Nonlinear quasi-static analysis of ultra-deep-water top-tension riser

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Abstract. In order to analyse the ultra-deep-water top-tension riser deformation in drilling conditions, a nonlinear quasi-static analysis model and equation are established. The riser in this model is regarded as a simply supported beam located in the vertical plane and is subjected to non-uniform axial and lateral forces. The model and the equation are solved by the finite element method. The effects of riser outside diameter, top tension ratio, sea surface current velocity, drag force coefficient, floating system drift distance and water depth on the riser lateral displacement are discussed. Results show that the riser lateral displacement increase with the increase in the sea surface current velocity, drag force coefficient and water depth, whereas decrease with the increase in the riser outside diameter, top tension ratio. The top tension ratio has an important influence on the riser deformation and it should be set reasonably under different circumstances. The drift of the floating system has a complicated influence on the riser deformation and it should avoid a large drift distance in the proceedings of drilling and production.

1 Introduction

Top-tension riser is the key equipment connecting subsea blowout preventer system and floating system (drilling platform or ship) in ultra-deep-water oil and natural gas drilling and production. The mechanical behavior of the riser is very complicated under the combined action of the top tension and the ocean loads. It is a prerequisite for reasonable riser structural design that accurately predicting the mechanical behavior of the riser.

Scholars have developed a number of mathematical models on riser static mechanical behavior[3-7,11,14-15]. They calculated the deformation of the riser by using finite-difference approximation, finite element method and so on. They also analyzed the influence of different parameters on the riser deformation. The dynamic mechanical behavior of the riser has also been analyzed by scholars[1-2,8-10,12-13].

The quasi-static characteristic refers to ignore the dynamic effect of the wave load and take the maximum load combined with the wave and current as a static load. The above mentioned researches are mostly concentrated on the shallow-water and deep-water riser; however, the research of ultra-deep-water top-tension riser quasi-static mechanical behavior is insufficient. Therefore, the purpose of this paper is to obtain the ultra-deep-water riser quasi-static mechanical behavior in drilling and production conditions.

2 Analysis model

In drilling and production conditions, the riser near the water surface is connected to the floating system through the upper flex joint and the riser close to the seabed is connected to the subsea blowout preventer stack through the lower flex joint. The riser at the top end can move with the drift of the floating system. The simplified top-tension riser quasi-static mechanical model in ultra deep-water can be expressed in figure 1.

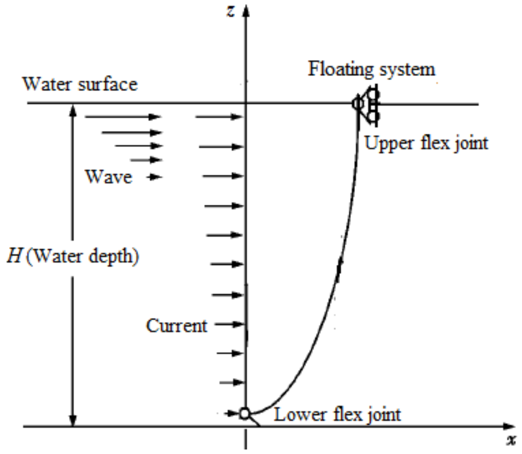

Figure 1: The simplified top-tension riser quasi-static mechanical model

The analysis model is regarded as a beam located in the vertical plane and subjected to both non-uniform axil force and lateral force. The riser differential control equation [5,7,14] can be represented as:

$$EI\frac{\mathrm{d}^4 x}{\mathrm{d}z^4} - T(z)\frac{\mathrm{d}^2 x}{\mathrm{d}z^2} - w\frac{\mathrm{d}x}{\mathrm{d}z} = F(z) \qquad (1)$$

Where, EI is the riser flexural rigidity, N·m^2; $T(z)$ is the riser effective axil force along the z axis, N; w is the per unit length wet weight of the riser, N/m; $F(z)$ is the lateral force along the z axis, N.

As is shown in figure 1, the boundary of equation (1) can be written as:

$$\begin{cases} x(0) = 0; EI\dfrac{\mathrm{d}^2 x(0)}{\mathrm{d}z^2} = K_\mathrm{b}\dfrac{\mathrm{d}x(0)}{dz} \\ x(d) = s_\mathrm{p}; EI\dfrac{\mathrm{d}^2 x(H)}{\mathrm{d}z^2} = K_\mathrm{u}\dfrac{\mathrm{d}x(H)}{dz} \end{cases} \qquad (2)$$

Where, K_b is the rotation stiffness of the lower flex joint, N·m/deg; K_u is the rotation stiffness of the upper flex joint, N·m/deg; H is the water depth, m; s_p is the floating system drift distance, m.

