

# Power system transient stability analysis based on branch potential characteristics

**Huan Han, Zengping Wang**

North China Electric Power University, Beijing, 102206 China

E-mail:hhan1993@163.com

**Abstract.** Branch potential function is proposed based on the power system network preserving model. The concept of thermodynamics-entropy, is introduced to describe spatial distribution characteristics of the branch potential energy. Branch potential energy was analysed in time and space domain., with transient stability index proposed accordingly. The larger disturbance energy line fault injected to grid is, the larger branch energy entropy will be, and the more energy accumulated on key branches is, the more prone to lose stability the system will be. Simulation results on IEEE system proved its feasibility.

## 1. Introduction

Power system is one of the most complex man-made strongly nonlinear, high dimensional dynamic large-scale industrial systems. The inter-connection power grid, the mature of power trading market, renewable energy accessible, the increase in electricity consumption and electrical equipment in system and household such as global energy interconnection, the internet of things, forcing the power system operation close to its stability limit<sup>[2,17]</sup>, power system online stability assessment and control strategy implementation is becoming increasingly difficult and important.<sup>[8,15]</sup>

Currently there are two methods in power system transient stability: time domain simulation and direct method. Time domain simulation method forms the system model of the power system components according to its topological relationship, obtaining the approximate solution of nonlinear differential equations, capable of accommodating various models, simple, being the major tool of the power system transient stability analysis<sup>[19]</sup>. Many commercial programs have been developed, such as PSASP, PTI, EUROSTAG and German VISTA procedures. But it exits lacking quantitative stability analysis, the sensitivity analysis to the system key parameters and a problem of large computation<sup>[4,23]</sup>. A numerical integration method of power system transient stability put forward<sup>[7]</sup> based on Padé approximation, proves to be of high accuracy and suitable to numerical analysis of transient stability.

Direct method capable of quantitative the stability extent analysing the stability problem in the point of energy, often need to reduce the network and results in the original network structure and parameters being “omitted”. Hence the impact network operation variables and structure variables imposed on the system transient stability fails to analyse. Since the application of Lyapunov function to the power system transient stability, the researching model are generally separated into network reduced model<sup>[1,14]</sup> and structure preserving model<sup>[3,15]</sup>.

After analysing the relationship between the variation rate and transient unbalance energy based on WAMS (Wide Area Measurement System), new multi-swing stability assessment criterion for on-line was proposed<sup>[6]</sup>.

System model establishment is the first step applying energy method to power system stability, then the corresponding transient energy function. In the complex dynamic system where various components



interact, as effective vulnerable transmission lines and generator group identification to the power system dynamic characteristic and the ideal location and estimation of control devices, simple and reliable of critical branches identification is vital to the successful online application of direct or mixed transient stability assessment methods as well<sup>[12]</sup>.

The relationship between the dynamic performance of the power system and the network structure is discussed using the structure preserving model without network simplification. The method of branch potential energy function is based on the structure-preserving model, mapping the global stability on local network (transmission lines), effectively integrating network topology, system dynamics and stability, providing convenience for online stability monitoring, sensitivity analysis of components and the subsequent analysis of system security and stability control measures.

In 1981, Bergen proposed a power system network structure-preserving model, analysing transient stability with Lyapunov function in the form of Lur'e-Postnikov while neglecting the impact of the fault imposed on transient stability<sup>[16]</sup>.

Decades of development and improvement, the energy function method has been used more widely.

The distribution of network energy in the system has the characteristics of accumulation, and the stability of some branches will determine the stability of the whole system to a great extent<sup>[9,11,13,21,22]</sup>. Some scholars introduce the concepts of local network potential energies and equivalent electrical centre into the critical line identification index, only the line angles, line power flows and bus voltage magnitudes at first swing are needed and fits for online assessment<sup>[10]</sup>.

The rest of this paper is structured as follow. Section II introduces branch potential function based on the structure-preserving network and relevant fundamental concept and theory essential to subsequent development. In section III, branch potential energy was analysed in space domain, with the entropy of thermodynamics concept introduced to describe its spatial distribution characteristics and accordingly transient stability index founded. Section IV validates the effectiveness of the proposed approach by simulation results and performance analysis on IEEE 4 generator 2 area system.

