic bearings

A Sytin[1], A Babin[1], S Vasin[2]

[1] Department of Mechatronics and International Engineering, Orel State University, Naugorskoe sh. 29, Orel, Russia, 302020
[2] Department of Design, Tula State University, pr. Lenina 92-9, Tula, Russia, 300600

E-mail: alex.mech.osu@gmail.com

Abstract. The present paper considers simulation of a rotor's dynamics behaviour on thrust foil gas dynamic bearings based on simultaneous solution of gas dynamics differential equations, equations of theory of elasticity, motion equations and some additional equations. A double-mass dynamic system was considered during the rotor's motion simulation which allows not only evaluation of rotor's dynamic behaviour, but also to evaluate the influence of operational and load parameters on the dynamics of the rotor-bearing system.

1. Introduction

Rotor systems are key elements of the majority of the military, research and industrial machines that are being developed nowadays. Bearings in them are used to maintain rotation axis steady relative to housing, for compensation of external forces, to decrease power loss due to friction and to damp vibrations. Various types of journal and thrust bearings are used, both roller-element and fluid-film, magnetic bearings and possible combinations of these, - they occupy a particular area of research because they determine and encapsulate functional peculiarities of a rotor machine. Due to a number of reasons, first off, interchangeability, the roller-element bearings are the most used, however are hardly applicable to highly loaded and/or high-speed rotors, where fluid-film bearings are virtually uncontested.

One of the possible ways of enhancing the performance of a rotor machine is increasing the rotation speed of the machine, and here of the most effective way of increasing the possible speed range is decreasing the viscosity of a lubricant. Bearings lubricated with gas have practically unlimited fleetness, high level of damping, operate with low power loss and heating and maintain a steady position of a rotor [1]. Here, the bearings with elastic elements, i.e. foil gas dynamic bearings, are the most promising. In the past 50 years a great variety of designs of foil bearings has been presented, however, the basic structure and basic elements generally remain intact. Foil gas dynamic bearings (FGDB) have a number of characteristics, complex of which provides these bearings with certain advantages in comparison to other types of bearings.

First of all, ambient air could be used as a lubricant for foil bearings, this simplifies maintenance and decreases dimensions of the lubrication systems quite significantly; it also provides possibility of operation under cryogenic conditions. This fact substantiates the use of FGDB in turbo-refrigerators, compressors, micro generators, and expanders in cryogenic and aircraft machinery. Secondly, the FGDB have dynamic properties that manifest themselves in stability of rotor's motion in a wide range

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

of rotation speeds given an acceptable level of balancing. Combination of damping properties of the lubricant film and elastic elements allows avoiding biharmonic vibrations due to vortex instabilities. It should be noted, that design of the FGDBs is implemented presently based on empiric data. The absence of specialized tools for calculation considering these bearings could also be noted – this objectively justifies fundamental and practical, theoretical and experimental research of FGDB. A few areas of research could be distinguished in this field: 1) application of low viscosity media (hydrogen, oxygen, neon, methane, etc.) which are used as lubricants in fuel components of aircrafts' driver nodes, in cooling systems, etc. [2,3]; 2) development of foil bearings with self-diagnostics functions and active control of load capacity and damping properties [4,5].

Dynamic behavior of a rotor machine, which is characterized by a specter of natural frequencies, vibration level and amplitude of vibrations, and external forces acting on the machine's elements, is determined by stiffness and damping of bearings [1]. Main reasons of internal axial forces' occurrence are design factors and lubricant's properties during transient regimes.

2. Simulation model of a FGDB

The present paper considers a thrust FGDB, calculation diagram of which is presented in the Figure 1.

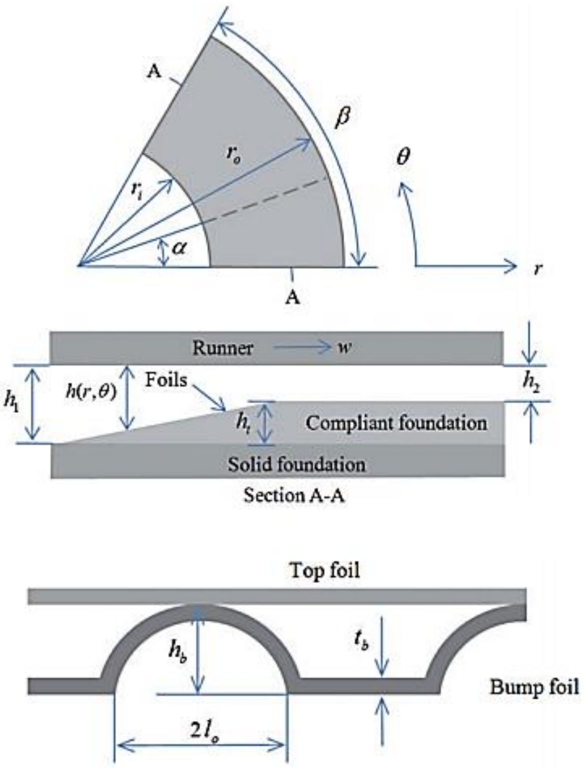

Figure 1. Calculation diagram of a thrust FGDB.