3 Riser force calculation

3.1 Effective axial tension

The effective axial tension on the riser cross-section considering the different pressures between the sea water and the internal fluid [6] can be represented as:

$$T(z) = T_\mathrm{top} - \int_z^H w\mathrm{d}z \qquad (3)$$

Where, T_top is the top tension generated at the top end of the riser, N.

The per unit length wet weight of the riser system can be represented as:

$$w = \frac{\pi}{4}\left[\rho_\mathrm{r}(D^2 - d^2) + \rho_\mathrm{f} d^2 - \rho_\mathrm{w} D^2\right]g \qquad (4)$$

Where, ρ_r is the density of riser, kg/m^3; ρ_f is the density of internal fluid, kg/m^3; ρ_w is the density of sea water, kg/m^3; D is the outer diameter of the riser, m; d is the inner diameter of the riser, m; g is the gravitational acceleration, m/s^2.

3.2 Lateral force

The combined wave and current per unit length force can be calculated by the Morison's equation [9,14,16]:

$$f(z) = 0.5C_{\mathrm{D}}\rho_{\mathrm{w}}D(u_{\mathrm{w}} + u_{\mathrm{c}})|u_{\mathrm{w}} + u_{\mathrm{c}}| + C_{\mathrm{M}}\rho_{\mathrm{w}}\pi D^2 a_{\mathrm{w}} / 4 \quad (5)$$

Where, C_{D} is the drag force coefficient, a dimensionless quantity; u_{w} is the horizontal velocity of sea wave particle, m/s; u_{c} is the current velocity, m/s; C_{M} is the inertia force coefficient, a dimensionless quantity; a_{w} is the horizontal acceleration of sea wave particle, m/s^2.

The current velocity under a certain depth can be represented as:

$$u_{\mathrm{c}}(z) = u_0(z/H) \quad (6)$$

Where, u_0 is the sea surface current velocity, m/s.

For ultra deep-water, the airy wave theory[14,16] is accurate enough to calculate the wave horizontal velocity, which is:

$$u_{\mathrm{w}} = \frac{gh}{2\omega}\mathrm{e}^{k(z-H)}\sin(kx - \omega t) \quad (7)$$

Where, h is the wave height, m; k is the wave number, a dimensionless quantity; ω is the wave circular frequency, rad/s.

The acceleration of wave particle points can be obtained by derivation of equation (7) with respect to t.

4 Model solution

Equation (1) is a complex nonlinear differential equation and it is difficult to get the solution by the analytical solution method. Therefore, in this paper, the finite element method was adopted. The vertical and horizontal bending beam element was adopted. The linear and cubic Hermite interpolation functions were used to discretize equation (1).

The shape function is shown as follows.

$$\begin{cases} N_1 = 1 - z/l; & N_2 = 1 - 3z^2/l^2 + 2z^3/l^3 \\ N_3 = z - 2z^2/l + z^3/l^2; & N_4 = z/l \\ N_5 = 3z^2/l^2 - 2z^3/l^3; & N_6 = -z^2/l + z^3/l^2 \end{cases} \quad (8)$$

Where, l is the element length.

The element stiffness matrix is composed of bending stiffness matrix and geometric stiffness matrix, which are caused by effective axial tension along the riser. Thus, the element stiffness matrix can be determined as:

$$[K]^{\mathrm{e}} = [K]^{\mathrm{e1}} + [K]^{\mathrm{e2}} \quad (9)$$

Where,

$$\begin{cases} [K]^{\mathrm{e1}} = \int_0^l \left(\frac{\partial^2 N}{\partial z^2}\right)^T EI \left(\frac{\partial^2 N}{\partial z^2}\right)\mathrm{d}z \\ [K]^{\mathrm{e2}} = \int_0^l \left(\frac{\partial N}{\partial z}\right)^T T(z) \left(\frac{\partial N}{\partial z}\right)\mathrm{d}z \end{cases} \quad (10)$$

So, equation (1) can be expressed as:

$$[K]\{u\} = \{f\} \quad (11)$$

Where, $[K]$ is the structural stiffness matrix; $\{u\}$ is the nodal displacement; $\{f\}$ is the load vectors of the node.

The Newton-Raphson iterative method was adopted to solve equation (11).

