

# Study on influence of coaxiality deviation on interfacial pressure of shrink-fit cylinder

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**Abstract:** The paper established the mathematical model of two-layer shrink-fit cylinder with proper coaxiality deviation based on the theory about elastic-plastic shrink-fit cylinder. An example was applied to show that the interfacial pressure value of the mathematical model was identical with the data of finite element analysis, which indicated the model conformed to actual condition and possessed some theoretical guidance for actual project. The application of the model indicated the coaxiality deviation of shrink-fit cylinder should be controlled strictly since the interfacial pressure between inner cylinder and outer one decreased gradually around the circumference.

## 1. Introduction

The hydraulic cylinder is an indispensable part of the hydraulic system, widely used in agriculture, engineering and other heavy machinery. During the hydraulic cylinder design process, not only to make the hydraulic cylinder have enough thrust, speed and effective stroke, which also need to have enough stiffness and strength, to make the design could sustain fluid pressure and other external forces. With the development of hydraulic equipment in the direction of high pressure and high power, the property of hydraulic cylinder should get higher requirements.

In order to improve the bearing capacity of the cylinder block and get uniform stress distribution, the most commonly method is use shrinkable sleeve technology for cylinder block to get prestress. In the elastoplastic theory, the premise of shrinkable sleeve cylinder theory is assume inside and outside cylinder coaxial completely (absolute ideal state). But the actual process of heat shrinkable sleeve, due to the affect of machining accuracy, mechanical vibration, temperature control error and other restriction, inner cylinder and outer cylinder inevitable will have axiality deviation, and must affect the cylinder's interface pressure. Therefore, for shrinkable sleeve cylinder precision industry, the axiality deviation of shrinkable sleeve cylinder pressure distribution regular is particularly important.

## 2. The mathematical model of hydraulic cylinder under axiality deviation

### 2.1. elastic-plastic shrinkable sleeve cylinder theory

Completely coaxial double-deck shrinkable sleeve cylinder plane strain is shown in figure 1. The shaded part is the theory of interference values relative of the outer and inner cylinder. The entire circumference of interference value are equal. Inside radius of inner cylinder, outer radius of inner cylinder and inside radius of outer tube respectively is  $r_a$ ,  $r_b$ ,  $r_c$ ; It is assumed that the material of cylinder body is ideally elastic-plastic, the elastic modulus is E, which inside and outside the axis of



the cylinder are in a same line; Make the inside radius ratio are  $K = r_c / r_a$ ,  $k_1 = r_b / r_a$ ,  $k_2 = r_c / r_b$ , by above conditions, **Timosenko** inferred the pressure of cylinder between inside and outside is [3]

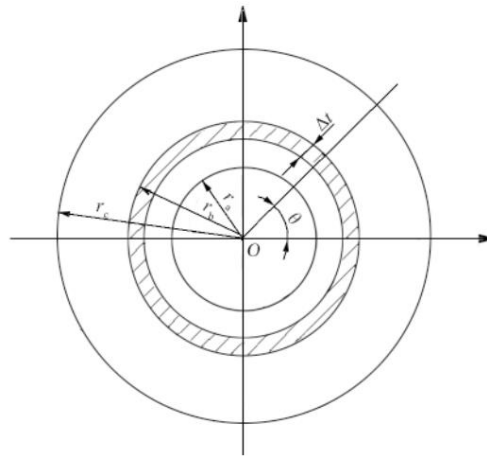
$$p_{1,2} = \frac{\Delta E}{2r_b k_1^2} \left[ \frac{(k^2 - k_1^2)(k^2 - 1)}{k^2 - 1} \right] \quad (1)$$

In lame formula the circumferential stress  $\sigma_t$  and radial stress  $\sigma_r$  respectively are

$$\sigma_t = \frac{p_i - p_u K^2}{K^2 - 1} + \frac{(p_i - p_0) K^2}{K^2 - 1} \left( \frac{r_a}{r} \right)^2, \quad (2)$$

$$\sigma_r = \frac{p_i - p_0 K^2}{K^2 - 1} - \frac{(p_i - p_u) K^2}{K^2 - 1} \left( \frac{r_a}{r} \right)^2, \quad (3)$$

In the formula:  $p_i$  is the cylinder's inner pressure;  $p_0$  is the cylinder's outer pressure.



