

# Determining of actual profile clearances and screw compressor rotor positions depending on working conditions

T N Mustafin<sup>1</sup>, R R Yakupov<sup>1</sup>, M S Khamidullin<sup>1</sup>, I G Khisameev<sup>1</sup>,  
L H Uybekova<sup>1</sup>, O Y Paranina<sup>1,2</sup>

<sup>1</sup>Kazan National Research Technological University, Russian Federation;

<sup>2</sup>HMS Group NIIturbokompressor, Russian Federation

Email: mustimur@rambler.ru

**Abstract.** In general, screw compressor characteristics depend on their working chamber clearances. One of the main of clearances in the working chamber of screw compressors are the clearances between rotors' profiles, which are produced due to a reducing of theoretical rotor profiles. This reducing is made to compensate thermal deformation, manufacturing errors and other negative factors, which occur during the screw compressor working cycles and may cause a rotors' teeth interference. This also may result in a transmission error, which increases the rotor profile clearance on one side of rotor tooth and reduces on other. A screw compressor rotor gearing mathematical model which can predict the actual rotor position and value of the transmission error was developed. An analysis of compressor working conditions influence on the transmission error value is done. Results of the presented work are used for working process mathematical models for improving their accuracy. They also may be used as basis for development of reducing theoretical profiles methods with the aim to reducing of the rotor profiles' clearances and improve screw compressor characteristics.

## 1. Introduction

The profiled rotor clearances are one of determining values that influence the energy characteristics of screw-type compressors. Many works are devoted to their determination and research [1–4]. Generally, they also determine the smoothness of transmission and affect to a certain extent the values of vibration. However, it should be noted that the majority of works determine these clearances in the rotors nominal position without taking into account the possible transmission error. Thus, the neglect of this factor results in overstating the clearances, on the one hand, and in understating the error introduced in the calculation, on the other hand. The calculation procedure itself of profiled clearances remains unchanged, while the calculation procedure of the relative position of the rotors requires clarification. Just this is the subject of this article.

## 2. Characteristic of rotors in operating mode of the transmission process

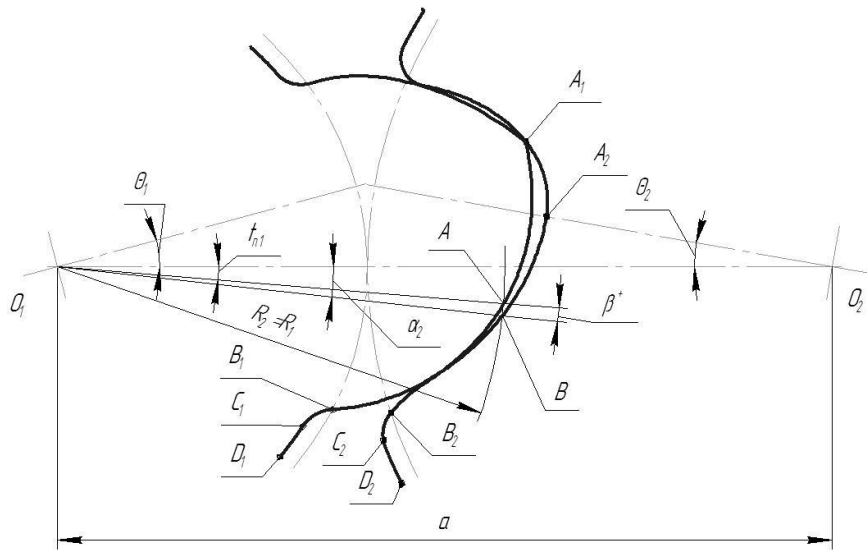
The positions of a free rotor are determined by the extreme positions upon contact of rotors along one of the sides or by some intermediate positions in the transition to one or another extreme position. For their determination, it is necessary to determine the frames, wherein a free rotor has the liberty to rotate until the contact with the female rotor is established (transmission error). This problem was



considered in [3, 4]. In order to determine the shift angle of the female rotor until the contact with the free rotor is established, we consider the position of the rotors when the male rotor is rotated at an angle  $\theta_1$  (figure 1), whereby the female rotor, respectively, should be rotated nominally at an angle:

$$\theta_2 = \theta_1 \frac{z_1}{z_2}, \quad (1)$$

where  $z_1, z_2$  – the number of teeth of the male and female rotors.



