An alternative method for centrifugal compressor loading factor modelling

Y Galerkin, A Drozdov, A Rekstin, K Soldatova
R&D Laboratory "Gas dynamics of turbo machines" Peter the Great St.Petersburg Polytechnic University, Polytechnical st. 29, St.Petersburg, Russia

Email: buck02@list.ru.

Abstract: The loading factor at design point is calculated by one or other empirical formula in classical design methods. Performance modelling as a whole is out of consideration. Test data of compressor stages demonstrates that loading factor versus flow coefficient at the impeller exit has a linear character independent of compressibility. Known Universal Modelling Method exploits this fact. Two points define the function – loading factor at design point and at zero flow rate. The proper formulae include empirical coefficients. A good modelling result is possible if the choice of coefficients is based on experience and close analogs. Earlier Y. Galerkin and K. Soldatova had proposed to define loading factor performance by the angle of its inclination to the ordinate axis and by the loading factor at zero flow rate. Simple and definite equations with four geometry parameters were proposed for loading factor performance calculated for inviscid flow. The authors of this publication have studied the test performance of thirteen stages of different types. The equations are proposed with universal empirical coefficients. The calculation error lies in the range of plus to minus 1,5%. The alternative model of a loading factor performance modelling is included in new versions of the Universal Modelling Method.

Nomenclature

b	width of channel
c_u	tangential component of absolute flow velocity
D	diameter
h_T	theoretical head, i.e. the head transferred to gas by impeller blades
K_{pd}	empirical coefficient of velocity diagram
K_μ	empirical coefficient of viscosity influence
l	length of blade
$\frac{l}{t}$	cascade solidity
$\bar{m}$	mass flow rate
M_u	blade Mach number
N_i	internal power, i.e. all power transferred to gas by an impeller
p	pressure
r	radius
u	tangential velocity

IOP Conf. Series: Materials Science and Engineering **232** (2017) 012039 doi:10.1088/1757-899X/232/1/012039

$X_{\psi t0}$	empirical coefficient
w	relative velocity
Γ	flow circulation
y	empirical coefficient
z	number of blades, empirical coefficient
β	angle between the relative speed and the reverse district direction
β_{bl}	blade angle
β_{df}	non-dimensional coefficient of disk friction
β_{lk}	non-dimensional leakage coefficient
β_{T}	angle of a loading factor performance inclination
φ	flow coefficient
η	polytrop~~h~~ic efficiency
λ	friction coefficient
ω	rotation speed
Φ	flow rate coefficient
ψ_{T}	loading factor.

Subscripts

0	impeller inlet
1	impeller blade row inlet
2	impeller outlet
ni	non incidence
inl	inlet
inv	inviscid flow
bl	blade
boud	boundary layer
imp	impeller
des	design
max	maximum
pd	center of pressure

1. Aim of the work

For flow path design, an instrument is necessary for calculation of centrifugal compressor gas dynamic performance. The authors use the primary design procedure described in [1]. The Universal Modelling Method presented in [2] is applied to calculate performance of a primary design and of possible better candidates. Calculated non-dimensional performance of a stage is presented in Figure 1.

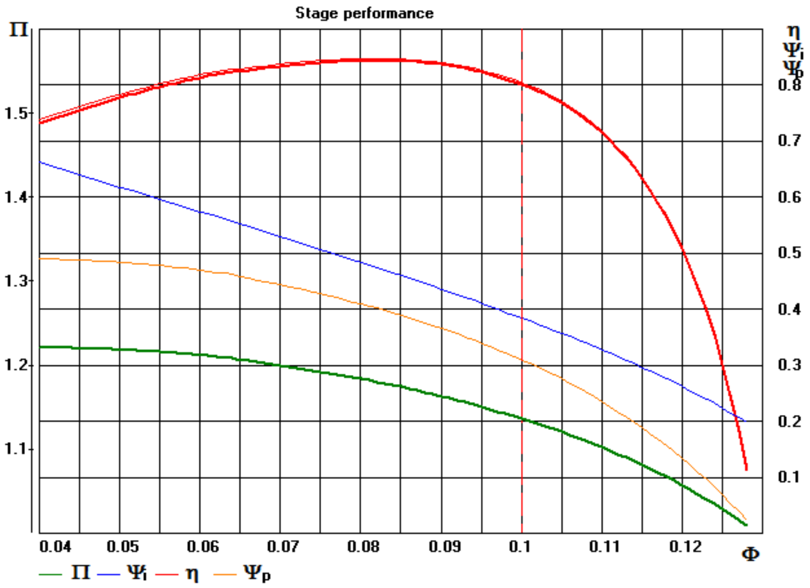

Figure 1. Calculated non-dimensional performance of a centrifugal compressor stage