Simulation of rotor's dynamic behavior is based on the following steps: 1) simulation of gas film considering the compliant surface of a bearing with certain properties, 2) estimation of dynamic coefficients of the film, i.e. stiffness and damping, 3) derivation of motion equations for a double-mass dynamic system to study the influence of top foil's mass on the calculation of rotor's displacement. Generally, simulation of the dynamic behavior of a rotor on thrust FGDB is based on numerical solution of equations of gas dynamics, theory of elasticity and differential equations of motion with some additional relations. The operational and geometric parameters of the simulated bearing are presented in the table 1 along with the notation used in the paper.

Table 1. Notation used in the paper.

r_i	Inner radius
r_o	Outer radius
α	Tapered region angular size
β	Single pad's angular size
z	Number of pads
h_1	Maximum film thickness
h_2	Minimum film thickness
h_b	Height of the pad, $= h_1 - h_2$
l_o	Half bump length
t_b	Thickness of the bump foil
t_f	Thickness of the top foil
s	Bump spacing
r, θ	Polar coordinates, radial and circumferential axes accordingly
ω	Angular velocity
$p(r,\theta)$	Pressure distribution over a single pad
p_a	Ambient pressure
μ	Viscosity
$h(r,\theta)$	Axial gap function
V_θ, V_r, V_y	Velocity components
ρ	Density of the gas
ν	Poisson's ratio
E	Young's modulus
R_y	Reaction force in the direction of film thickness
R_{y0}	Reaction force in the equilibrium position, $= mg$
c_y, b_y	Stiffness and damping coefficients
$\Delta Y,\ \Delta\dot{Y}$	Displacement and velocity increments
$F(t)$	External load

2.1. Simulation of gas film on a compliant foundation

Simulation of the gas film is based on numerical solution of the steady-state Reynolds equation. Here a compressible medium is considered in a non-isothermal formulation. The axial gap is simulated using the Bezier curves, the procedure is presented in more detail in [6]. This allows one to avoid calculation error in the region where the tapered land turns to plain surface, on the contrary to the known and assumed correct procedure in [7]. The governing Reynolds equation is as follows:

$$\frac{\partial}{\partial r}\left[\frac{\rho h^3}{\mu}\frac{\partial p}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[\frac{\rho h^3}{\mu}\frac{\partial p}{r\partial\theta}\right]=6V_\theta\frac{\partial(\rho h)}{\partial\theta}+12\rho V_y \tag{1}$$

Numerical solution of (1) has been implemented using the finite difference method. In order to simulate the deformation of the elastic elements, the following additional relations have to be introduced. The axial gap considering local deformation of the elastic element:

$$h(r,\theta) = h_0 + w \tag{2}$$

where h_0 – initial local film thickness, w – deformation of the foil.

Stiffness of the bump foil is, according to [7], governed by the following relation:

$$c_b = \frac{Et_b^{\,3}}{2(1-\nu^2)l_o^{\,3}}$$

so, the compliance rate of the bump foil is

$$k = \frac{2s}{E}\left(\frac{l_o}{t_b}\right)^3(1-\nu^2)$$

Finally, to calculate the deformed top foil, the following transformation of (2) takes place:

$$\begin{gathered} h(r,\theta) = h_0 + K1(p - p_a) \\ K1 = \frac{\delta \cdot h_2}{p_a} \\ \delta = \frac{2p_a s}{h_2 \cdot E}\left(\frac{l_o}{t_b}\right)^3(1-\nu^2) \end{gathered} \tag{3}$$

Equations (1) and (3) are solved iteratively using the finite difference method given an accuracy criterion ε expressed in terms of load capacity on the previous and current iterations. The load capacity of a thrust bearing could be determined using the following relation:

$$R = z\iint_{\theta r} p(r,\theta) r d\theta dr$$

Once the reaction force of the gas film given the deformation of the foil is determined, the dynamic coefficients of stiffness and damping could be obtained. This is based on the premise, that the reaction force, non-linear in reality, could be linearized using Tailor series in close proximity to the equilibrium position [8]. Equilibrium position in the present case is the position where all elastic and gas dynamic forces are equal to the rotor's weight *mg*.