5 Application and case study

5.1 Validity of the analysis model

The analysis mode of this study has also been validated against the Abaqus (Beam23H) result of drilling riser in ultra-deep-water. The riser outer diameter is 533.4mm, the riser wall thickness is 25.4mm, the elastic modulus of the riser is 210GPa, the riser density is 7850kg/m^3, the water depth is 2000m, the sea water density is 1030kg/m^3, the wave height is 8m, the wave period is 10s, the sea surface current velocity is 1.5m/s, the internal fluid density is 1600kg/m^3, the drag force coefficient is 1.2, the inertia force coefficient is 2.0, the top tension is 1.6*G* (*G* is the riser system wet weight), the floating system drift distance is 0. In order to calculate the maximum value of the riser deflection, the rotational stiffness of the upper flex joint and the lower flex joint are all set to zero.

The comparison of the lateral displacement between the numerical result and the Abaqus result is shown in figure 2. It is found that the numerical result agree well with the Abaqus result. The numerical result are slight larger than Abaqus result. The slight difference between the numerical and the Abaqus result may be attributed to computational errors.

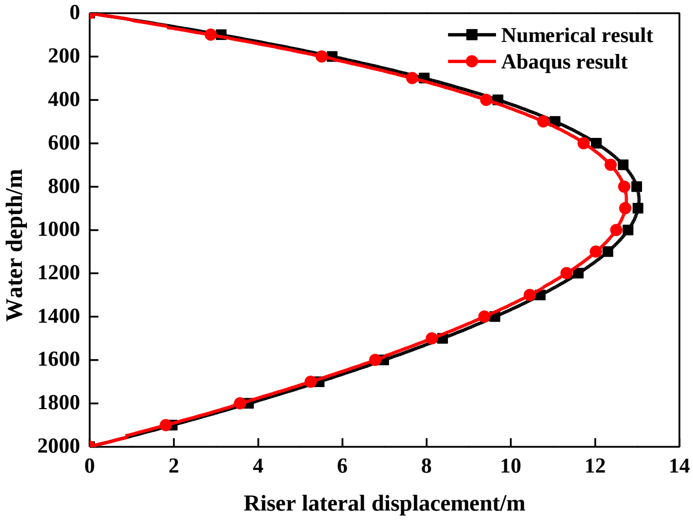

Figure 2: Comparison of the numerical result and Abaqus result

5.2 Analysis of the calculation result

As is shown in figure 2, the riser lateral displacement increases at first along the water depth direction and reaches the maximum value of 13.04m at the water depth of 860m and then decreases. The reason for this phenomenon is that although the lateral force is relatively large in the top area, but the influence of the axial force greater than the influence of the lateral force; so, the lateral displacement is relatively small. With the increase of the water depth, the influence of the lateral force gradually greater than the influence of the axial force on the riser lateral displacement; so, the lateral displacement is relatively large. With the continuous increase of the water depth, the lateral force is very small in the bottom area, the influence of the axial force greater than the influence of the lateral force; so, the lateral displacement is relatively small.

5.3 Analysis of quasi-static influencing factors

In the process of drilling, there are a number of influencing factors, such as riser size, and so on, which could affect riser quasi-static mechanical behavior. In order to better guide the quasi-static mechanical behavior of the riser in ultra-deep-water environment, six influencing factors are discussed as follows.

5.3.1 Riser outside diameter. In case the other parameters are constant (as defined in 5.1), the variations of riser deflection with riser outside diameter are shown in figure 3. The variation of the riser outside diameter has an influence on the riser lateral displacement. The effect of this influence has two aspects: first, the weight of the riser system increases with the increase in the outside diameter, which causes the axial tension increasing; second, with the increase of the outside diameter, the flexural rigidity of the riser increases. Both effects have a tendency to reduce the riser deflection. Therefore, as is shown in figure 3, with the increase of the riser outside diameter, the riser lateral displacement decreases gradually. Moreover, the maximum positions of the riser lateral displacement for these models are all at the water depth of 860m.