Efficiency η and work coefficient ψ_i are necessary for calculation of polytrophic coefficient and pressure ratio:

$$\psi_p = \psi_i \times \eta. \tag{1}$$

$$\pi = \left(1 + (k-1)\psi_i M_u^2\right)^{\frac{k}{k-1}\eta}. \tag{2}$$

Efficiency prediction is possible with new versions of the Universal Modelling PC programs without special skill and experience [2-4]. Prediction of the work coefficient performance $\psi_i = f(\Phi)$ still requires experience and intuition. The authors' aim is to offer an alternative way of the work coefficient performance calculation that is simple and precise. Presented work is based on ideas and results of Y. Galerkin and K. Soldatova who had studied loading factor performances of impellers with inviscid flows.

2. Scheme of work coefficient modelling

In accordance with the scheme proposed previously in [5] the head transmitted to gas by an impeller h_i consists of three parts. The main part of engine's mechanical head h_T is transferred to gas by blades of the impeller. Two additional parts appear due to parasitic losses. The part of the head h_{lc} is lost in labyrinth seals. The friction on outer surfaces of hub and shroud transmits additional mechanical head h_{df} but this head does not increase gas pressure:

$$h_i = h_T + h_{df} + h_{lc}. \tag{3}$$

Non-dimensional presentation of this equation:

$$\psi_i = \psi_T\left(1 + \beta_{df} + \beta_{lk}\right). \tag{4}$$

For an impeller with large flow coefficient $\Phi_{des} \approx 0{,}15$ the sum $\beta_{df} + \beta_{lk}$ is less than 0,01. For an impeller with small flow coefficient $\Phi_{des} \approx 0{,}015$ this sum is about 0,055-0,065. It is not large part of a head coefficient. Semi-empirical formulae in [1] and CFD-calculations [9] are good instruments for modelling of these coefficients. Therefore the main problem is modelling of a loading factor.

As a rule there is no velocity tangential component at an impeller inlet in industrial compressors. Then impeller blades transfer the head to a gas in accordance with the Euler equation:

$$h_T = c_{u2} u_2. \tag{5}$$

This head is presented by the non-dimensional loading factor:

$$\psi_T = c_{u2} / u_2. \tag{6}$$

The modelling of a loading factor performance is facilitated by the fact that it is a linear function – Figure 2.

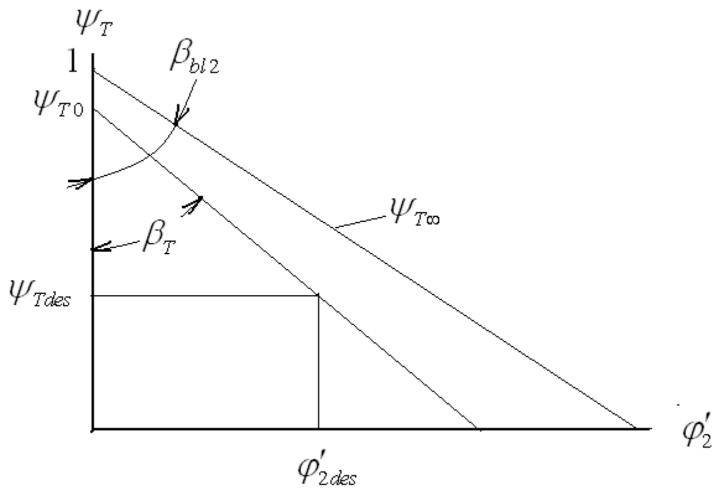

Figure 2. Linear function of a loading factor for ideal and real impellers

The linear nature of it is evident for an “ideal” impeller with infinite number of infinitely thin blades:

$$\psi_{T\infty} = 1 - \varphi_2' ctg \beta_{bl2} \tag{7}$$

The similar equation is valid for real impellers:

$$\psi_T = 1 - \varphi_2' ctg \beta_2. \tag{8}$$

The flow angle β_2 is not constant versus flow coefficient. Anyway test data for industrial compressors demonstrates practically a linear characteristic of work input versus flow rate [5]. Model stages’ tests demonstrated linear characteristic of the function $\psi_T = f(\varphi_2')$ independent of blade Mach number [10].

