

# Crosswind stability of FSAE race car considering the location of the pressure center

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**Abstract.** An 8-DOF vehicle dynamic model of FSAE race car was established, including the lateral motion, pitch motion, roll motion, yaw motion and four tires rotation. The model of aerodynamic lateral force and pressure center model were set up based on the vehicle speed and crosswind parameters. The simulation model was built by Simulink, to analyse the crosswind stability for straight-line condition. Results showed that crosswind influences the yawing velocity and sideslip angle seriously.

## 1. Introduction

It is obviously that the crosswind influences the vehicle steering characteristic, and causes the tire cornering. The driver should control the steering wheel accurately and efficiently to keep the right running tracks in high crosswind, which will increase driver's burden [1-3]. For FSAE race car, with the speed increased, large rear wing end plates installed as well as the pressure center drifted, the crosswind influence to vehicle stability is increased. So, study of the crosswind stability is meaningful to improve the race car handling stability, decrease the driver burden and improve the vehicle security [4-6]. At present, the research of vehicle crosswind stability is still in the primary stage, especially when considering the location of pressure center. Basically, the location of pressure center is assumed in most researches, and most papers only focus on regular pattern, which ignore the impact extent of crosswind. This paper establishes the accurate pressure center model to study the crosswind stability.

## 2. Vehicle dynamic model

### 2.1. 8-DOF vehicle model

Figure 1 shows the vehicle main parameters in a turn condition. The gravity center is shown at point  $C$ , roll center at point  $O_m$  and pressure center at point  $O_w$ . Four tires are numbered from 1 to 4, and  $F_{xi}$ ,  $F_{yi}$ ,  $F_{zi}$  are used to express the longitudinal force, lateral force and vertical force separately. Based on the vehicle coordinate showed in figure 1, vehicle model is expressed as follow:



$$\left. \begin{aligned}
& (F_{x1} + F_{x2}) \sin \delta + (F_{y1} + F_{y2}) \cos \delta + F_{y3} + F_{y4} + F_w = m(\dot{v} + u\gamma) - m_s h_s \ddot{\phi} \\
& ((F_{x2} - F_{x1}) \cos \delta + 0.5(F_{y1} - F_{y2}) \sin \delta) B_f + (F_{x1} + F_{x2}) \sin \delta \cdot a - (F_{y3} + F_{y4}) b \\
& + (F_{y1} + F_{y2}) \cos \delta \cdot a + 0.5(F_{x4} - F_{x3}) B_r + F_w l_w = I_z \dot{\gamma} + I_{zx} \ddot{\phi} - I_{yx} \dot{\phi}^2 + I_{zy} \ddot{\theta} - I_{xy} \dot{\theta}^2 \\
& m_s g h_s \phi - F_w h_{mw} - k_\phi \phi - c_\phi \dot{\phi} = I_x \ddot{\phi} + I_{xz} \dot{\gamma} - I_{yz} \gamma^2 + I_{xy} \ddot{\theta} + I_{zy} \dot{\theta}^2 - m_s (\dot{v} + u\gamma) h_s \\
& F_d h_w - F_w l_w - k_\theta \theta - c_\theta \dot{\theta} = I_y \ddot{\theta} + I_{yz} \dot{\gamma} + I_{xz} \gamma^2 + I_{yx} \ddot{\phi} - I_{zx} \dot{\phi}^2
\end{aligned} \right\} \quad (1)$$