**Figure 1.** Polar coordinate of shrink-fit cylinder without coaxiality deviation.

Assumption the cylinder body without internal pressure and external pressure when study of interface pressure of the cylinder. In the absence of external and internal pressure, the maximum pressure of shrink-fit interface is appeared between the outer wall of inner cylinder and the inner wall of outside cylinder[4].

## 2.2. The mathematical model of the double-deck shrinkable sleeve cylinder which have axiality deviation

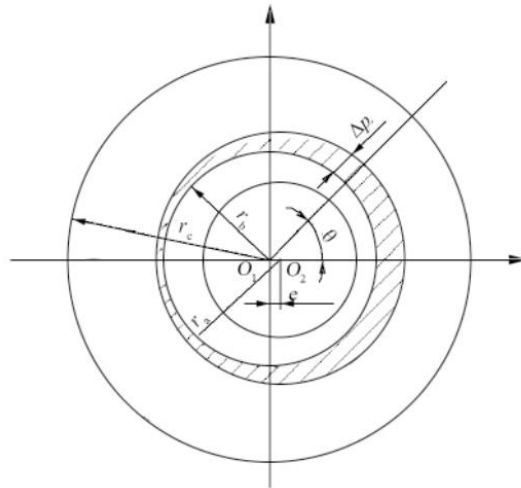
On the basis of the principle of completely coaxial double-deck shrinkable sleeve cylinder. Establish the math model of exist axiality deviation double-deck shrinkable sleeve cylinder. As the length-diameter of the cylinder block is relatively large, also assume the cylinder material evenly and it has the ideal elastic-plastic material. Exist axiality deviation cylinder inside and outside cylinder axis is still a straight line, and the outer wall of inner tube with the inner wall of the outer tube is always stay in touch. In order to get the relationship of actual interference values  $\Delta p$  and uniform variable angle  $\theta$ , put the coaxial deviation of cylinder model into the polar coordinate, see figure 2. Make the outer wall of off-axis shrinkable sleeve cylinder become rigid surface, and the center of the circle in the original point, put the inner cylinder center in increasing x, get  $e$  is the eccentricity of inside and outside cylinder,  $(e, 0)$  is the inner cylinder center in polar coordinate. The shadow part shows the interference values of inner tube relative to outer tube

$(r_{bi})_i$  is the radius of outer cylinder,  $\rho_{oi}$  is polar radius, and the polar equation of inner wall belong to outer cylinder is gotten

$$\rho_{oi} = (r_{bi})_i, \quad (4)$$

Assume outer wall of inner cylinder is  $(r_{bi})_o$ , the polar radius is  $\rho_{io}$ , the same can get the polar equation of outer wall belong to inner cylinder

$$(\rho_{io} \cos \theta)^2 + (\rho_{io} \sin \theta)^2 = (r_{bi})^2 \quad (5)$$



**Figure 2.** Polar coordinate of shrink-fit cylinder with coaxality deviation.

Due to the interference values of Partial axis cylinder is distribution axisymmetric about X, so just need to study range  $\theta \in [0, 180]$ . From the formula(5),  $e \cos \theta$  within the range can be positive or negative, due to the  $(r_{bi})_0 \gg e$ , so  $[(r_{bi})_0^2 - e^2 \sin^2 \theta]^{1/2}$  is positive definite, and  $\rho_{oi} > 0$ . After above conditions, we can get the outer wall of inner cylinder's polar radius

$$\rho_{oi} = e \cos \theta + [(r_{bi})_0^2 - e^2 \sin^2 \theta]^{1/2} \quad (6)$$