The existing way of modelling [2-4] exploits this fact. Two values of a loading factor are necessary to determine a linear performance $\psi_T = f(\varphi_2')$.

In the method described in [1] these two values are a loading factor at a design flow rate $\psi_{T\,des}$ and at zero flow rate ψ_{T0}. The design flow rate corresponds to non-incidence inlet of the critical streamline. This condition is $\beta_{1cr} = \beta_{bl1}$. Exit and inlet velocity triangles demonstrate~~s~~ influence of blades' load and blockade on a critical streamline direction – Figure 3.

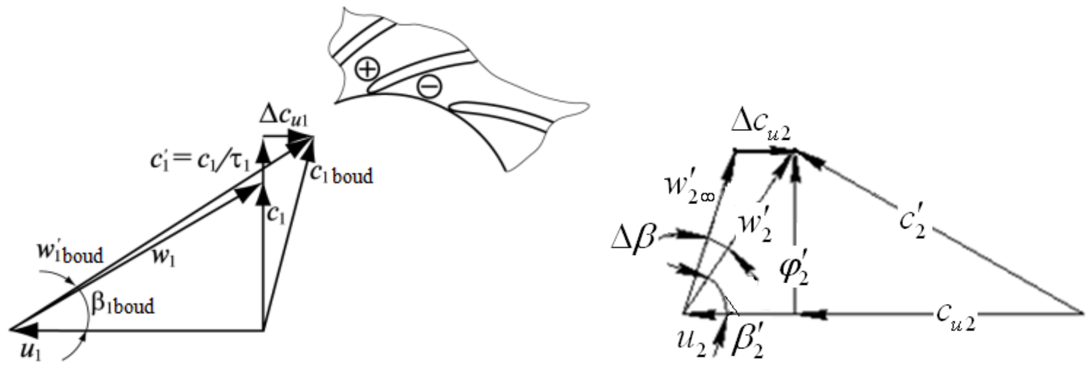

Figure 3. Inlet and exit velocity triangles at design flow rate

The absolute velocity c_1 is accelerating near a blade cascade inlet due to a blade blockade: $c_1' = c_1 / \tau_1$. A critical streamline turns its direction to a suction side of a blade where pressure is less than on a pressure side. Symbols "plus/minus" in the Figure 3 demonstrate difference of pressures on blade surfaces. A critical streamline obtains a tangential velocity Δc_{u1}. The scheme of the Δc_{u1} calculation is based on the idea of representing a blade by a swirl with the circulation $\Gamma_{bl} = 2\pi r_2 c_{u2} / z$. The swirl induces velocities depending of distance from its center. The equation to calculate tangential velocity induced on the radius r_1 where the blades start is presented in non-dimensional way:

$$\Delta\bar{c}_{u1} = \frac{\Psi_{\mathrm{T\,des}}}{z\left(\bar{r}_{pc} - \bar{r}_1\right)}. \tag{9}$$

Here $\bar{r}_{pc} = r_{pc} / r_2$ is a relative distance from an impeller axis to a center of a velocity diagram, Figure 4.

A loading factor at design flow rate is calculated on the same principle. The velocity triangle is shown above in the Figure 2. In accordance with Kutta-Zoukovsky postulate a critical streamline leaves blades in direction of an angle β_{bl2}. Due to a blade load a pitch-averaged velocity got a tangential velocity component:

$$\Delta\bar{c}_{u2\,des} = -K_{\mu} \frac{\Psi_{\mathrm{T}\,des}}{z\left(1 - \bar{r}_{pc}\right)}. \tag{10}$$

The equation for a loading factor is:

$$\psi_{T\,des} = \frac{1-\varphi'_{2des} ctg\beta_{bl2}}{1+K_\mu \dfrac{1}{z\left(1-\bar{r}_{pc}\right)}}. \tag{11}$$

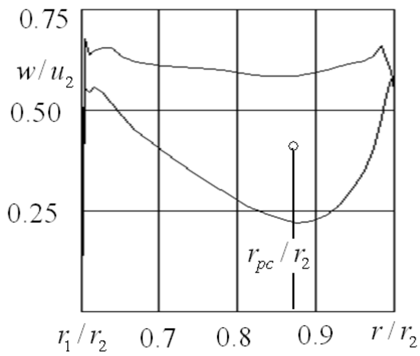

Figure 4. Velocity diagram at design flow rate (inviscid flow)