Where  $a$  is the distance between gravity center and the front axle;  $b$  is the distance between the gravity center and the rear axle;  $\gamma$  is the vehicle yaw velocity;  $m$  is the vehicle total mass;  $m_s$  is the sprung mass.  $u$  and  $v$  are the vehicle longitudinal and lateral speeds, respectively.  $F_w$  and  $F_d$  are aerodynamic lateral force and resistance, respectively.  $B_f$  and  $B_r$  are the front wheel tread and rear wheel tread, respectively.  $l_w$  and  $h_w$  are the longitudinal and vertical distance between gravity center and pressure center, respectively.  $\delta$ ,  $\phi$ ,  $\theta$ ,  $\beta$  are front wheel angle, vehicle roll angle, pitch angle and sideslip angle, respectively.  $k_\phi$  and  $k_\theta$  are the rolling angle stiffness and pitching angle stiffness, respectively;  $c_\phi$  and  $c_\theta$  are the rolling angle damping coefficient and pitching angle damping coefficient, respectively.  $I_x$ ,  $I_y$  and  $I_z$  are the moment of inertia for sprung mass about  $x$  axis,  $y$  axis and  $z$  axis, respectively;  $I_{xy}$ ,  $I_{yz}$  and  $I_{zx}$  are the moment of inertia for sprung mass about  $xoy$  plane,  $yoz$  plane and  $zox$  plane.

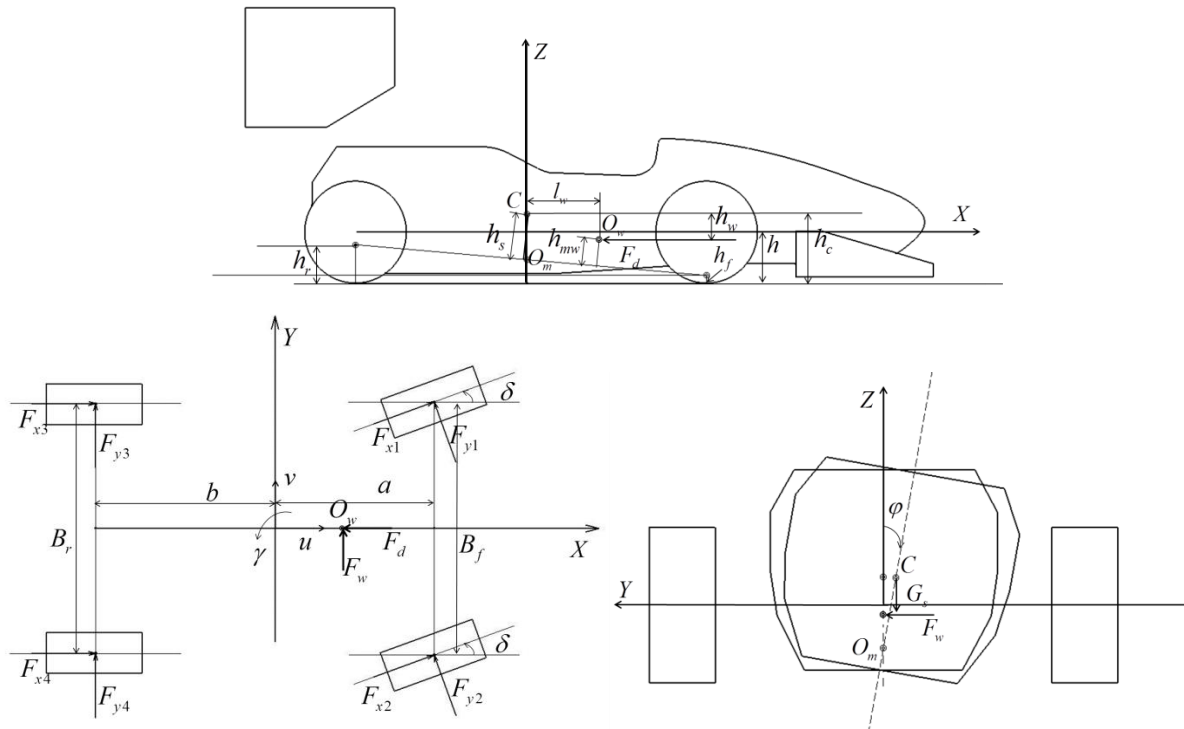


Figure 1. Vehicle dynamic parameters diagram for 8-DOF dynamic model.

