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Influence of driving voltage of liquid crystal on modulation phase

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Abstract. Based on the elastic theory and the dynamics equation of liquid crystal, we use Finite-Difference iterative method to calculate the liquid crystal molecules director distributions under the effect of electric field. According to the director distributions, this paper gets the relationship between LCD modulation phase and the driving voltage. The results of simulation proves that with driving voltage varying from 0 to 5v and the crystal modulation phase varies from 0 to 4π.

1 Introduction

The response characteristic of liquid crystal has important theoretical and practical significance for driver design and performance analysis of device. The elastic theory describing the basic theory of liquid crystal physical properties is put forward by C.W.Oseen in the 1930 s, it treats the liquid crystal as elastic continuous medium[1]. After that Frank proposed curvature elasticity theory which imitates solid elastic deformation theory and introduces the proportion coefficient - Hooke coefficient of solid elastic deformation to describe the director variation under the external field, namely the LCD continuum elastic deformation theory[2-3].

The continuum elasticity theory of liquid crystal describes the distribution function of the liquid crystal director with the position under the external force in the case of the boundary condition. In normal circumstances, there is no analytic solution of the distribution function, only the numerical calculation method can be used to calculate the distribution of the liquid crystal. In the numerical calculation method of liquid crystal director, there are Newton method, relaxation method and differential iteration method. The Newton method uses the angle and twist angle of the director to describe the spatial orientation of the liquid crystal director. It is suitable for the simple twist series type liquid crystal devices. The relaxation method is more general than the Newton method, and solves a variety of liquid crystal devices easily, but the relaxation method needs to introduce time parameters and viscosity coefficient. Differential iteration and Newton method are the same, using the tilt and twist angle to describe the liquid crystal, and spatial orientation of the vector is the same as relaxation method which is applicable to different liquid crystal devices. Therefore, the differential iteration method is used to simulate and calculate the position distribution of liquid crystal director with the position under the electric field force.

LCD director distribution under the effect of electric field can be drawn according to the elastic theory and electric free energy formula. It solves the partial differential equation of director dynamic

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response. The equation only has analytical solution under some approximate conditions, normally. Using numerical calculation can get the numerical solution with different boundary conditions and initial conditions. This article chooses different iterative method to numerical analyze and simulate the LCD director distribution under electric field.

2 Director distribution and numerical simulation of liquid crystal in electric field

2.1 Liquid crystal director distribution equation

Starting from the view of the liquid crystal free energy, the continuum elasticity theory get the distribution of director is obtained by minimizing the free energy of the system. The total free energy density of the liquid crystal is expressed as the elastic free energy and the free energy of electric field. The total free energy can be expressed by the following equation:

$$F = F_g - F_e = \int_{V_0} \left(f_g - f_e \right) d\tau \qquad (1)$$

f_g is the liquid crystal deformation free energy density, f_e is the liquid crystal free density of external electric field. The specific expression is shown as equation (2) and (3):

$$f_e = \frac{1}{2}\left[K_{11}\left(\nabla \bullet \hat{n}\right)^2 + K_{22}\left(\hat{n}\bullet\nabla\times\hat{n}\right)^2 + K_{33}\left(\hat{n}\times\nabla\times\hat{n}\right)^2 \right] \qquad (2)$$

$$f_e = \frac{1}{2} D \bullet E \qquad (3)$$

For pure phase modulation LCD device, the twist angle constant is 0, the total free energy is calculated by the following equation:

$$\begin{aligned} F = \int_{V_0} & \left(K_{11}\cos^2\theta + K_{33}\sin^2\theta \right)\left(\frac{\partial\theta}{\partial z} \right)^2 \\ & -(\varepsilon_\perp + \Delta\varepsilon\sin^2\theta)\left(\frac{dV}{dz} \right)^2 d\tau \end{aligned} \qquad (4)$$

In the above equation, τ is volume element, V_0 is the integral volume range, θ is the director deflection angle, Z is the position in the liquid crystal cell, and V is the driving voltage. When the minimum free energy is required, the following extreme conditions need to be satisfied as the following equation:

$$\begin{aligned} & \left(K_{11}\cos^2\theta + K_{33}\sin^2\theta \right)\frac{d^2\theta}{dz^2} \\ & + \left(K_{11} - K_{33} \right)\sin\theta\cos\theta\left(\frac{d\theta}{dz} \right)^2 \\ & + \Delta\varepsilon E^2 \sin\theta\cos\theta = 0 \end{aligned} \qquad (5)$$

$$\begin{aligned} & \left(\varepsilon_\parallel \sin^2\theta + \varepsilon_\perp \cos^2\theta \right)\frac{d^2V}{dz^2} + \\ & 2\Delta\varepsilon\sin\theta\cos\theta\frac{dV}{dz}\frac{d\theta}{dz} = 0 \end{aligned} \qquad (6)$$

The two partial differential equations above are the basic equations to describe the distribution $\theta(z)$ and voltage distribution $V(z)$ of twisted nematic liquid crystal director. There is not analytical solution, only through reasonable approximation to find its approximate solution or by numerical method to find its numerical solution.