The empirical coefficient $K_\mu > 1$ represents influence or real viscid character of flow. Its value for different types of impellers may be between 1,5-2,3. There is no satisfactory correlation with an impeller configuration. The close analog is necessary. The alternative is to model a loading factor performance by values of ψ_{T0} and angle β_T that are shown in Figure 2:

$$\psi_T = \psi_{T0} - \varphi'_2 ctg\beta_T. \tag{12}$$

The aim of this work is to model loading factor performance of impellers in real viscid flow on the basis of several impeller geometry parameters.

Test data for model stages and factory test data of several pipeline compressors were reduced. The information on the objects of modelling is presented in Table 1. All stages and compressors were tested at M_u=0,60 or 0,80. Model stage names mean the following. For example, 0,0604-0,527-0,290 means that the design flow rate coefficient of the stage is Φ_{des} = 0,0604, design loading factor is $\psi_{T\,des}$ = 0527, hub ratio is D_h/D_2 = 0,29. Names of compressors are given by their manufacturers on their own principles. Data on compressors are taken from [4]. Symbol * means that 2D impeller has an arc blade mean line. Mean lines are designed by the Method presented in [1] in other cases. In columns 3-9 geometry parameters of impellers are presented, they are included in the presented below equations for loading factor performances modelling.

Table 1. Parameters of stages and compressors used for modelling

1	2	3	4	5	6	7	8	9	10	11
	Parameter/ /stage	b_2/b_1	z	D_1/D_2	l/t	β^0_{bl1}	β^0_{bl2}	$\bar{\delta}_{bl}$	K_μ	$X_{\psi T0}$
1	0.0373-0.482-0.373*	0.754	15	0.514	3.423	25	30.3	0.0133	2.0	2.1
2	0.0455-0.539-0.350*	0.589	13	0.565	2.40	25	34	0.02	1.05	2.3
3	0.0480-0.49-0.290	0.573	11	0.534	2.461	23	30	0.014	1.65	1.3

4	0.0604-0.527-0.290	0.618	13	0.592	1.916	22	47	0.0115	1.45	0.9
5	0.0653-0.409-0.290	0.657	11	0.570	1.967	28	32	0.014	1.7	1.9
6	0.0692-0.476-0.290	0.707	11	0.570	2.062	25	32	0.0138	1.2	1.7
7	0.0685-0.68-0.345	0.539	17	0.577	1.864	27	79	0.0128	1.34	0.4
8	16-76-1.6*	0.693	11	0.5765	1.987	26	32	0.0121	0.95	2.17
9	108-5-1*	0.587	13	0.5651	2.399	21	38	0.02	1.5	1.02
10	GPA-76-1.7	0.614	13	0.583	2.227	24.8	35.8	0.0143	2.62	1.7
11	61-1.64	0.549	19	0.548	2.518	25.5	67	0.0138	1.62	0.95
12	650-1.37-76	0.711	11	0.570	1.967	28	32	0.0138	1.6	1.4
13	16-76-1.7	0.360	25	0.5	2.996	30	104	0.0089	1.34	0.4

The sample of data on model stages from the "IDENT" program is presented in Figure 5. The dimensions that are necessary for calculation of efficiency and a loading factor performances are presented in the database. The measured values of these parameters are black figures and lines. The efficiency calculated according to the 6th version of model with a universal set of coefficients are red figures and lines. The values $X_{\psi T0}$ (the coefficient for an empirical equation for ψ_{T0} calculation) and K_{μ} are picked up individually for each stage for the best compliance to the measured performances. These values are presented in columns 10, 11 in the table 1. The program calculates also values β_T and ψ_{T0} corresponding to preset values $X_{\psi T0}$ and K_{μ}.