## 2. Branch potential function

### 2.1 Network preserving model

If there are  $m$  generators,  $n_0$  nodes,  $l_0$  branches in a multi-machine system, the rotor motion equation of generator  $i$  based on the Centre of Inertia (COI) reference frame can be expressed as below:

$$\begin{cases} \frac{d\delta_i}{dt} = \omega_i \\ M_i \frac{d\omega_i}{dt} = P_{Mi} - P_{Ei} - \frac{M_i}{M_T} P_{COI} \end{cases} \quad (1)$$

Where  $i$  starts from 1,2, ...,  $m$ ;  $\delta_i$  and  $\omega_i$  are angular and speed of generator  $i$  relative to the centre of inertia respectively,  $M_i$  is inertia coefficient of generator  $i$ ;  $M_T$  is the system inertia coefficient, the sum of the inertia coefficients of every generator;  $P_{mi}$  and  $P_{ei}$  are input mechanical power and output electromagnetic power of generator  $i$  respectively;  $P_{COI}$  is the acceleration power of centre of inertia.

First, the dynamic model of multi-machine power system is established, without the conventional procedure of contracting the load impedance to the transmission network. Other aspects are the same assumptions as the classical model<sup>[18]</sup>. The network of buses connected by transmission lines can be described by the power flow equations.

The active power injection of generator and active demand of load is equivalent to the power injection of bus. Introducing fictitious buses representing the generator internal buses to the original network, which is connected to the original network through the transient reactance of the generator. In augmented networks, the total number of system buses is  $n=m+n_0$ , and the total number of branches is  $l=l_0+m$ . The voltage phase angle of the buses  $i$  and  $j$  of the connection branch  $k$  is denoted as  $\delta_i$  and  $\delta_j$ . The generator adopts the classical model, and the load adopts the constant impedance model, then we can reach

$$P_{Ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^m (E_i E_j B_{ij} \sin \delta_{ij} + E_i E_j G_{ij} \cos \delta_{ij}) \quad (2)$$

$$P_{COI} = \sum_{i=1}^m (P_{Mi} - P_{Ei}) \quad (3)$$

where  $i=1,2, \dots, n$ ;  $1, \dots, m$  are the generator internal potential buses;  $m+1, \dots, 2m$  are generator outlet buses;  $2m+1, \dots, n$  are load buses;  $E_i$  and  $E_j$  are the internal potential of  $i$  and  $j$  respectively;  $B_{ij}$  and  $G_{ij}$  are conductance and susceptance between  $i$  and  $j$  respectively;  $\delta_{ij}$  is the angle between phasor  $E_i$  and  $E_j$ .

The transmission capacity of branch power is determined by the potential difference between the network buses, which can be divided into active power transmission and reactive power transmission both limited by branch thermal constraint and buses voltage constraint, and accordingly branch potential energy also has its limit.

## 2.2 Potential energy function

Taking direct method as a reference, according to the integral principle of potential energy function based on the network topology model, the potential energy function of each branch of the network can be expressed as

$$E_{ij} = \int_{(\delta_{ij}^s, U_{ij}^s)}^{(\delta_{ij}, U_{ij})} [f_{pij}, f_{qij}] \cdot \begin{bmatrix} d\delta_{ij} \\ dU_{ij} \end{bmatrix} \quad (4)$$

Power flow of branch  $ij$  is:

$$P_{ij} = G_{ij} U_i^2 - U_i U_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (5)$$

$$Q_{ij} = -U_i^2 B_{ij} + U_i U_j (B_{ij} \cos \delta_{ij} - G_{ij} \sin \delta_{ij}) \quad (6)$$

