# 3D modeling and curvature analysis of orthogonal face gear considering the rounding tooth top of shaper cutter

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**Abstract.** Aiming at the lack of the theory of transitional surface of face gear, the tooth profile equation of the orthogonal face gear is derived according to the differential geometry and meshing principle; In this paper, a tooth surface structure is proposed, which is a smooth surface of the tooth root transition part. A method of generating transition surface is presented, which takes into account the rounding tooth top of gear shaper cutter; the smooth transition surface equation of face gear tooth root is derived. The 3D solid model of the orthogonal face gear generated by two kinds of structures, the top angle of the gear shaper cutter and the addendum of the gear shaper cutter are established. The curvature analysis shows that the radius of curvature of the orthogonal face gear cut by the rounding tooth top of shaper cutter is larger than the other. The influence of the main parameters on the surface curvature of the face gear transmission is analyzed, the conclusion has reference value for the strength calculation and analysis of the face gear transmission.

## 1. Introduction

The face gear transmission is a kind of transmission of cylindrical gear and face gear, it is used to realize the motion and power transmission between intersecting axes and intersecting axes. As a new type of gear transmission, the face gear has many advantages, such as simple structure, light weight, large transmission coincidence, small vibration, low noise and so on, it has broad application prospects in the important transmission devices such as helicopters [1-3].

The face gear is generally made of a cutter with involute tooth surface. The tooth surface of face gear is composed of two parts: the working tooth surface and the transition surface, the working tooth surface is formed by the involute profile of the gear shaper cutter, The transitional surface is made up of the top point of the gear shaper cutter or the top of the tooth, the transitional surface is made up of the rounding tooth top of shaper cutter or the angle tooth top of shaper cutter [4-5]. The structure of the transition surface has a certain influence on the strength of the face gear. Therefore, it is of great significance to improve the reliability of the face gear transmission by adopting some measures to improve the structure of the transitional surface of the face gear.

At present, the domestic and foreign scholars have carried on the related research to the face gear tooth surface generation. In foreign countries, Litvin[2] et al. have studied the gear transmission and the gear tooth surface generation. At home, Wu Xuehui et al.[6] have studied the transition surface of the face gear, which takes into account the rounding tooth top of shaper cutter, but they did not give a detailed surface gear surface equation. Lu Wenlong et al. [7] have studied on the tooth surface curvature of orthogonal face gear drive, the variation law of the main curvature is obtained. There are few reports on the comparative analysis and the analysis of the curvature of the transition surface of

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the gear cutter with the top angle of the tooth and the fillet of the tooth. Therefore, according to the theory of differential geometry and meshing, the tooth surface equation of orthogonal face gear is obtained. Then, the tooth tip fillet equation of gear shaper cutter is deduced, the transition surface equation of orthogonal face gear is obtained by a series of coordinate transformation. On this basis, the accurate model of the orthogonal face gear is generated in consideration of the influence of the rounding tooth top of shaper cutter. Finally, the curvature of the transitional surface of the face gear is analyzed, the influence of the transmission parameters of the face gear on the curvature is revealed.

## 2. Geometric model of orthogonal face gear

*2.1 Machining coordinate system and tool tooth surface equation*

The orthogonal face gear transmission is a kind of gear transmission which is engaged with the bevel gear. The cutting processing of Gear tooth surface of orthogonal face gear tooth surface gear is based on the gear cutting knife and the processed surface of the gear cutting into the system, and its processing meshing principle as shown in figure1. Based on the envelope principle of face gear, the meshing transmission of orthogonal face gear is analyzed by using four standard frames, $S_{S0}$ ($O_{S0}X_{S0}Y_{S0}Z_{S0}$) and $S_{20}$ ($O_{20}X_{20}Y_{20}Z_{20}$) is the two fixed frame, $S_S$ ($O_SX_SY_SZ_S$) is a moving frame and is combined with the tool S. It rotates with the tool. $S_2$($O_2X_2Y_2Z_2$) is a moving frame which is fixed with the orthogonal face gear 2. It rotates with the face gear 2. $Z_{S0}$ and $Z_S$ is the tool rotation axis, $Z_2$ and $Z_{20}$ is the face gear rotation axis, its intersection point is O. $Z_{S0}$ ($Z_S$) and $Z_{20}$ ($Z_2$) included angle of γ=90 degrees, as shown in figure2.