$$R_y = R_{y0} + K_y \Delta Y + B_y \Delta\dot{Y}$$

In the Figure 2 and Figure 3, the calculation results of stiffness and damping coefficients are presented for a following setup: inner radius – 0.025 m, outer radius – 0.05 m, angular velocity – 8000 rad/s, minimum film thickness – 10 μm, viscosity of ambient air – $1\cdot10^{-5}$ Pa·s, bump foil's height – 1.75 mm, bump foil's thickness – 0.125 mm, bump spacing – 4.75 mm, half bump length – 1.75 mm, top foil's thickness – 0.125 mm, Poisson's ratio – 0.29, Young's modulus - $214\cdot10^9$ Pa. Angular size of the tapered land region is 0.6β.

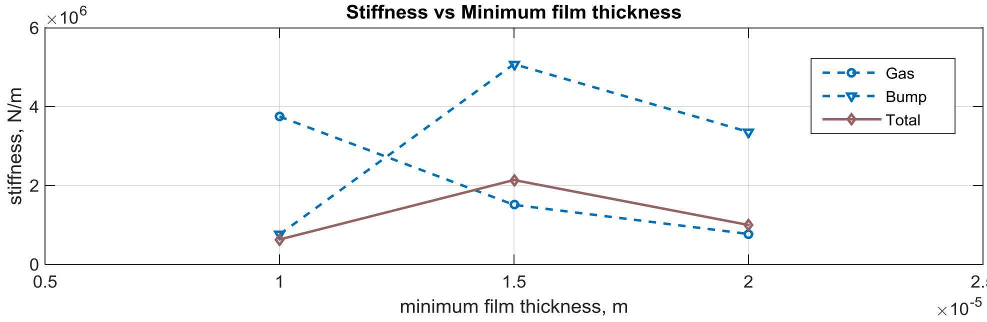

Figure 2. Stiffness of gas film, bump foil and total stiffness vs minimum film thickness.

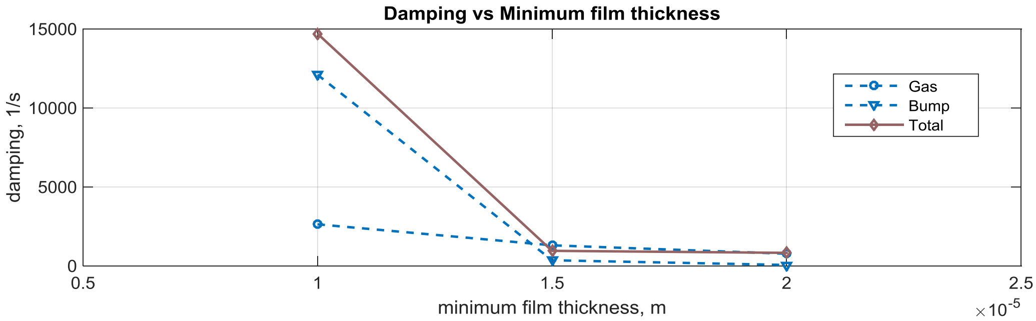

Figure 3. Stiffness of gas film, bump foil and total stiffness vs minimum film thickness.

It could be seen, that dynamic coefficients of a thrust FGDB depend significantly on the position of a rotor, i.e. the influence of presence of elastic elements is stronger at smaller values of minimum film thickness. Thus, problems of design of thrust FGDB always take specific operational and geometric parameters of a rotor machine into account.

2.2. Single- and double-mass dynamic system simulation

To study the influence of the top foil's mass on the dynamic behaviour of a rotor two models have been developed, the diagrams are presented in the Figure 4. Indexes of *c* and *b* correspond to gas film (index '1') and bump foil (index '2').

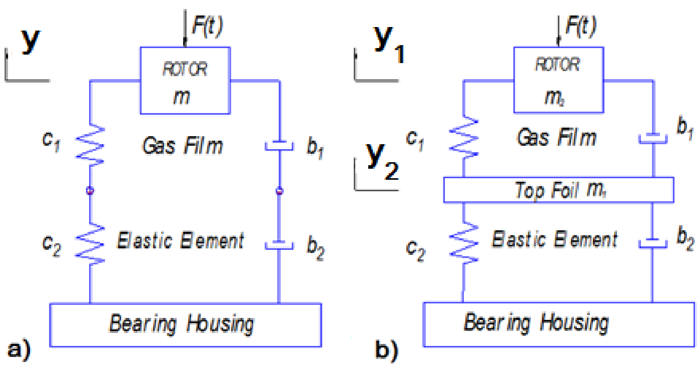

Figure 4. Single-mass (a) and double-mass (b) dynamic systems.