$P_{ij}$  and  $Q_{ij}$  are the instantaneous power transmission of branch  $i, j$ . The value under the initial steady-state condition is chosen as the calculation starting point. According to the branch power transmission relation, the variation of the branch power transmission is:

$$f_{pij} = P_{ij} - P_{ij}^s \quad (7)$$

$$f_{qij} = U_{ij}^{-1} (Q_{ij} - Q_{ij}^s) \quad (8)$$

formula above substituted in (4), expression of branch potential energy can be reached:

$$E_{ij} = \int_{(\delta_{ij}^s, U_{ij}^s)}^{(\delta_{ij}, U_{ij})} [f_{pij}, f_{qij}] \cdot \begin{bmatrix} d\delta_{ij} \\ dU_{ij} \end{bmatrix} \quad (9)$$

$$= \int_{\delta_{ij}^s}^{\delta_{ij}} (P_{ij} - P_{ij}^s) d\delta_{ij} + \int_{U_{ij}^s}^{U_{ij}} U_{ij}^{-1} (Q_{ij} - Q_{ij}^s) dU_{ij} \quad (10)$$

$$= \int_{\delta_{ij}^s}^{\delta_{ij}} [G_{ij} U_i^2 - U_i U_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) - P_{ij}^s] d\delta_{ij} \\ + \int_{U_{ij}^s}^{U_{ij}} \left[ \frac{U_i U_j (B_{ij} \cos \delta_{ij} - G_{ij} \sin \delta_{ij}) - U_i^2 B_{ij} - Q_{ij}^s}{U_{ij}} \right] dU_{ij} \quad (11)$$

Where  $E_{ij}$  is the branch potential between buses  $i$  and  $j$ ;  $\delta_i^s, \delta_j^s, U_i^s, U_j^s, P_{ij}^s, Q_{ij}^s$  are the initial steady-state values for corresponding variables respectively.

The phase difference at the ends of branch  $k$  can be expressed as  $\sigma_k = \delta_i - \delta_j$ , then

$$E_k = \int_{\delta_k^s}^{\delta_k} (P_k - P_k^s) d\delta_k \quad (12)$$

The branch potential energy takes the initial steady state as the reference point, which reflects the instantaneous deviation effect of corresponding electrical parameters relative to the initial steady state, and the cumulative effect of these variables in the energy domain.

### 2.3 Energy entropy

Entropy, a function of state in the concept of thermodynamic, represents the extent of randomness of the distribution of system energy in space domain. Shannon proposed the entropy of information<sup>[5]</sup> to measure the degree of order and disorder of discrete systems. When  $N$  states of a system occur with equal probability, the system disorder extent is the highest and the information entropy is the largest. When the system is in the unique state, the system has the lowest degree of disorder degree and the information entropy is minimum<sup>[20]</sup>.

Power system is a complex self-organized energy homeostasis system, its internal energy has certain rules under specific operation mode: energy of system distribution is stable and orderly under equilibrium state; while there is a disturbance or fault occurrence, its energy distribution heading towards the direction of disorder. Therefore, entropy is introduced into the electric power system to measure the uniformity extent of the potential energy distribution in the power system.

Combine entropy with simplified branch potential, we can get the energy entropy of the power system. Entropy represents the extent of randomness of the distribution of system energy in space domain.

The ratio of the branch energy of branch  $k$  to the total branch energy is

$$\mu_k = \frac{E_k}{E} \quad (13)$$

The branch with maximum branch energy is taken as reference, then the distribution ratio of the branch potential energy will be

$$\mu_k = \frac{\frac{E_k}{E}}{\frac{E_{max}}{E}} = \frac{E_k}{E_{max}} \quad (14)$$

Then we can get energy entropy of the power system

$$H = \sum_{k=1}^N (\mu_k \ln \mu_k + \frac{1}{e}) \quad (15)$$

Where  $\mu_k$  is the distribution ratio of the branch  $i$  potential energy;  $N$  is the total number of branches;  $E_{max}$  is the branch with lowest potential. The value of  $\mu_k$  is between 0 and 1, which will cause excessive concentration of the branch energy when it is too large or too small. The proposed energy entropy index of the power system is greater than 0, we can assume that the greater the value is, the more likely the branch potential energy distribution is concentrated in specific few branches.