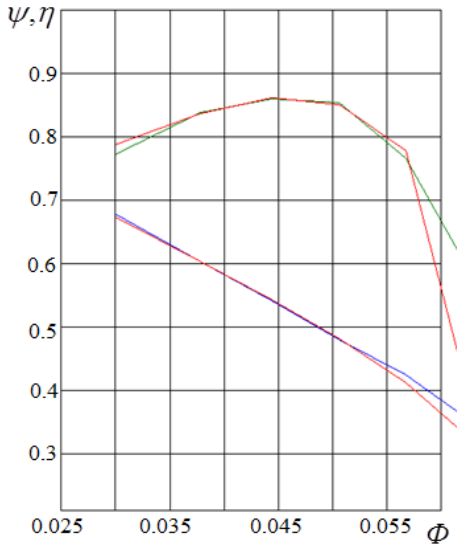

Figure 5. An example of information from the database of the IDENT program. (stage No. 3, from the table 1)

The authors have studied all impeller geometry parameters as possible arguments in functions $\beta_T, \psi_{T0} = f\left(\varphi_2', \bar{F}\right)$, where $\bar{F}$ - symbol of geometry parameters that define an impeller flow path. Main dimensions of a typical industrial impeller in meridian plane are shown in Figure 6.

Among dimensions shown in Figure 6 only the ratio b_2 / b_1 influences functions $\beta_T, \psi_{T0} = f\left(\varphi_2', \bar{F}\right)$. The most important parameter is a blade exit angle β_{bl2}. Ratio of a blade length l to average pitch t presents in functions too. This ratio is calculated by the formula from [5]:

$$1/t = z\frac{\lg\frac{D_2}{D_1}}{2.73\sin\left(\frac{\beta_{bl2}+\beta_{bl1}}{2}\right)}. \quad (13)$$

Most of 2D impellers designed by the authors have either an arc mean line of blades, or a mean line ~~is~~ optimized by analysis of velocity diagrams. Two mean lines and corresponding velocity diagrams are presented in Figure 7.

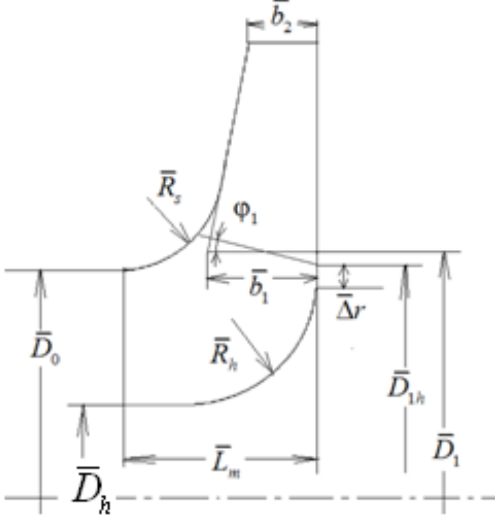

Figure 6. Main dimensions of a typical industrial impeller in meridian plane

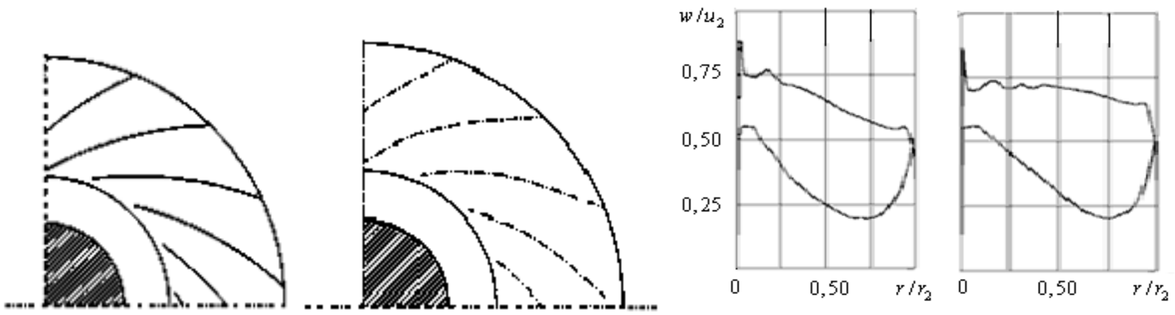

Figure 7. Mean lines and velocity diagrams of 2D impellers. Left – arc mean line, right – optimized mean line

Two groups of approximating formulae are developed for two types of blade mean line.