The stiffness and damping coefficients according to the corresponding calculation diagram in the Figure 4 are calculated as follows:

$$b_{total} = b_1 + b_2$$
$$\frac{1}{c_{total}} = \frac{1}{c_1} + \frac{1}{c_2}$$

The corresponding equations of motion are as follows: 1) single-mass dynamic system:

$$m\ddot{Y} = R_{y0} + c_y(\Delta Y) + b_y(\Delta\dot{Y}) - F(t) \tag{4}$$

where
c_y and b_y are total stiffness and damping accordingly,
m – rotor's mass.
2) double-mass dynamic system:

$$\begin{cases} m_1\ddot{Y}_1 = c_1(\Delta Y_1 - \Delta Y_2) + b_1(\Delta\dot{Y}_1 - \Delta\dot{Y}_2) - F(t) \\ m_2\ddot{Y}_2 = c_2(\Delta Y_2) + c_1(\Delta Y_2 - \Delta Y_1) + b_2(\Delta\dot{Y}_2) + b_1(\Delta\dot{Y}_2 - \Delta\dot{Y}_1) \end{cases} \tag{5}$$

where indexes '1' and '2' correspond to the rotor and the foil accordingly.

Equations (4) and (5) are solved using the ode23 solver in Matlab, with the following initial conditions and assumptions: equilibrium position of the rotor is defined by the equilibrium of forces, the equilibrium position of the top foil is the maximum deformation of the foil which is balanced by the gas dynamic forces and elastic forces in the bump foil. Thus, the following initial conditions have been set: initial position of the rotor is 50 µm, where the action of forces in the lubricant is virtually not deforming the foil, initial position of the foil is defined by the equilibrium position of the foil, which is taken -0.5 µm relative to 0 µm displacement at the initial moment in time. Initial velocities of both the rotor and the foil are zero.

The bearing with the same parameters as in par. 2.1 is considered. The rotor's mass is 1.75 kg, free fall acceleration is 9.8 m/s, which results in the steady-state minimum film thickness of 10 µm. The mass of a single top foil is 0.001 kg.

To study the dynamic behavior of the system, two types of external forces were considered – impulse of 8 N (Figure 5) and periodic force of 4 N at approx. 60 Hz (Figure 6).

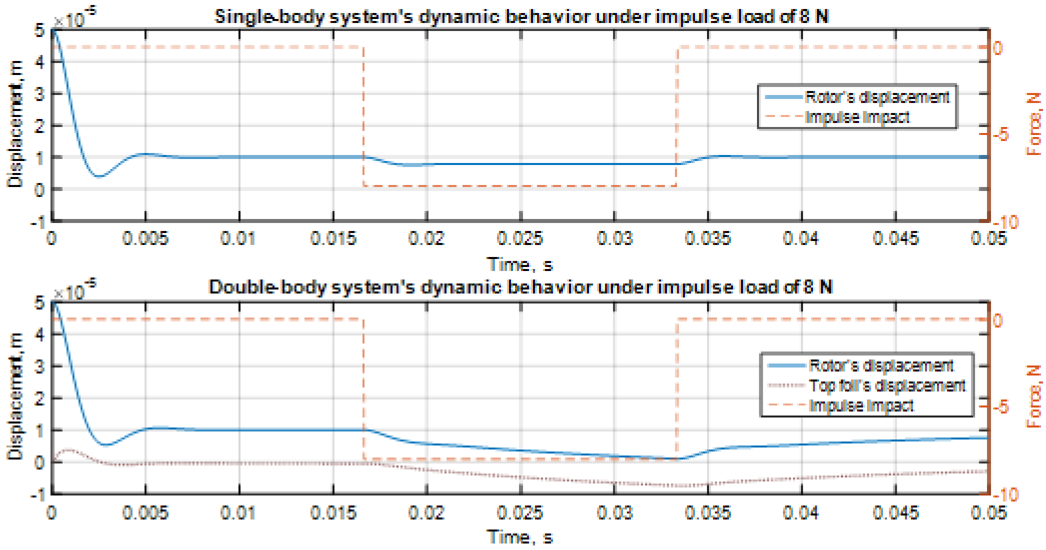

Figure 5. Dynamic behavior of a thrust FGDB under an impulse load.