**Figure 1.** Diagram of shift angle determination of the male rotor until contacting the female rotor

Select in the  $A_1D_1$  segment the arbitrary point  $A$  with the angular coordinate  $\alpha_1$ . Find on the front part of the profile of the female rotor ( $A_2D_2$  segment), rotated by the angle  $\theta_2$ , the point  $B$ , whose radial coordinate  $R_2$  is equal to  $R_1$ . The difference  $\beta$  of the angular coordinates of points  $A$  and  $B$  will give the value of the additional-turn angle of the male rotor until the point  $A$  of the female rotor profile is contacted

$$\beta = \alpha_2 - t_{n1}. \quad (2)$$

After finding the minimum of the function (2) by the variable  $\alpha_2$ , we will obtain the value of the additional turn of the male rotor until its contact with the female rotor profile for the considered position:

$$\beta_{\min \alpha}^+ = f(\theta_1). \quad (3)$$

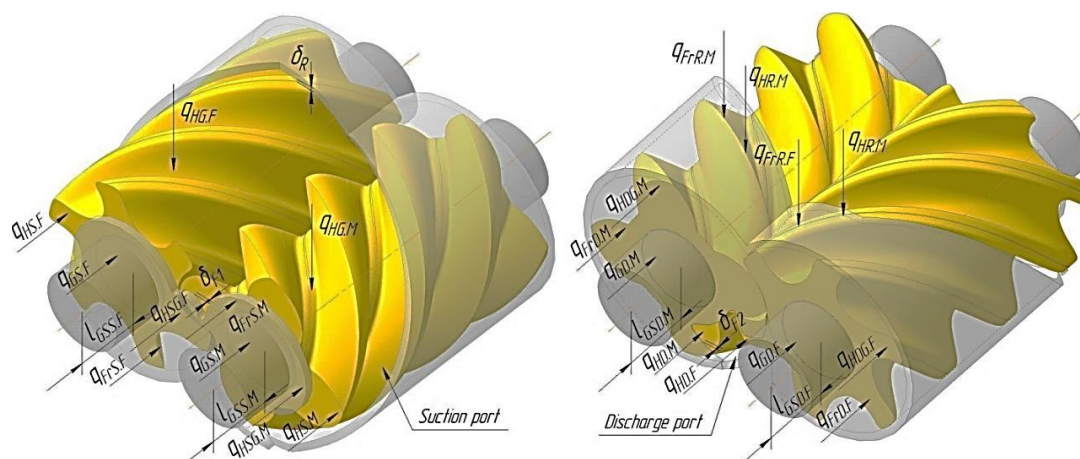
From the relation (3), determine the minimum shift angle of the male rotor  $\beta_{\min}^+$  for the entire rotation range  $\theta_1$  within the existence of the contact line along the front side of the rotor tooth profile.

Similarly, the additional-turn angle  $\beta_{\min}^-$  is calculated.

The rotor profile should be accepted in the calculation by taking into account the thermal deformations, whose method of calculation is illustrated in details in [3, 4]. The thermal load on the compressor rotors is cyclical. However, the period of a single cycle is short enough, so the temperature pattern of the rotor remains practically unchanged over time. Then, the thermal state of the rotors is determined by the stationary heat-transfer equation:

The boundary conditions for the equation (4) are determined by the calculation model (figure 2) on the basis of equation  $\lambda_M \frac{\partial T}{\partial n} = q$ , where  $\lambda_M$  – thermal conductivity of the rotor material,  $\frac{\partial T}{\partial n}$  – derivative on the normal to the rotor's surface affected by the heat flow  $q$ .