## 2.2. Tire model

Magic Formula is a mathematical formula that describes the tire characteristics for the interaction forces, which is applied to solve the longitudinal force and lateral force in this paper. Besides, in order to combine tire model and vehicle dynamic model, this paper establish the vertical force model, tire longitudinal slip ratio model and tire sideslip angle model, which are shown in follows:

$$\left. \begin{aligned} F_{z1} &= 0.5(mgb + k_\theta \theta + c_\theta \dot{\theta})L^{-1} - (k_{\phi f} \phi + c_{\phi f} \dot{\phi})B_f^{-1} - b(h_{uf} m_u \dot{v} + h_f m_s \dot{v})(LB_f)^{-1} \\ F_{z2} &= 0.5(mgb + k_\theta \theta + c_\theta \dot{\theta})L^{-1} + (k_{\phi f} \phi + c_{\phi f} \dot{\phi})B_f^{-1} + b(h_{uf} m_u \dot{v} + h_f m_s \dot{v})(LB_f)^{-1} \\ F_{z3} &= 0.5(mga - k_\theta \theta - c_\theta \dot{\theta})L^{-1} - (k_{\phi r} \phi + c_{\phi r} \dot{\phi})B_r^{-1} - a(h_{ur} m_u \dot{v} + h_r m_s \dot{v})(LB_r)^{-1} \\ F_{z4} &= 0.5(mga - k_\theta \theta - c_\theta \dot{\theta})L^{-1} + (k_{\phi r} \phi + c_{\phi r} \dot{\phi})B_r^{-1} + a(h_{ur} m_u \dot{v} + h_r m_s \dot{v})(LB_r)^{-1} \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} s_b &= (u_i - \omega r)u_i^{-1} \\ u_1 &= (u - 0.5B_f \gamma) \cos \delta + (v + a\gamma) \sin \delta \\ u_2 &= (u + 0.5B_f \gamma) \cos \delta + (v + a\gamma) \sin \delta \\ u_3 &= u - 0.5B_r \gamma \\ u_4 &= u + 0.5B_r \gamma \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \alpha_1 &= \delta - \arctan((v + a\gamma)(u - 0.5B_f \gamma)^{-1}) \\ \alpha_2 &= \delta - \arctan((v + a\gamma)(u + 0.5B_f \gamma)^{-1}) \\ \alpha_3 &= \arctan((v - b\gamma)(u - 0.5B_r \gamma)^{-1}) \\ \alpha_4 &= \arctan((v - b\gamma)(u + 0.5B_r \gamma)^{-1}) \end{aligned} \right\} \quad (4)$$

Where  $L$  is wheelbase;  $s_b$  is tire longitudinal slip ratio;  $\gamma_i$  tire sideslip angle;  $\omega$  is tire rotational speed.

### 3. Aerodynamic lateral force

Race car runs through the 6m crosswind at a constant longitudinal speed, and the crosswind speed is also constant for 22m/s, which is shown in figure 2. There are four points, for time  $t_0$ , car begin through the crosswind; for time  $t_1$ , whole car enter the crosswind; for time  $t_2$ , car begin to leave the crosswind; for time  $t_3$ , car leave the crosswind completely.

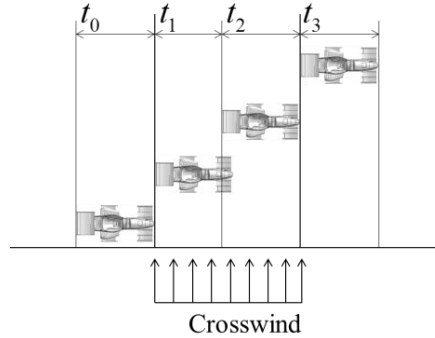


Figure 2. Race car runs through the crosswind.

Vehicle speed, wind speed and vehicle appearance both have influence on aerodynamic lateral force, which are expressed as follow:

$$F_w = 0.5\rho((v_w \cos \beta)^2 + u^2 + v^2)c_y A \quad (5)$$

Where  $v_w$  is crosswind speed;  $C_y$  is aerodynamic lateral force coefficient;  $A$  is vehicle frontal area. For mula (5) is only fit for time stage  $t_1$  to  $t_2$ , which could not express aerodynamic lateral force for time stages  $t_0$  to  $t_1$  and  $t_2$  to  $t_3$ . So, following formula is built to fit for these two time stages.