As shown in figure 1, when the shrinkable sleeve cylinder inside and outside axis completely coaxial, make the inner wall of outer tube become rigid surface, and the theory interference values is  $\Delta t$  (the shadow part), so the relationship between inner radius of outer cylinder and outer radius of inner cylinder is

$$(r_{bi})_0 = (r_{bo})_i + \Delta t \quad (7)$$

The different angles for actual interference values

$$\Delta p = \rho_{io} - \rho_{oi} \quad (8)$$

Take(4),(6),(7)into(8), and make

$$\left\{ [(r_{bo})_i + \Delta t]^2 - e^2 \sin^2 \theta \right\}^{1/2} = C$$

Then simplified got

$$\Delta p = e \cos \theta - (r_{bo})_i + C \quad (9)$$

Assume that the inner wall of outer cylinder is rigid surface and not eccentric, the delamination radius of off-axis cylinder is  $r_b = (r_{bo})_i$ ; due to  $e = r_a$ ; In order to simplify the problem, the inner and outer

cylinder circle can be as a doublication, take (9) to (1) can get the shrinkable sleeve interface stress of the eccentric shaft cylinder

$$p_{1,2} = \frac{[e \cos \theta - (r_{b0})_1 + C] E \left[ \frac{(K^2 - K_1^2)(K_1^2 - 1)}{K^2 - 1} \right]}{2(r_{b0})_1 K_1^2} \quad (10)$$

Make the work inner pressure  $p_i = 0$  when research interface stress, and basis on the lame formula (2), (3), by superimposing the tube outer wall stress and the interfacial residual stress, get when  $p_i = 0$ , the outer wall of inner cylinder circular stress and radial stress are

$$(\sigma_u)_o = -p_{1,2}^\Delta \frac{K_1^2 + 1}{K_1^2 - 1} \quad (11)$$

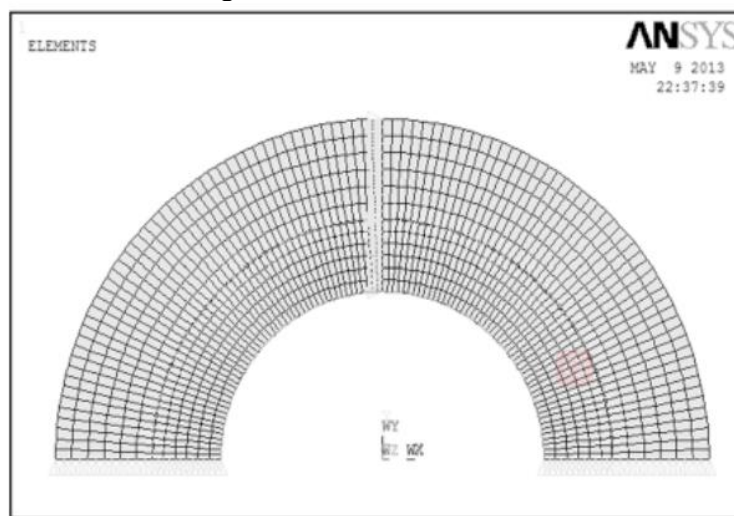
$$(\sigma_N)_o = -p_{1,2}^\Delta \quad (12)$$

### 3. shrinkable sleeve cylinder real-case analysis

#### 3.1. Finite element analysis of cylinder body under the axially deviation

An imported shrinkable sleeve hydraulic cylinder inside and outside the tube material is OCr17Ni4CuNb, the elastic modulus is  $E = 213\text{GPa}$ . The shrinkable sleeve cylinder is optimized through the equal strength theory: inner cylinder  $r_b = 32.7\text{mm}$ , delamination radius  $r_c = 46.5\text{mm}$ , outer cylinder radius  $r_b = 23.0\text{mm}$ , compared with outer tube the best interference quantity of inner cylinder is  $0.0768\text{mm}$ .