Thereby, the heat gains resulting from the heat transfer of the gas-oil medium with the rotors' surface in the working space should be separated from the heat transfer of the end surfaces of the rotors with gas-oil medium in the increasing spaces through the end surfaces of the suction and discharge ports; the heat gain resulting from the friction of the rotors in the gas-oil medium in the end and radial clearances, as well as the heat gain along the shaft areas from the bearings. The procedure of their determination is presented in [5, 6].



**Figure 2.** Calculation model for determining the boundary conditions

Equations describing the ratio of motions and strains were used for calculation of thermal deformations:

$$(\varepsilon_x - \beta_R \cdot (T - T_{st})) = \frac{\sigma_x - \nu \cdot (\sigma_Y + \sigma_Z)}{E}, \quad (5)$$

$$(\varepsilon_Y - \beta_R \cdot (T - T_{St})) = \frac{\sigma_Y - \nu \cdot (\sigma_X + \sigma_Z)}{E}, \quad (6)$$

$$(\varepsilon_z - \beta_R \cdot (T - T_{St})) = \frac{\sigma_z - \nu \cdot (\sigma_x + \sigma_y)}{E} \quad (7)$$

$$\gamma_{\text{XY}} = \frac{\tau_{\text{XY}}}{G}, \quad (8)$$

$$\gamma_{\text{xz}} = \frac{\tau_{\text{xz}}}{G}, \quad (9)$$

$$\gamma_{\text{YZ}} = \frac{\tau_{\text{YZ}}}{G}, \quad (10)$$

3

material under tension;  $\nu$  – Poisson ratio;  $\sigma$ ,  $\tau$  – normal and shearing stresses, accordingly;  $\varepsilon$ ,  $\gamma$  – relative elongation and relative shear deformation,  $T_{St}$  – standard rotor temperature; it was accepted to be equal to 293 °K. Thus, the absence of axial movement of the rotor mid-section and the absence of rotors deformation by external forces (they were neglected because they were small [5, 6]) were set as boundary conditions.

### 3. Dynamics of the free rotor and the possibility of its recovery from direct contact with the female rotor

The change in the additional-turn angle of the free rotor leads to it being forced to rotate with certain acceleration. This can be caused both by a direct contact of the rotors and by gas forces rotation torque. The screw-type compressor running at nominal rating typically encounters the last one: when driving by the male rotor - additional turn of the female rotor until the contact with the main rotor in the direction of rotors rotation. The value of this additional-turn angle can be calculated by the foregoing procedure. Therefore, for the rotors to continue being in transmission, the possible acceleration of rotor due to gas forces must exceed the required acceleration related to the variation of the additional-turn angle. This is performed upon the following condition:

$$\frac{d^2\theta_2}{d\tau^2} \geq \frac{d^2\beta_2}{d\theta_1^2} \cdot \omega_1^2. \quad (11)$$

The approach to determining  $\frac{d^2\theta_2}{d\tau^2}$  is considered in [5]; the following development of this approach is presented further on. From the dynamics equation, it can be written:

$$\frac{d^2\theta_2}{d\tau^2} = \frac{M_{GF} + M_{AF} - M_R - M_{F1} - M_{F2} - M_{MEC}}{J_2}, \quad (12)$$

where  $J_2$  – moment of inertia of a free rotor;  $M_{GF}$  – the moment produced by gas forces is determined on the basis of the indicator (P-V) diagram by using the "subtend chords" method [3, 7];  $M_{MEC}$  – moment of resistance of the mechanical components of the compressor such as seals, balance pistons and bearings (its value is mainly determined by the design of the mechanical components of the rotor);  $M_R$ ,  $M_{F1,2}$  – friction torques in the compressed medium in the radial and axial clearances, respectively;  $M_{AF}$  – moment created by adhesion forces. It is not difficult to show that in this case the term "adhesion" may be replaced by the term "cohesion" that is close to it. Then, this moment will be proportional to the surface tension coefficient, the contact area and the wetting angle; at the same time it should be noted that this moment will be much less than the other moments of forces, which in turn permits to ignore them in the further calculations:  $M_{AF} \approx 0$ .