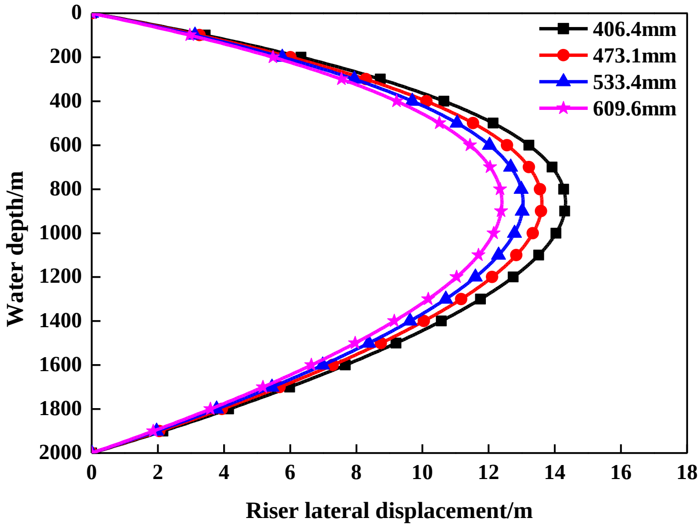

Figure 3: Variation of riser lateral displacement with riser outside diameter

5.3.2 Top tension ratio. In case the other parameters are constant, the variations of riser deflection with top tension are shown in figure 4. As analyzed in 5.3.1, the top tension has a tendency to reduce the riser deflection. Therefore, as can be seen from figure 4, the riser lateral displacement decreases gradually with the increase in the top tension ratio. In 1.2*G* top tension, the maximum riser later displacement up to 20.45m, but in 2.0*G* top tension, the maximum riser later displacement is only 9.71m. Moreover, with the increase of the top tension ratio, the depth of the maximum riser lateral displacement decreases gradually. In 1.2*G* top tension, the depth of the maximum riser lateral displacement is 950m, but in 2.0*G* top tension, the depth is 825m.

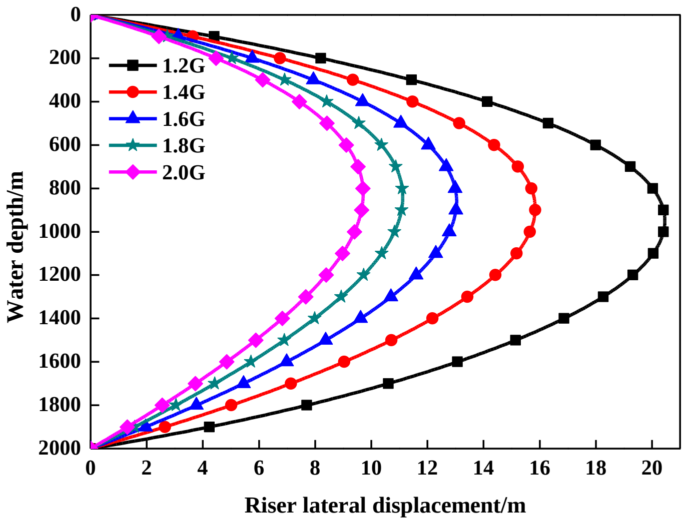

Figure 4: Variation of riser lateral displacement with top tension ratio

5.3.3 Sea surface current velocity. In case the other parameters are constant, the variations of riser deflection with sea surface current velocity are shown in figure 5. As can be seen from equation (6) and equation (7), the lateral force will increases with the increase in the surface current velocity, which has a positive influence on the riser deflection. Therefore, as can be seen from figure 5, with the increase of the surface current velocity, the lateral displacement increase significantly. Moreover, both the maximum positions of the riser lateral displacement for these models are all at the water depth of 860m.

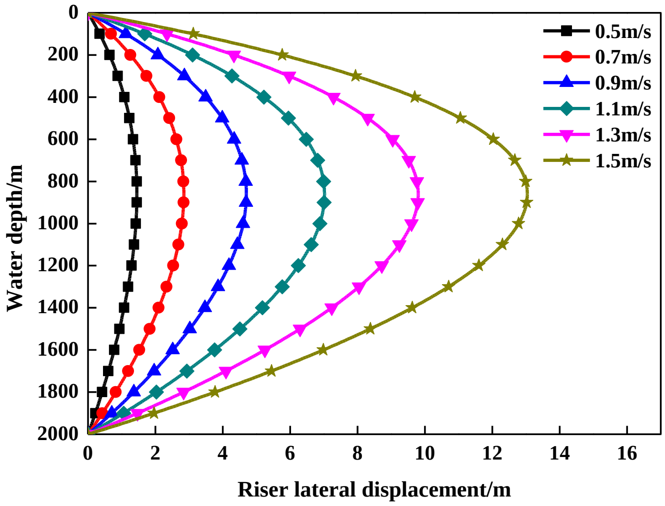

Figure 5: Variation of riser lateral displacement with sea surface current velocity

5.3.4 Drag force coefficient. In case the other parameters are constant, the variations of riser deflection with drag force coefficient are shown in figure 6. As is shown in figure 6, with the increase of the drag force coefficient, the riser lateral displacement increases significantly. The reason for this phenomenon is that the lateral force increases with the increase in the drag force coefficient, which can be seen from equation (5). Moreover, the maximum positions of the riser lateral displacement for these models are all at the water depth of 860m.