### 3. Spatial distribution characteristics of branch potential energy

When fault occurs in power system, disturbance energy will be generated and injected into the network, then transformed into the form of branch potential energy and distributed in the power grid, which will bring the whole system develop in the direction of disorder. The spatial distribution of branch energy in power system shows not the consumptive ability of branches to disturbance energy, but also reflects the degree of cumulativeness of disturbance energy. If the branch energy distributes in each branch uniformly or heads towards in that direction, the system is stable; if the branch energy distribution of each branch is of great difference, that is, gathering in a few branches, the system stability depends on largely the stability of that few branches, gradually decreasing and likely to tear down from the cut set formed by these branches, resulting in the loss of power system stability.

Because the branch potential energy is time-varying, it is inaccurate to study it from a certain time section. Considering that there is a cumulative effect of branch energy time-varying property in transient process, so the distribution of the branch potential energy in a certain time interval can be used to study its distribution.

When fault occur at branch  $l$ , the accumulative energy of branch  $k$  in transient process can be expressed as:

$$E_{l-k} = \int_{t_c}^{t_s} E_k(t) dt \quad (16)$$

Where  $E_k(t)$  is instantaneous branch potential energy value of branch  $k$  at time  $t$ ;  $t_c$  is fault clearing time;  $t_s$  is the time when post fault system reaches stable equilibrium point. While it is unfavourable for calculation in practical application, use simulation terminal time instead.

Therefore, the total disturbance energy generated in each branch when fault occurs at the line  $l$  is:

$$E_l = \sum_{\substack{k=1 \\ k \neq l}}^n E_{l-k} \quad (17)$$

The ratio of the disturbance energy generated in line  $k$  to the total disturbance energy of the line  $l$  fault is

$$\mu_{l-k} = \frac{E_{l-k}}{E_l} \quad (18)$$

The influence degree of the line  $l$  fault to each branch of the whole system:

$$\mu_{l-k} = \frac{\frac{E_{l-k}}{E_l}}{\frac{\max E_{l-k}}{E_l}} = \frac{E_{l-k}}{\max E_{l-k}} \quad (19)$$

Define the uniformity index of the system branch energy distribution of line  $l$  fault as follow:

$$F_l = - \sum_{\substack{k=1 \\ k \neq l}}^N \mu_{l-k} \ln \mu_{l-k} \quad (20)$$

Define the branch energy distribution entropy of the system when the line  $l$  fault as:

$$H_l = \sum_{\substack{k=1 \\ k \neq l}}^N (\mu_{l-k} \ln \mu_{l-k} + \frac{1}{e}) \quad (21)$$

When fault occurs on line  $l$  and the disturbance energy injected to all branches is fixed, the branch energy uniformity index  $F_l$  and the branch energy distribution entropy  $H_l$  reflect the spatial distribution characteristics of the disturbance energy in the network. The smaller value of  $F_l$ , or the larger value of  $H_l$  indicates more concentrated the disturbance energy in a few branches, which means the greater the risk of system instability; the greater the value of  $F_l$ , or the smaller the value of  $H_l$ , shows enough absorptive capacity of the disturbance energy that the branch system can accommodate, which means branch potential energy distributed uniformly and the system is relatively have good stability.

Based on this, the power system transient stability index  $H_l$  is presented when fault occur at the branch  $l$ .

#### 4. Simulation

MATLAB is used as a simulation tool, and IEEE 4 generator system is used as an example to testify the effectiveness of the method.