According to the calculation diagram, the moments of friction in the compressed medium in the end clearances depend on the operating mode:

$$M_{F1,2} = \begin{cases} \frac{\pi \cdot \omega_2 \cdot \eta_{MIX} \cdot R_{eq}^4}{2 \cdot \delta_{F1,2}} \cdot \left(1 - \frac{r_R^4}{R_{eq}^4}\right), & \text{if } Re_{F1,2} < 10^4 \\ \frac{\rho_{MIX} \cdot \omega_2^3}{4} \cdot (c'_{M1,2} \cdot R_{eq}^5 - c''_{M1,2} \cdot r_R^5), & \text{if } 10^6 \geq Re_{F1,2} \geq 10^4 \end{cases}, \quad (13)$$

where  $c_{M1,2} = 0,0277 \cdot \text{Re}_F^{-0,2} \cdot \left( \frac{\delta_{F1,2}}{R} \right)^{-0,2}$  – friction moment coefficient; for determining  $c_M''$  as the characteristic value,  $r_R$  ( $R = r_R$ ) is used, in all other cases –  $R_{eq}$  ( $R = R_{eq}$ );  $\text{Re}_F = \frac{R^2 \cdot \omega_2 \cdot \rho_{MIX}}{\eta_{MIX}}$  – Reynolds number;  $\omega_2 = \omega_1 \cdot \frac{z_1}{z_2}$  – nominal angular speed of a free rotor;  $\rho_{MIX}$ ,  $\eta_{MIX}$  – respectively, the density and kinematic viscosity of the gas-oil mixture in the working space of the compressor;  $\delta_{F1}$ ,  $\delta_{F2}$  – end clearances between the rotor and the housing on the suction and discharge sides, respectively. The equivalent radius is determined as  $R_{eq} = \left( \frac{1}{2 \cdot \pi} \int_0^{2 \cdot \pi} (r(t;0))^4 \cdot dt \right)^{\frac{1}{4}}$ , where  $r(t;0)$  is determined for a certain fixed rotation angle  $\theta$ , for which it is expedient to take the initial position:  $\theta = 0$ .

Similarly, the friction torques in the compressed medium in the radial clearance can be written as:

$$M_R = \begin{cases} \frac{2 \cdot \pi \cdot \xi \cdot \omega_2 \cdot \eta_{MIX} \cdot R_{2*}^3 \cdot z_2 \cdot b_R}{(1 + \xi) \cdot \delta_R}, & \text{if } \text{Re}_R < 2500 \\ \pi \cdot c_R \cdot \rho_{MIX} \cdot \omega_2^2 \cdot z_2 \cdot b_R \cdot R_{2*}^4, & \text{if } 10^5 \geq \text{Re}_R \geq 2500 \end{cases}, \quad (14)$$

where  $c_R \approx 0,0076 \cdot \text{Re}_R^{-0,25}$  – friction coefficient,  $\text{Re}_R = \frac{\omega_2 \cdot R_{2*} \cdot \delta_R \cdot \rho_{MIX}}{\eta_{MIX}}$  – Reynolds number,  $b_R$  – rotor tooth thickness in the direction of Z axis,  $R_{2*}$  – outer diameter of the rotor,  $\delta_R$  – radial clearance between the rotor and the housing of the compressor,  $\xi = \frac{m_{OIL}}{m_{GAS}}$  – gas-oil ratio.

The thermodynamic parameters of the mixture are determined by the following relations:

$$\eta_{MIX} = \left( \frac{1}{1 + \xi} \right) \cdot \eta_{GAS} + \left( \frac{\xi}{1 + \xi} \right) \cdot \eta_{OIL}, \quad (15)$$

$$\rho_{MIX} = (1 + \xi) \cdot \rho_{GAS}. \quad (16)$$

Thereby the suction temperature for the clearance on the suction side and the discharge temperature for the clearance on the discharge side can be taken as a determining temperature for the equation (19) in the preliminary calculations. An arithmetic average temperature between suction and discharge can be taken as a determining temperature for the equation (14). In order to simplify the calculations, the value  $\xi$  can be assumed constant for all parts of the rotor.