$$F_w = 0.5\rho v_w^2 c_y A A_y(t) A_s^{-1} \quad (6)$$

Where  $A_s$  is vehicle side area, and  $A_y(t)$  is the side stress area which changed with time. According to the vehicle three-dimensional model,  $A_y(t)$  is obtained and got the fitting formula, shows as follow:

$$A_y(t) = \begin{cases} 0.0875(ut)^2 + 0.3293(ut) - 0.06629 & 0 \leq t < t_1 = L_0 u^{-1} \\ 1.709 & t_1 \leq t < t_2 = 6u^{-1} \\ -0.0875(u(t-t_2))^2 - 0.3293(u(t-t_2)) + 1.7196 & t_2 \leq t < t_3 = (6+L_0)u^{-1} \end{cases} \quad (7)$$

The vehicle longitudinal speed is 23.3m/s, which is got from the vehicle acceleration match event. And based on these calculation data, the aerodynamic lateral force is shown in figure 3.

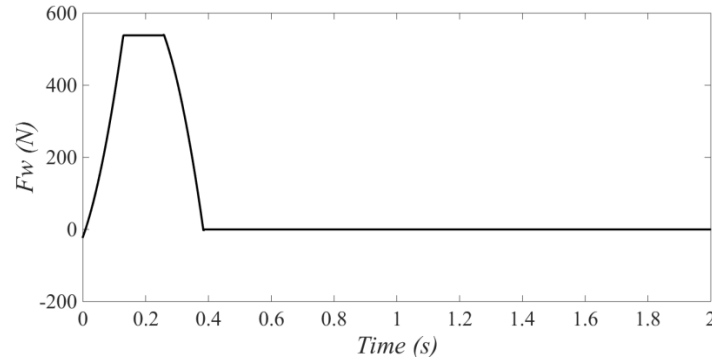


Figure 3. Aerodynamic lateral force-time curve.

#### 4. Pressure center model

Pressure center could be seen as the aerodynamic resultant point, including aerodynamic resistance, aerodynamic lift and aerodynamic lateral force. The pressure center is the same location with vehicle centroid [7], when there only has aerodynamic lateral force. In this paper, there are two pressure center,  $O_{w1}$  and  $O_{w2}$ .  $O_{w1}$  is the resistance and lift combined function point that can be got from CFD analyse.  $O_{w2}$  is aerodynamic lateral force function point, which can be measured by vehicle three-dimensional model. Following are pressure center location fitting formulas:

$$l_{w2}(t) = \begin{cases} -0.0184(ut)^2 - 0.5146(ut) + 1.7319 & 0 \leq t < t_1 \\ 0.069004 & t_1 \leq t < t_2 \\ -0.0336(u(t-t_2))^2 - 0.3252(u(t-t_2)) + 0.077 & t_2 \leq t < t_3 \end{cases} \quad (8)$$

$$h_{w2}(t) = \begin{cases} 0.0161l_i^5 - 0.0115l_i^4 + 0.2618l_i^3 - 0.1977l_i^2 - 0.04l_i + 0.018 & 0 \leq t < t_1 \\ -0.209871 & t_1 \leq t < t_2 \\ -0.031l_g^6 + 0.2216l_g^5 - 0.5976l_g^4 + 0.73l_g^3 - 0.4022l_g^2 + 0.4022l_g - 0.2212 & t_2 \leq t < t_3 \end{cases} \quad (9)$$