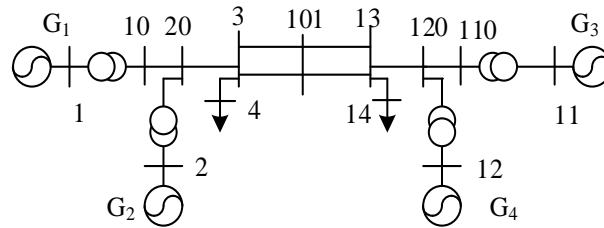


Figure 1: IEEE four-machine two-area testing system

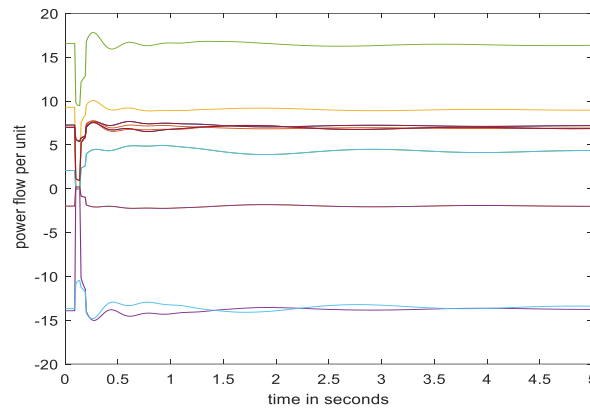


Figure 2: branch power variation of stable system

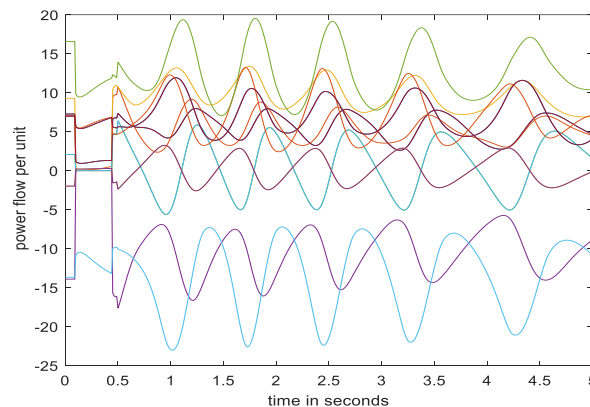


Figure 3: branch power variation of unstable system

Figure 2 and 3 are branch power flow variation with time when fault applied to line 3-101 at 0.10s, cleared at 0.20s and 0.30s respectively. From figure 2 and 3 we know that the vulnerable branches that leading the system towards instability are the same in stable and unstable case.

Table 1: Transient stability index based on branch potential distribution entropy

Fault line	$E_l$	$F_l$	$H_l$	CCT
3-4	0.0223	5.8075	0.8143	0.31
3-101	0.1863	5.9806	0.6412	0.28
2-20	0.2540	6.0399	0.5819	0.27
1-10	0.3378	6.2505	0.3713	0.24
1-G1	0.5782	6.3280	0.2938	0.22
2-G2	0.5921	6.3926	0.2292	0.22
10-20	0.7108	6.5723	0.0495	0.19
3-20	0.8728	6.6109	0.0109	0.17

Table 1 shows that there is a positive correlative between  $E_l$  and  $F_l$ , and the negative association between CCT and  $H_l$ . The difference of CCT reveals the vulnerability of branches mainly caused by network connection. The relatively high impedance of the branch with a transformer has a negative impact on the system stability. Because the 3-101 is double circuit, the structure is good for its stability and the transient stability margin is higher when fault occurred on that line. Line 3-20 is of high importance because it is the key branch connecting the two group of generators. Based on Branch Energy distribution entropy, the transient stability index of  $H_l$  technically consistent with critical fault clearing time.

It is obvious that the greater the value of is  $H_l$ , the more likely the branch potential energy distribution is concentrated in specific few branches. The correctness of the method proposed in this paper illustrates that it can be applied to transient stability analysis.

## 5. Conclusion

The relationship between the dynamic performance of the power system and the network structure is discussed using the structure preserving model without network simplification. A method of power system transient stability assessment analysing stability trend from network distribution, propagation and consumption of disturbance energy based on spatial characteristics of branch energy is proposed.

Combine complex entropy theory with power system to evaluate the accumulative degree and distribution of disturbance energy injected by fault, effectively integrating network topology, system dynamics and stability, disturbance energy impact index is established to as well as branch stability margin.

Numerical results are consistent with the critical clear time in fault scenarios and validates the accuracy of proposed method. Simulation results verify that the larger disturbance energy line fault injected to grid is, the larger branch energy entropy will be, and the more energy accumulated on key branch cutsets is, the more prone to lose stability the system will be.

Reliability and computational simplicity have made this method suitable for online transient stability assessment. providing convenience for online stability monitoring, sensitivity analysis of components and the subsequent analysis of system security and stability control measures.

### Acknowledgements

This research is funded by the Key Project of the National Natural Science Foundation of China (no. 51637005).

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