In compliance with the inequality (11), the design angle of rotation of the female rotor is determined as:

$$\theta_2 = \theta_1 \frac{z_1}{z_2} + \beta_{\min a}^+ \quad (17)$$

At the moment, when the compliance with the inequality (11) ends, the position of the female rotor should be determined by integrating the differential equation (12) with the following boundary conditions, taking the time of separation as a reference point:

$$\begin{cases} \theta_2(0) = \theta_1 \frac{z_1}{z_2} + \beta_{\min a}^+ \\ \frac{d\theta_2}{d\tau}(0) = \omega_1 \frac{z_1}{z_2} \end{cases} \quad (18)$$

The relation between the integration time  $\tau$  and the angle of rotation of the male rotor is determined by the expression

$$\omega_1 = \frac{d\theta_1}{d\tau}. \quad (19)$$

The integration is carried out until the rotors contact again. Thereby, at the moment of contact due to the elasticity of impact a rebound will occur of the rotors relatively to each other. However, its value is determined by virtue of the low relative velocity of the contact points and the high value of the damping capacity of the greasy layer. Therefore, the conditions for continuation of the integration will have the form:

$$\theta_1 \frac{z_1}{z_2} - \beta_{\min a}^- < \theta_2 < \theta_1 \frac{z_1}{z_2} + \beta_{\min a}^+. \quad (20)$$

When coming into contact on the back side of tooth, the recess condition (11) will change its sign to the opposite:

$$\frac{d^2\theta_2}{d\tau^2} \leq \frac{d^2\beta_2}{d\theta_1^2} \cdot \omega_1^2 \quad (21)$$

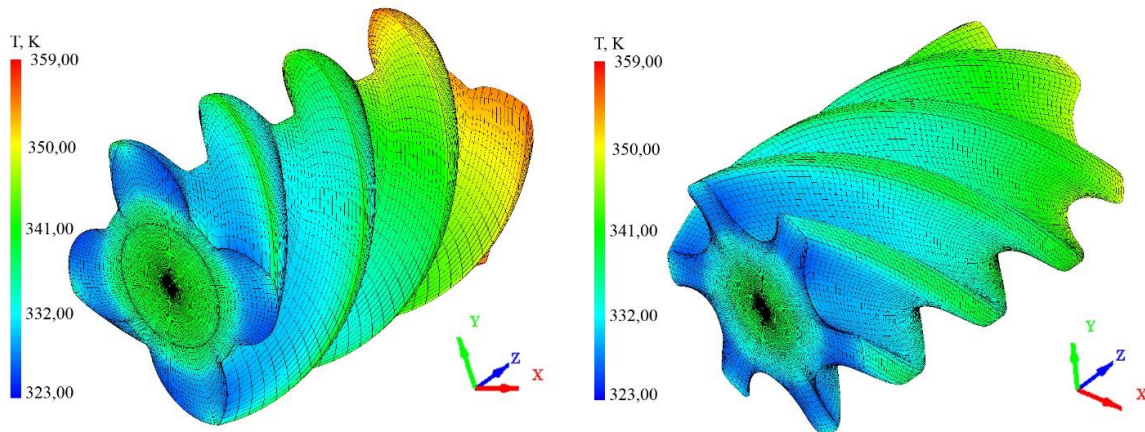
It should be noted also the fact that, under normal operating conditions, the rotors loss of contact occurs only at the moment of teeth disengagement. In this case, the dynamics of the female rotor in a free state can be neglected and its position is determined by the maximum angle of the possible deviation, which is the additional-turn angle.