The loading factor at zero flow rate:
- for impellers with optimized mean line:

$$\psi_{T0} = 0.964 + 0,002\left(\frac{\beta_{bl2}}{40}\right)^4 - 0.068\left[\ln\left(\frac{l}{t}\right)\right]^3 - 0.00011\left[\ln\left(\frac{b_2}{b_1}\right)\right]^{-4}. \quad (14)$$

- for impellers with arc mean line:

$$\psi_{T0} = 0.747 + 0{,}097\left(\frac{\beta_{bl2}}{40}\right)^4 - 0.008\left[\ln\left(\frac{l}{t}\right)\right]^7 - 20.247\left[\ln\left(\frac{b_2}{b_1}\right)\right]^9. \tag{15}$$

Angle β_T^0:

- for impellers with optimized mean line:

$$\beta_T^0 = 0.534\frac{\beta_{bl2}}{40} + 0.028\left(\frac{l}{t}\right)^6 - 3.577\left[\ln\left(\frac{b_2}{b_1}\right)\right]^{-1}. \tag{16}$$

- for impellers with arc mean line:

$$\beta_T^0 = 20.311\left(\frac{\beta_{bl2}}{40}\right)^{-2} + 3.595\left(\frac{l}{t}\right) - 0.106\left[\ln\left(\frac{b_2}{b_1}\right)\right]^{-4}. \tag{17}$$

Figures 8, 9 demonstrate accuracy of approximation.

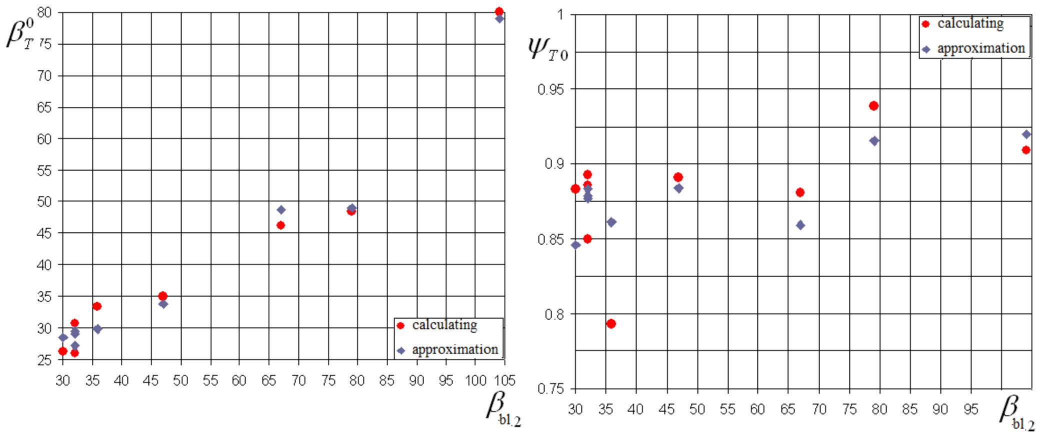

Figure 8. Comparison of the measured and calculated β_T and ψ_{T0} for impellers with different β_{bl2} (optimized mean line)

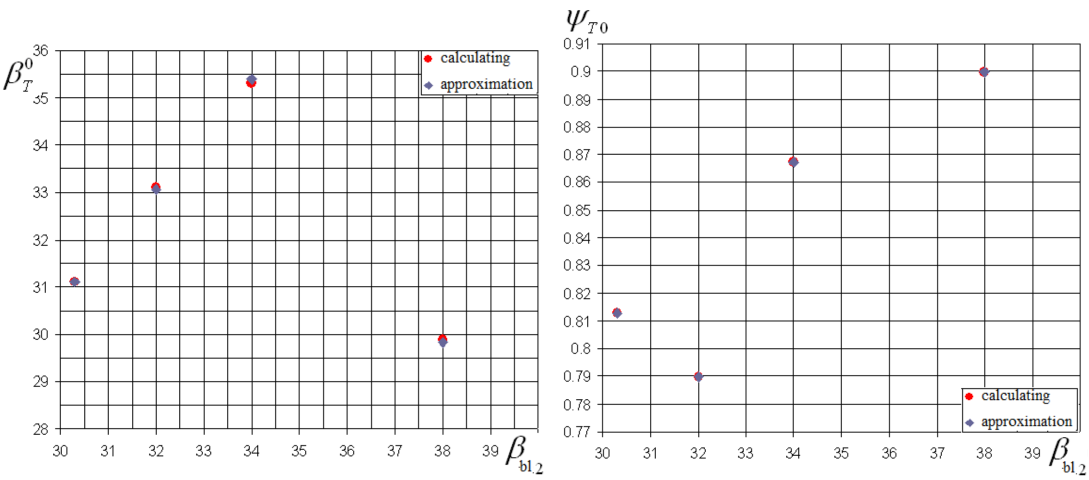

Figure 9. Comparison of the measured and calculated β_T and ψ_{T0} for impellers with different β_{bl2} (arc mean line)

Modelling accuracy is satisfactory for an exception of the 1st stage of 6-stage pipeline compressor GPA-76-1.7 (line 10 Table 1). For impellers of this compressor the empirical coefficient is K_μ = 2.62 (eq. 10). It is an unusually large value. The authors have no explanation for this fact. The modelling error in all other cases is within admissible limits. Figure 10 presents the comparison of performance of one of model stages.