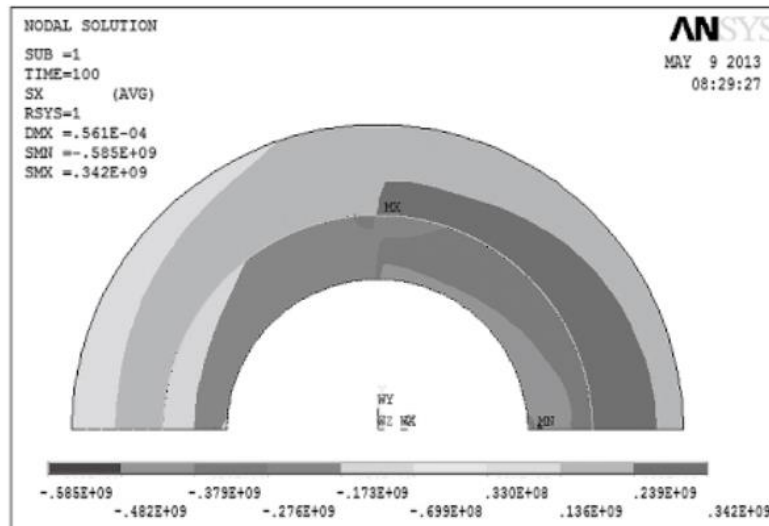
The inner tube radius of the cylinder's finite element model is  $23.0\text{mm}$ , the outer radius of inner tube is  $32.7768\text{mm}$ , the inner radius of outer tube is  $32.7\text{mm}$ , the outer radius of outer tube is  $46.5\text{mm}$ , the eccentricity of inside and outside tube relatively is  $0.03\text{mm}$ . choose the PLANE183 unit and use mapping method to generate mesh, the recycling number is 1200, through the contact wizard to establish contact and set the contact parameters. When research the situation of cylinder's prestress distribution, add displacement limit in X,Y direction, the inner pressure  $P_i = 0$  means not have inner-pressure. The mesh model shown in figure 3.



**Figure 3.** Meshed finite element model.

The situation of interface pressure distribution shown in figure 4. As the inner wall of outer tube give a prestress to the outer wall of inner tube, the maximum interface pressure always exist in the

junction between inner and outer cylinder, so the interfacial pressure is key. From formula (12) we known ,when inner stress  $P_i = 0$  , the radial force of cylinder equal the interface pressure of shrinkable sleeve .



**Figure 4.** Distribution of interfacial pressure (radial pressure ) of shrink-fit cylinder.

Use the same angle as the step length in the circumference of the inner wall belongs out tube, extract the radial numerical of finite element model. (Summary in table 1)

**Table 1** Comparison of theoretical interfacial pressure value and FEM one of shrink-fit cylinder with coaxiality deviation.

Angle /°C	Interference value /mm	Interference stress simulation value / MPa	Interface stress theoretical value /MPa	Relative error value of theoretical and simulation /%
0.00	0.1068	368.91	355.40	3.80
10.91	0.1063	368.54	353.59	4.23
21.82	0.1046	362.91	348.24	4.21
32.73	0.1020	353.23	339.54	4.03
43.64	0.0985	343.06	327.79	4.66
54.55	0.0942	332.31	313.44	6.02
65.45	0.0893	301.57	297.00	1.54
76.36	0.0839	269.23	279.06	3.52
87.27	0.0782	235.53	260.27	9.51
98.18	0.0725	265.15	241.31	9.88
109.09	0.0670	231.12	222.87	3.70
120.00	0.0618	198.53	205.62	3.45
130.91	0.0571	175.81	190.16	7.55
141.82	0.0532	160.35	177.08	9.45
152.73	0.0501	148.05	166.82	11.25
174.55	0.0480	140.72	159.78	11.93
163.64	0.0469	138.77	156.19	11.15