#### 4. Example of the use of the presented procedure and analysis of the obtained results

A standard screw compressor was selected as the object of the research, designed in V.B. Shnepp NIIturbokompressor (HMS GROUP) CJSC (the successor of SKBK (Special Boiler Design Bureau)) with 250 mm rotor diameter, 340 mm length of the profiled section of the rotors, 440 mm and 660 mm screw pitch for the male and female rotors, respectively, and transmission ratio of 4/6, which is analogous to the profiles described in [8]. The following values were taken for end clearances between the rotor and the housing: on the suction side – 0.5 mm, on the discharge side – 0.06 mm. The value of 0.2 mm was taken as the radial clearance. As a mode being studied, the following parameters were selected: the mode of compressor operation in the air with compression rate from 1 to 9 at (pressure ratio  $P = 9$ ), suction temperature 25 °C, discharge temperature 90 °C, temperature of the injected oil 40 °C and volume ratio equal to 4.5. The gas-oil ratio was taken equal to 3.8.

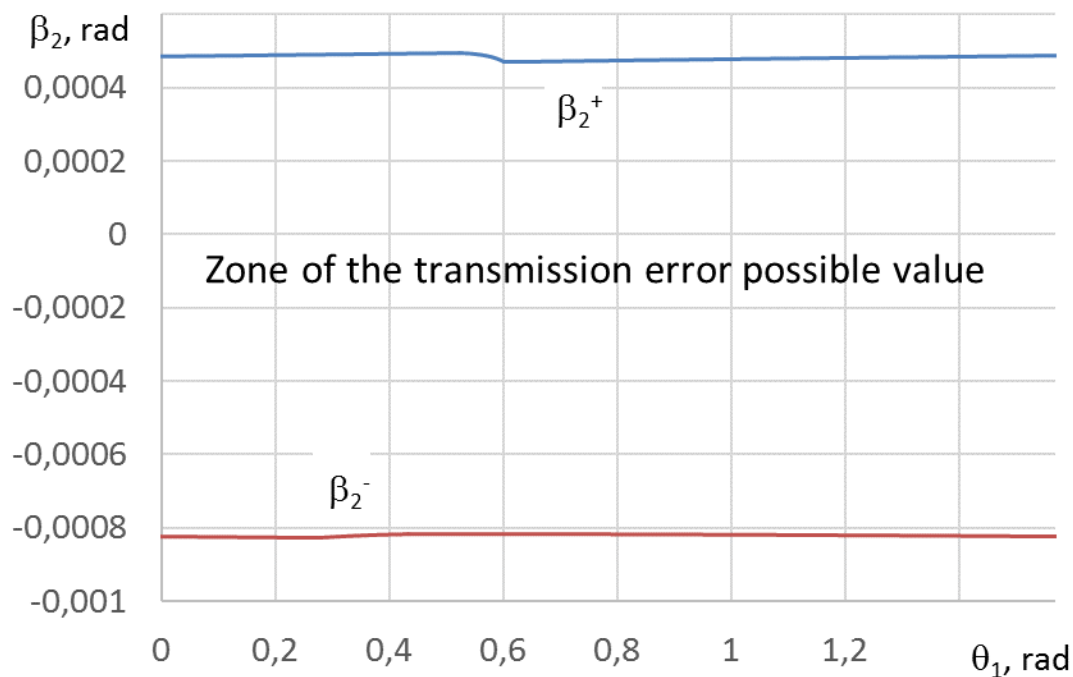
The calculation results of the temperature patterns of the female rotor, as well as the dependency diagrams characterizing the transmission in the compressor operation, are shown in figure 3 and figure 4. The calculations are presented for the case where the compressor design does not provide a suction port on the radial housing boring