2.2 Numerical simulation

The calculation of the liquid crystal director with position distribution is of great significance to design the liquid crystal device. Using differential replace formula (5) and (6), we get the iterative formula is shown as equation (7) and (8):

$$V_i = \frac{V_{i+1}+V_{i-1}}{2} - \frac{1}{4}\frac{\Delta\varepsilon \sin\theta_i \cos\theta_i \left(V_{i+1}+V_{i-1}\right)\left(\theta_{i+1}+\theta_{i-1}\right)}{\varepsilon_{\parallel}\sin^2\theta_i + \varepsilon_{\perp}\cos^2\theta_i} \tag{7}$$

$$\theta_i = \frac{\theta_{i+1}+\theta_{i-1}}{2} + \frac{1}{16}\sin 2\theta_i \frac{\left(K_{33}-K_{11}\right)\left(\theta_{i+1}-\theta_{i-1}\right)^2 + \Delta\varepsilon\left(V_{i+1}-V_{i-1}\right)^2}{\left(K_{11}+\left(K_{33}-K_{11}\right)\sin^2\theta_i\right)} \tag{8}$$

Using YM8 models of liquid crystal, the elasticity and dielectric constants in the material are $K_{11}=1.37\times10^{-12}$, $K_{22}=7.0\times10^{-12}$, $K_{33}=1.68\times10^{-12}$, $\varepsilon_{\parallel}=3.186\times10^{-12}$, $\varepsilon_{\perp}=9.116\times10^{-12}$,and the thickness of the liquid crystal cell is $d=10\mu m$. The distribution of the liquid crystal director is shown in Figure 1 by using the Gaussian-Saier iteration.

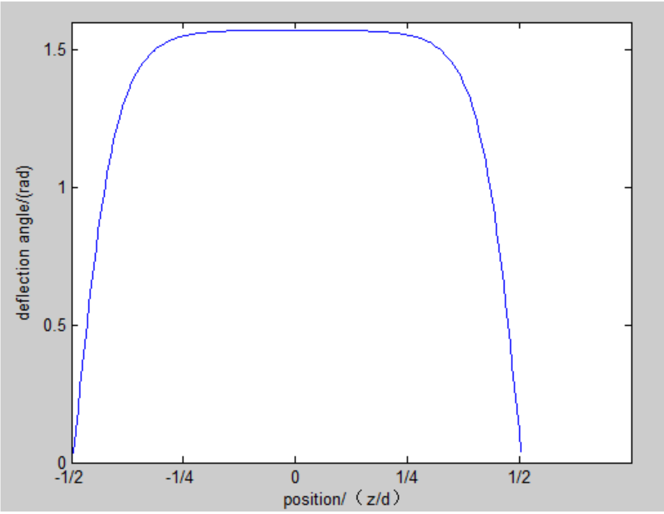

Figure 1 The relationship between the tilt angle and the z-axis position

3 Relationship between optical path difference and the driving voltage of LCD cell

The direction of the liquid crystal molecules and electro-optical characteristics will change under the external field. When the voltage applied to the liquid layer is greater than Freedericksz threshold, the liquid crystal molecules will deflect and change the refractive index of the liquid crystal layer, the change in refractive index will bring the phase change of the incident light.

3.1 Electrically controlled birefringence of liquid crystal

The liquid crystal is a uniaxial crystal whose optical axis is in the long axis direction of the liquid crystal molecule. In addition to the analytical method, the optical properties of crystal can be visually represented by geometrical shapes, including ellipsoid, wave vector, normal surface and light surface et. These geometries can visually show the direction relationship between the vectors of the light waves and the spatial distribution of the refractive indices corresponding to the propagation directions. Using these graphics solute the problem of light propagation simply and the refractive index ellipsoid is the most commonly used geometric method. For any particular crystal, the refractive index ellipsoid is uniquely determined by the main dielectric constant or the main refractive index of the crystal.

Birefrigent effect occurs when a bunch of monochromatic light is incident on the liquid crystal surface, where a bunch of refracted light follows the normal light (o light) of the refractive law of the light, whose refractive index is constant; Another may not in the incident surface, and its refractive index is not constant, this beam is called unusual light (e light)[5-6]. The refractive index of the e light and o light propagate in the liquid crystal can be expressed by the refractive index ellipsoid, as shown in Figure 2.