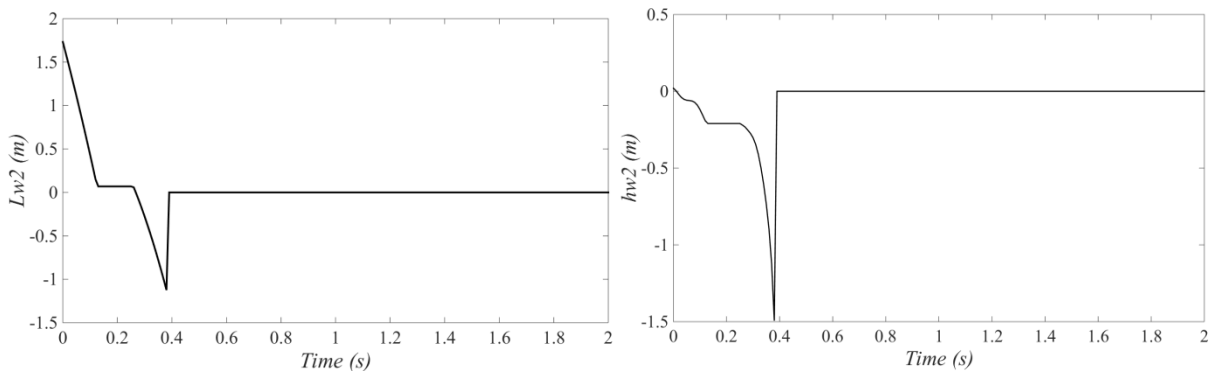


Figure 4. Pressure center location-time curve.

#### 5. Simulation

Simulation is analysed by Simulink, there are some matters should be attention:

- Sideslip angle and yaw velocity are used to estimating the vehicle crosswind stability.

- Tire rotational speed is 107rad/s, which is because tire longitudinal slip ratio is less than 5% in constant longitudinal speed [8].
- There need to set delay function for vertical load between tire model and vehicle model.
- Front wheel angle is 0, to keep the longitudinal straight motion.
- There are two conditions, with crosswind and without crosswind. Results are following:

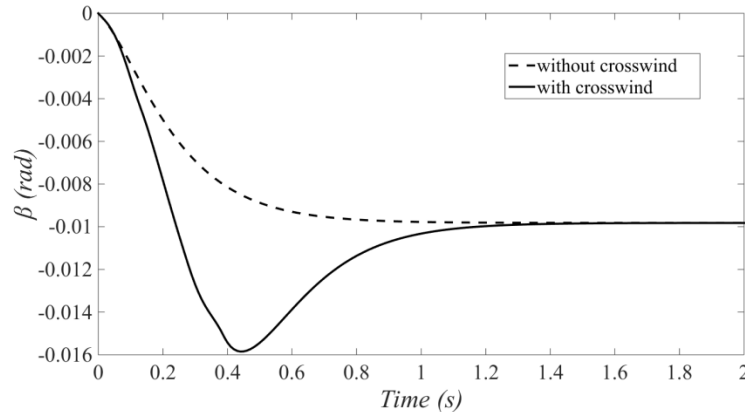


Figure 5. Sideslip angle-time curves for with or without crosswind conditions.

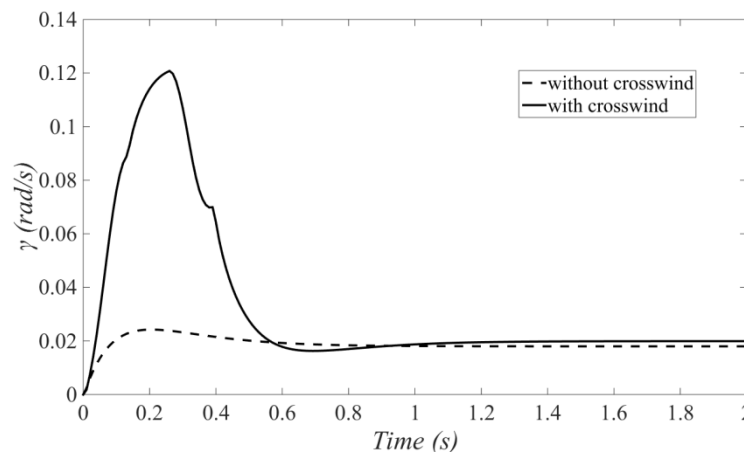


Figure 6. Yaw velocity curves for with or without crosswind conditions.

## 6. Conclusion

Although the crosswind acts only 0.3863s, both vehicle sideslip angle and yaw velocity change significantly, which shows that crosswind affect the vehicle stability seriously. The vehicle sideslip angle is not zero without crosswind, which means there are some vehicle parameters need adjustment. Based on these results, active control technology could be designed to improve the vehicle stability.

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