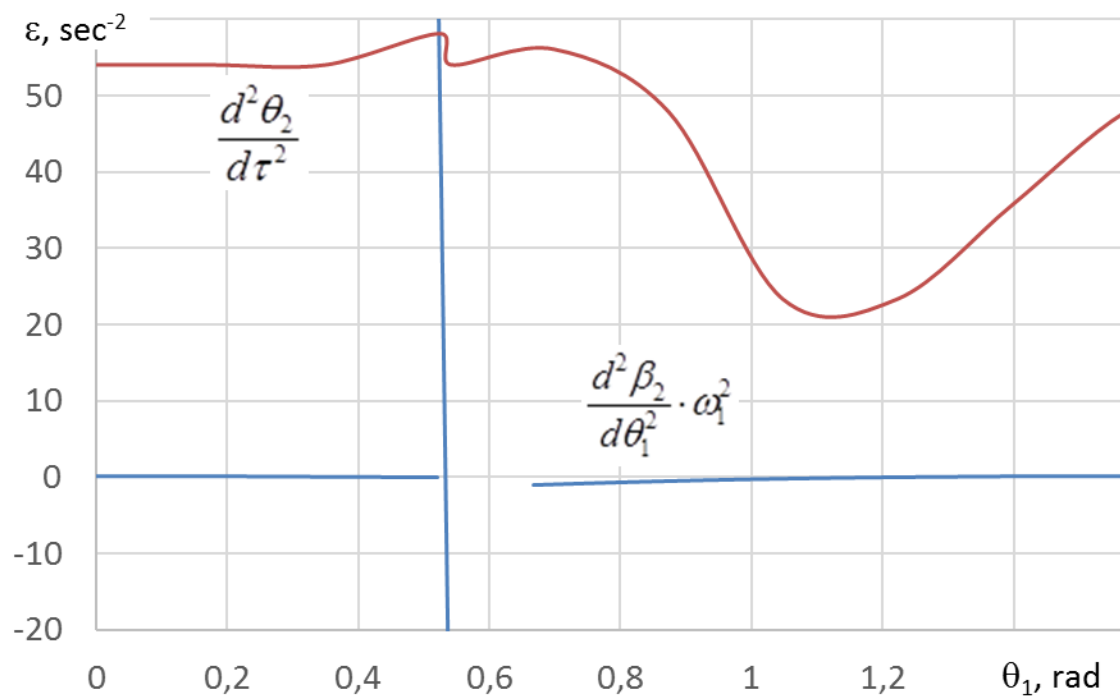


**Figure 3.** Design temperature patterns of the rotors

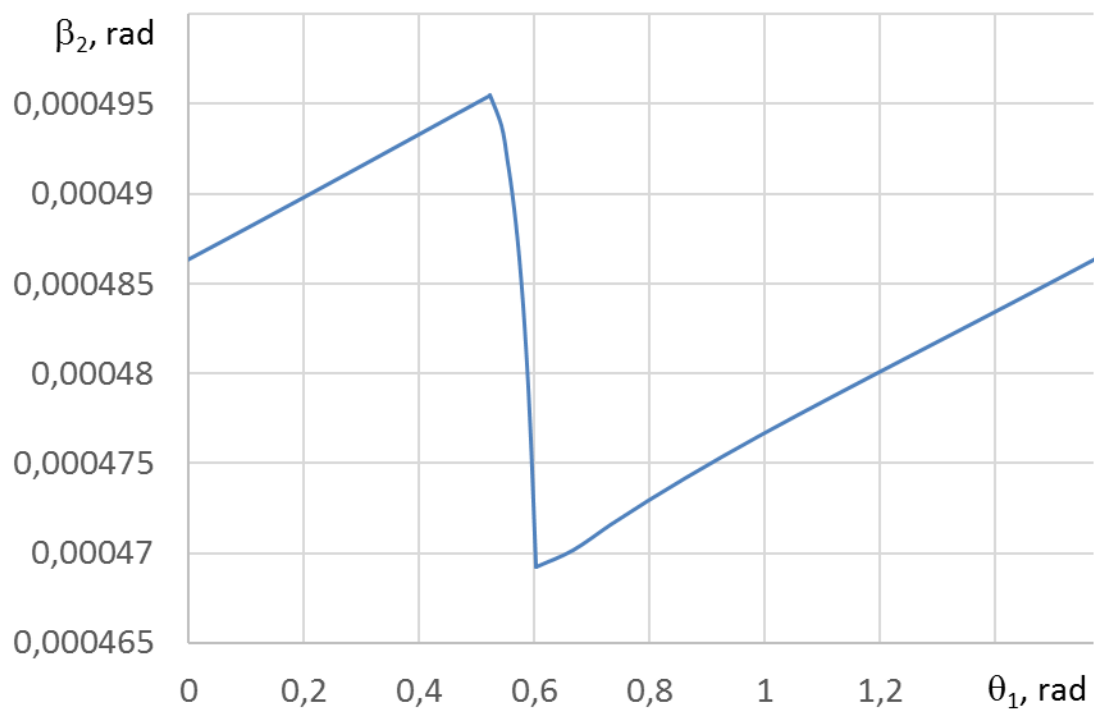
The dependences characterizing the transmission are shown in figure 5. This figure shows that the second derivative of the shift angle has two gaps. The first gap is at the moment of teeth relocation in the transmission, the second one – at the moment when the contact point starts running along the rotor axis (at the initial moment the contact occurs in the initial cross section of the rotor). Analyzing this diagram, it can be concluded that the process of contact loss in transmission for the mode being studied occurs only at the moment of teeth relocation. For the rest of the time, the female rotor is pressed to the male rotor. It provides, in turn, that the required adjustment of the female rotor rotation angle, characterizing the transmission error, is determined by the  $\beta_{\min\alpha}^+$  values. The dependence in its final form is shown in figure 6.