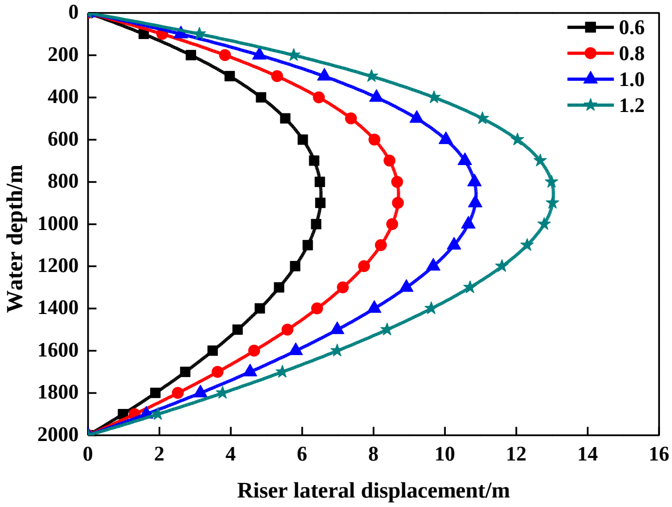

Figure 6: Variation of riser lateral displacement with drag force coefficient

5.3.5 Floating system drift distance. In case the other parameters are constant, the variations of riser deflection with floating system drift distance are shown in figure 7. As is shown in figure 7, there is a significant decrease in the depth of the maximum riser lateral displacement with the increase in the drift distance.

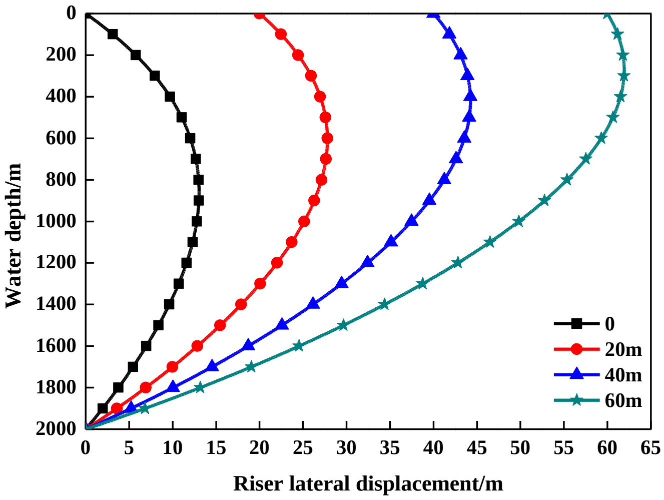

Figure 7: Variation of riser lateral displacement with floating system drift distance

5.3.6 Water depth. In case the other parameters are constant, the variations of riser deflection with water depth are shown in figure 8.As is shown in figure 8, with the increase of the water depth, the riser lateral displacement increases gradually. In addition, the depth of the maximum riser lateral displacement is roughly in a linear relationship with the water depth.

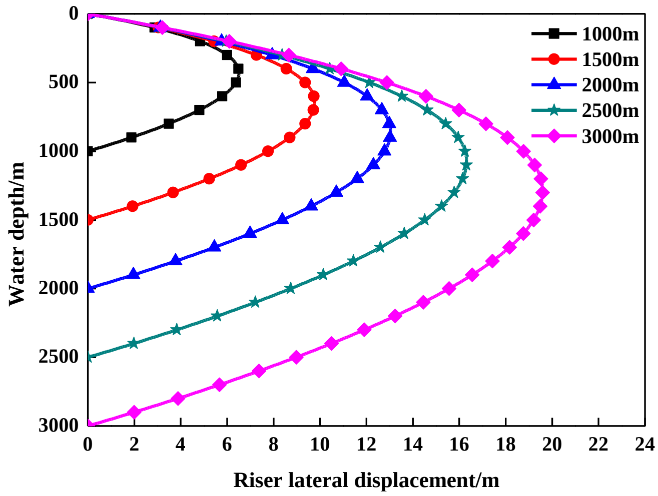

Figure 8: Variation of riser lateral displacement with water depth

6 Conclusions

(1) In the case where the floating system has no drift, with the increase of the top tension ratio, the riser lateral displacement decrease gradually and the depth of the maximum riser lateral displacement decreases gradually. The top tension has a significant influence on the riser deformation and the depth of the maximum riser lateral displacement is only related to the top tension ratio.
(2) In the case where the floating system has no drift, with the increase of the lateral force due to the change in the environmental parameters, the riser lateral displacement increase significantly.
(3) In the case where the floating system has no drift, the variation of the riser outside diameter has a significant influence on the riser lateral displacement.
(4) The floating system drift and the water depth have an important influence on the riser deformation. Therefore, the riser structural parameters should be designed rationally according to the water depth, and the floating system drift distance should be strictly controlled in the actual operating environment.

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