Graphics with individual adjustment of the loading factor performance are presented on the left. Right – loading factor is calculated by eq. (14, 16). Efficiency performance is calculated by 6th version of the Math model in both cases. Despite visually noticeable difference of load performances, an error of calculation $\psi_{T\,des}$ is 1,2%. It means that an impeller designed by this method will have +1,2% of head input. It is acceptable for design practice.

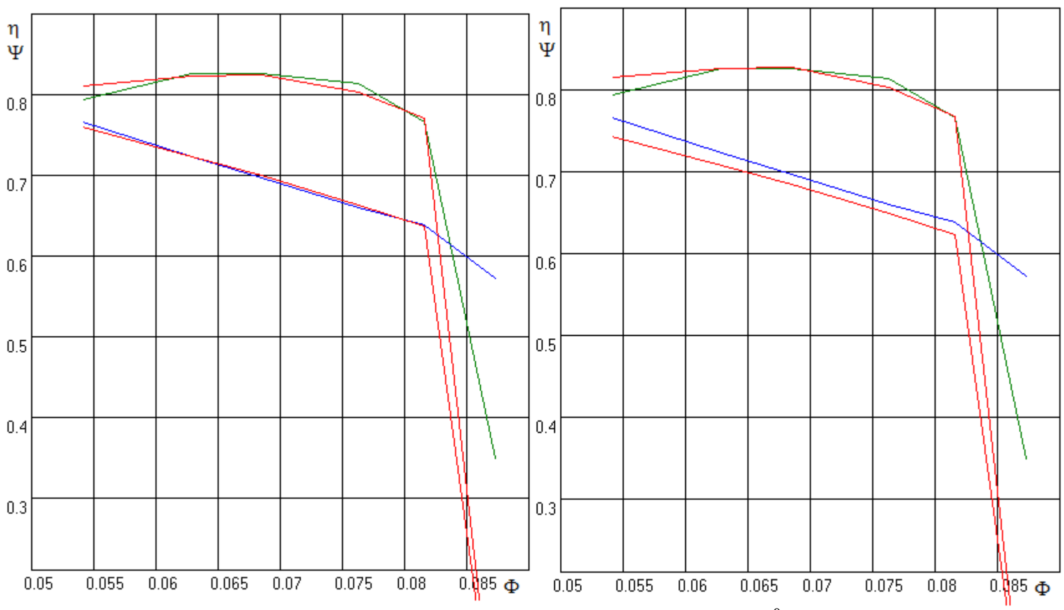

Figure 10. Model stage performance comparison (#3, β_{bl2}=79^0). Green – measured, red – calculated. Right – loading factor is calculated by eq. (15, 17)

IOP Conf. Series: Materials Science and Engineering **232** (2017) 012039 doi:10.1088/1757-899X/232/1/012039

Conclusion

The authors plan to apply the presented method of a loading factor performance calculation in parallel with the existing method in their future projects. In case of a positive result the new method will be incorporated into the Math models. This method is simple and does not require users' high experience and corrections with analogs.

Acknowledgment

The reported study was funded by RFBR according to the research project № 16-08-00624 A.

References

[1] Galerkin. Y.B. Turbocompressors. // LTD information and publishing center. - Moscow. – 2010. – P.650. [Russian].
[2] Galerkin Y., Soldatova K. The application of the Universal Modelling Method to development of centrifugal compressor model stages. Conference "Compressors and their Systems". – 477-487. – London. – 2013.
[3] Galerkin Y., Drozdov A. New generation of Universal modelling for centrifugal compressors calculation. Conference "Compressors and their Systems". – London. – 2015.
[4] Galerkin Y., Rekstin A., Soldatova K., Drozdov A. Universal modelling method – the instrument for centrifugal compressor gas dynamic design. ASME Gas Turbine India Conference. – 2015. - GTINDIA2015-1202.
[5] Ris V. Centrifugal compressors. // Leningrad. – 1964. [Russian].