**Figure 4.** Possible deviations of the rotation angle of the female rotor



**Figure 5.** Transmission characteristic curve



**Figure 6.** Real deviation value of the rotation angle of the female rotor



## 5. Conclusion

The presented methodology permits to do forecasting by calculating the real position of the rotors in the transmission. Its development may clarify the calculation of the operating process of the screw-type compressor and form the basis for calculation of the contact stresses and for vibration forecasting. At the same time, it can be used already at the design study stage.

## Acknowledgments

This study is a part of scientific investigations supported and funded by V.B. Shnepp NIIturbokompressor (HMS GROUP) CJSC.

## References

- [1] Buckney D, Kovacevic A, Stosic N 2015 Rotor clearance design and evaluation for an oil injected twin screw compressor *IOP Conf. Ser.: Mater. Sci. Eng.*, 90
- [2] Buckney D, Kovacevic A, Stosic N 2014 Accounting for Local Thermal Distortions in a Chamber Model for Twin Screw Compressors *International Compressor Engineering Conference*. Paper 2306.
- [3] Yakupov R R, Mustafin T N, Nalimov V N, Khamidullin M S and Khisameev I G 2013 Discussion of actual profile clearances' calculation method in rotary compressors in the absence of rotor timing units *8th Int. Conf. on Compressors and their Systems (London: City University)* 209
- [4] Yakupov R R, Mustafin T N, Nalimov V N, Khamidullin M S and Khisameev I G 2014 Analysis of transmission error depending on compressor working conditions *J of Proc. Mechanical Engineering (Proc. of the Institution of Mechanical Engineers) Part E (London: SAGE Publications)*
- [5] Mustafin T N, Yakupov R R, Burmistrov A V, Khamidullin M S, Khisameev I G 2015 Analysis of the screw compressor rotors' non-uniform thermal field effect on transmission error *IOP Conf. Ser.: Mater. Sci. Eng.* 90
- [6] Mustafin T N and etc. 2016 Analysis of influence of screw compressor construction parameters and working condition on rotor temperature fields *Procedia Engineering* **152**,
- [7] Adams G P and Soedel W 1994 Dynamic simulation of rotor Contact Forces in Twin Screw Compressors *Proc. Int. Compressor Engineering Conference*, (West Lafayette: Purdue University)
- [8] Khisameev I G and Maksimov V A 2000 *Dvukhrotornyye vintovyye I pryamozubyye kompressory. Teoriya, raschet i proyektirovaniye (Twin rotors screw and spur compressors. Theory, calculation and design (in Russian))* (Kazan: Fen)