### 3.2.The calculation of shrinkable sleeve cylinder mathematical model in axially deviation

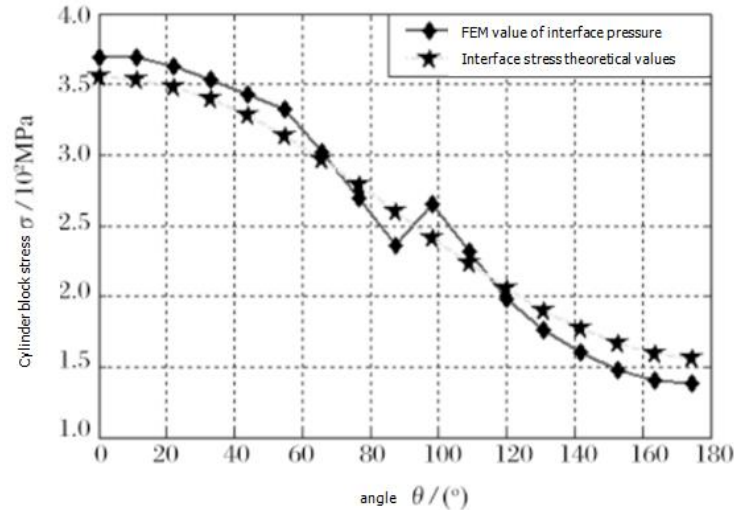
The outer tube eccentricity of this import shrink-fit sleeve cylinder is  $e = 0.03$  mm, therein

$$K = \frac{r_c}{r_a} = \frac{46.5}{23} = 2.0217, \quad K_1 = \frac{r_b}{r_0} = \frac{32.7}{23} = 1.4317, \quad K = \frac{r_c}{r_0} = \frac{46.5}{23} = 1.4220,$$

The elasticity modulus of cylinder material is  $E = 2.13 \times 10^5$  MPa, the inner wall radius of outer tube is  $(r_{bi})_0 = 23$  mm, the theory of interference value  $\Delta t = 0.0768$  mm,  $\theta = 10.91^\circ$ , take them into formula (9),(10),(12); respectively obtain the shrinkable sleeve pressure  $p_{1.2}^\Delta$  in angle  $\theta$ , The actual interference value  $\Delta p_{(\theta=10.9^\circ)}$ , and radial stress value about outer wall of inner tube  $(\sigma_{ri})_0$ . Through above calculate of mathematical model, the interface pressure of the node corresponding to finite element model and the corresponding interference value are shown in table 1.

### 3.3.data processing

Select the whole finite element model's node that in the outer wall of inner cylinder, use same angle as the step length in the circumference of out wall inner tube and extract the radial stress of the finite element model. Through the mathematical model's calculation and gather the corresponding amount of interference, we can compared the two aspect and verified; from table 1 we known, only in  $\theta = 152.73^\circ$ ,  $\theta = 163.64^\circ$ ,  $\theta = 173.55^\circ$ , the relative error more than 10%, the other angle's relative error within 10%. In  $[0, 180]$  the average value of all node's relative error is 6.46%. Shown in figure 5, in  $[0, 180]$  entire range, the calculate value of finite element and the mathematical cylinder model of shrinkable sleeve cylinder axiality deviation are essentially agree, so the model can be applied to practical engineering.



**Figure 5** Comparison of calculation stress value by model and FEM one.

### 3.4.conclusion analysis

(1) The experimental example of axiality deviation of shrinkable sleeve cylinder mathematical model showing the average relative error is 6.46% that compared the theory value of interface pressure model with finite element calculation. The theory value of model with finite element calculation are essentially agree.

(2) The outer wall of inner tube's interface pressure uneven distribution in counterclockwise direction and have the decrease trend in the  $[0, 180]$  when the shrinkable sleeve cylinder exist axiality

deviation. The maximum interface pressure  $(p_{1.2}^{\Delta})_{\max}$  is near  $\theta = 0^{\circ}$ , the minimum interface pressure  $(p_{1.2}^{\Delta})_{\min}$  is near  $\theta = 180^{\circ}$ , above two values are differ to 199MPa, so for the precise shrink-fit cylinder the coaxility deviation must be strict controlled.

#### 4. Epilogue

The model truly reflect the affection of coaxility deviation to the shrink-fit cylinder's interface pressure, which could provide certain theoretical guiding significance for high demand precision shrink-fit cylinder. In order to improve the model, we should select the axiality deviation values for equal step in the optimal range of interference values, and make the model can comprehensively reflect the influence of axiality deviation on shrinkable sleeve cylinder's interface pressure; through increase the grid number of finite element model to improve the accuracy of the finite element calculate value.

#### Appendix

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