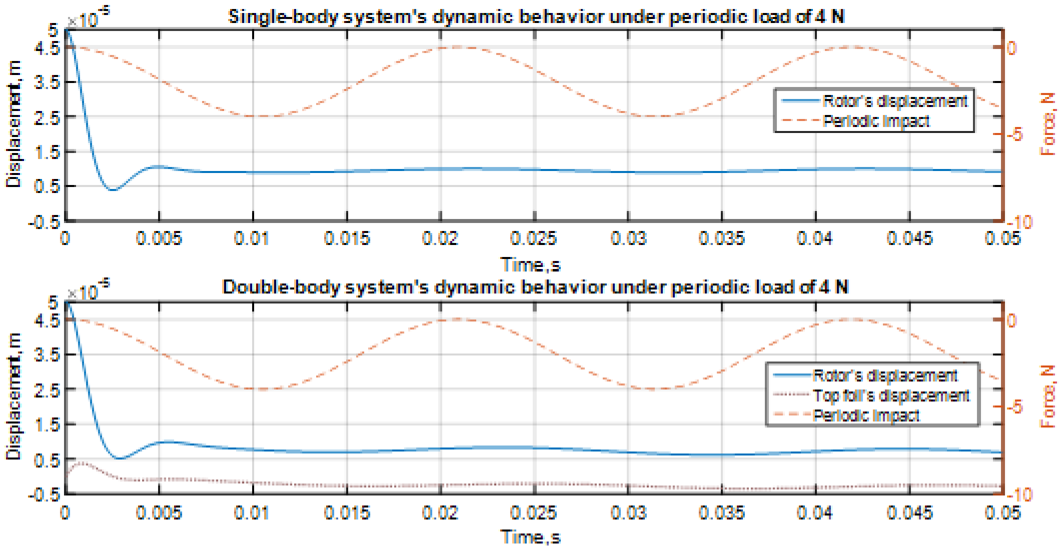

Figure 6. Dynamic behavior of a thrust FGDB under a periodic load.

3. Conclusions

It could be seen from the Figure 5 and the Figure 6, that introduction of the top foil's mass to the dynamic system does not influence the displacement of the rotor significantly. In comparison to the rotor's weight, the top foil usually weights several orders of magnitude less. As for the computational performance, in case of the single-mass dynamic system, the ode23 function returned results in less than a second, whereas in the case of the double-mass dynamic system the time it took for the program to return results was more than 15 min depending on various parameters. So, a conclusion could be drawn, that when performing the dynamic analysis of a rotor-bearing system with thrust FGDBs, top foil's mass could be neglected. The developed single-mass model could be used to study the dynamic response of a thrust FGDB to various types of external impact. Further research involves a series of experiments to verify the obtained numerical results.

References

[1] M V Korovchinsky 1959 *Theoretical foundations of sliding bearings* (M .: Mashgiz) p 404
[2] V O Tyurin, L A Savin, A V Sytin 2016 Lubrication of petal gas-dynamic bearings with low-viscosity cryogenic liquids *Bulletin of BSTU* (Bryansk: Bryansk State Technical University) **2** (50), pp 13-20
[3] M Braun, R Wheeler III, R Hendricks 1987A fully coupled variable properties thermohydraulic model for a cryogenic hydrostatic journal bearing *Journal of Tribology* **109** pp 405 1987
[4] A V Sytin, V O Tyurin, S D Antonov 2015 Active control of mechatronic foil bearing of rotor-supporting units of high-speed turbomachines *Fundamental and applied problems of technology and technology* **3** (311) pp 59-65
[5] G Aguirre, F Al-Bender, H Van Brussel 2008 Dynamic stiffness compensation with active aerostatic thrust bearings *Proceedings of ISMA* pp 105-118
[6] A Babin, L Savin 2016 Increase of Energy Efficiency by Means of Application of Active Bearings *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering* **10, 9** pp 1667 – 1671
[7] K Feng, Z Guo, Z Zhao 2015 Parametric study on static and dynamic characteristics of bump-type gas foil thrust bearing for oil-free turbomachinery *Proceedings of the Institution of Mechanical Engineers Part J Journal of Engineering Tribology 1994-1996 (vols 208-210)* **229** (10)

IOP Conf. Series: Materials Science and Engineering **233** (2017) 012045 doi:10.1088/1757-899X/233/1/012045

[8] H Heshmat, J A Walowit, O Pinkus 1983 Analysis of Gas Lubricated Foil Journal Bearings *Journal of Lubrication Technology* **105** pp 647-655
[9] L A Savin, O V Solomin 2006 *Modeling of rotor systems with fluid-film bearings: monograph* (M.: Engineering 1) p 444

Acknowledgments
The present research was implemented as a part of the RSF project No. 16-19-00186 ‘Planning of optimal energy efficient rotor trajectories in mechatronic modules with complex rheology medium’.