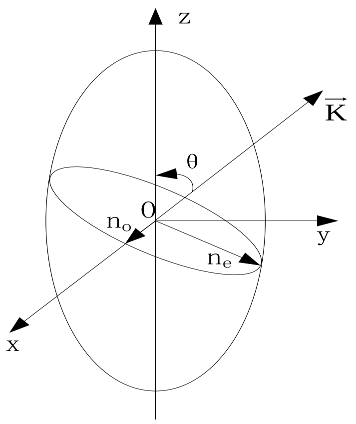

Figure 2 Refractive index ellipsoid

The axial length along z axis is $n_{\|}$ and the direction of the x and y axis is $n_{\perp}$. If the incident light deflects θ from Z axis, the refractive index components of e-light and o-light are expressed as the following equation[7].

$$\begin{cases} \dfrac{1}{n_e{}^2} = \dfrac{\cos^2\theta}{n_{\perp}^2} + \dfrac{\sin^2\theta}{n_{\|}^2} \\ n_o = n_{\perp} \end{cases} \tag{9}$$

n_e and n_o are the refractive index component of e-light and o-light respectively. Simplify equation (9), the refractive index components are expressed as the follow equation(10).

$$\frac{1}{n_e{}^2} - \frac{1}{n_o^2} = \frac{\sin^2\theta}{n_{\|}^2} - \frac{1-\cos^2\theta}{n_{\perp}^2} \tag{10}$$

Finished birefringence expression is calculated by the following equation.

$$\Delta n = n_e - n_o = \left(n_{\|} - n_{\perp}\right)\sin^2\theta \tag{11}$$

The optical path difference caused by the $\theta(x)$ in the thickness of entire cell is calculated as equation (12).

$$\Delta \approx \left(n_{\|} - n_{\perp}\right)\int_{-d/2}^{d/2} \theta(x)\mathrm{d}x \tag{12}$$

3.2 The relationship between the modulation phase and driving voltage

According to the relationship between the liquid crystal molecular director and position and the formula (12), the correspondence between the optical path difference and the voltage is as shown in Figure 3.

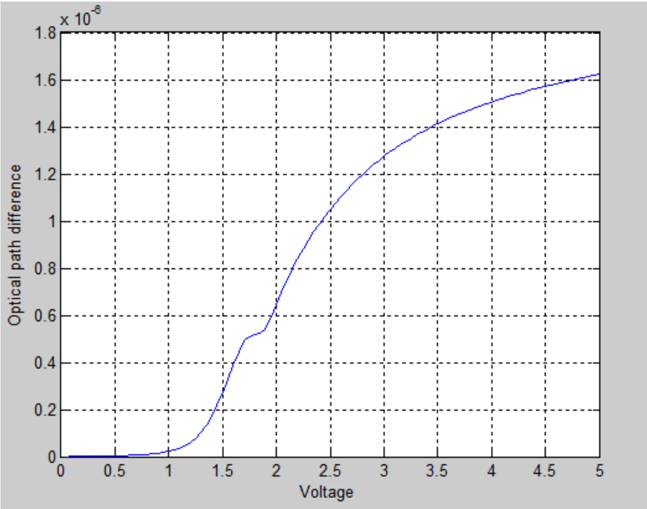

Figure 3 The relationship between the optical path difference and driving voltage

In Figure 2, the threshold voltage is about 0.6V, the modulated phase upward trend when the drive is greater than the threshold voltage. The relationship between the phase and the optical path difference of the liquid crystal grating is shown in Equation (13).

$$\varphi = 2\pi L / \lambda \qquad (13)$$

According to the above analysis, using 671nm laser wavelength, the simulation of the relationship between the liquid crystal modulation phase and driving voltage is shown in Figure 4.

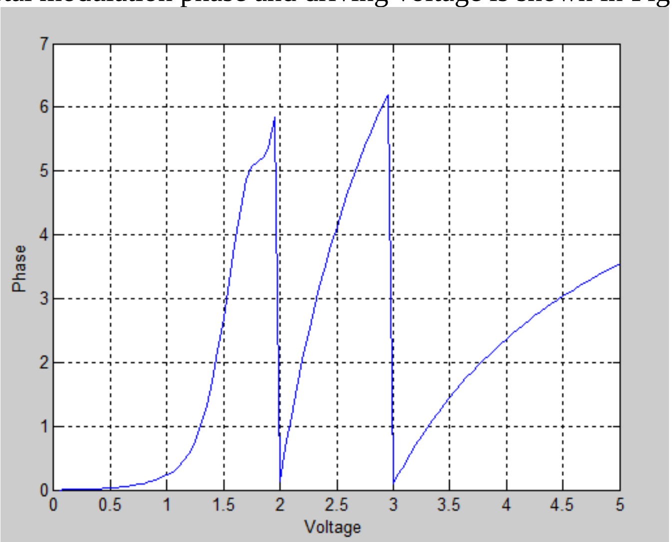

Figure 4 Relationship between the LCD modulation phase and driving voltage

In Figure 4, the drive voltage changes from 0 to 5V, the liquid crystal modulation phase changes from 0 to 4π.

4 Conclusion

The numerical solution of the director distribution under the action of the electric field is obtained by the differential iteration method. According to the electro-optic effect, the relationship between the optical path difference and the voltage of the liquid crystal cell is deduced. The simulation results show when the driving voltage changes from 0 to 5V, the optical path difference of the liquid crystal cell changes from 0 to 4π.

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