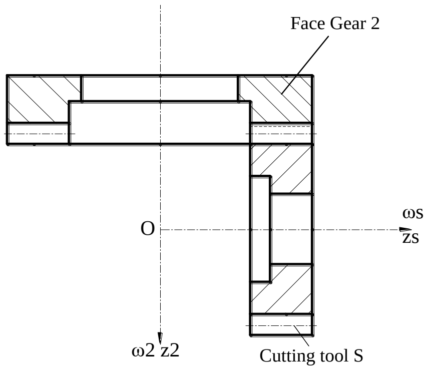



**Figure 1.** Schematic diagram of processing gear of orthogonal face gear

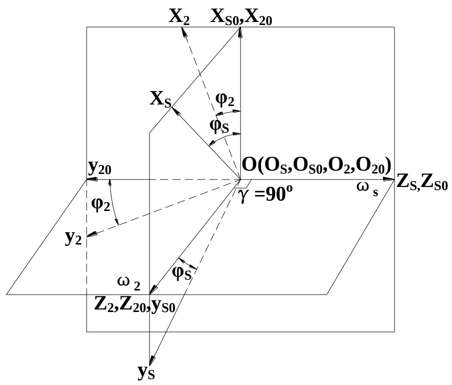



**Figure 2.** Machining coordinate system of orthogonal face gear

According to figure.2, the coordinate transformation matrix from the frame $S_S$ to the frame $S_2$ can be expressed as

$$M_{2,S}=M_{2,20}\bullet M_{20,S0}\bullet M_{S0,S}$$
$$=\begin{bmatrix} \cos\varphi_2\cos\varphi_s & -\sin\varphi_2\cos\varphi_2 & -\sin\varphi_2 & 0 \\ -\cos\varphi_s\sin\varphi_2 & \sin\varphi_s\sin\varphi_2 & -\cos\varphi_2 & 0 \\ \sin\varphi_s & \cos\varphi_s & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Where $\varphi_s$ is the turning angle of the tool S, and $\varphi_2$ is the turning angle of the face gear 2. The relationship between the parameters of the gear tooth surface of the gear slotting cutter is established. Under the motion frame $S_S$ ($O_SX_SY_SZ_S$) which fixed the tool S, the tool tooth surface equation is

$$\vec{r}_s\left(u_{s,}\theta_s\right)=\begin{bmatrix} x_s & y_s & z_s & t \end{bmatrix}^T$$

$$=\begin{bmatrix} \pm r_{bs}\left[\sin\left(\theta_{s0}+\theta_s\right)-\theta_s\cos\left(\theta_{s0}+\theta_s\right)\right] \\ -r_{bs}\left[\cos\left(\theta_{s0}+\theta_s\right)+\theta_s\sin\left(\theta_{s0}+\theta_s\right)\right] \\ u_s \\ 1 \end{bmatrix} \tag{2}$$

In the above formula, $\theta_{S0}=\pi/(2N_s)-inv\alpha_S$, where $r_{bs}$ is the tool base circle radius, $\theta_s$ is azimuth of the tooth, $\theta_{s0}$ is the alveolar angle. $u_s$ is Z direction variable of the tool. $N_s$ is cutter tooth number. $\alpha_s$ is pressure angle of the tool. $inv\alpha_s$ is the involute function. "$\pm$"of the equation (2) respectively with the involute $\gamma-\gamma$ and $\beta-\beta$ corresponding.

The unitary normal of the cutter tooth surface is

$$\overrightarrow{n_s}=\begin{bmatrix} n_{sx} \\ n_{sy} \\ n_{sz} \end{bmatrix}=\frac{\partial\overrightarrow{r_s}/\partial\theta_s\times\partial\overrightarrow{r_s}/\partial u_s}{\left|\partial\overrightarrow{r_s}/\partial\theta_s\times\partial\overrightarrow{r_s}/\partial u_s\right|}=\begin{bmatrix} \mp\cos(\theta_{s0}+\theta_s) \\ -\sin(\theta_{s0}+\theta_s) \\ 0 \end{bmatrix} \tag{3}$$

*2.2 The establishment of tooth surface equation of orthogonal face gear*

Assuming any point $N(x_s,y_s,z_s)$ on the tooth surface of the tool, the diameter of the vector $\vec{r_s}$ is listed below

$$\vec{r_s}=\begin{bmatrix} x_s & y_s & z_s \end{bmatrix}^T=x_s\vec{i_s}+y_s\vec{j_s}+z_s\vec{k_s} \tag{4}$$

Where $\vec{i_s}$ , $\vec{j_s}$ , $\vec{k_s}$ are unit vector of three directions in the frame $S_s$ .

The relative velocity vector $\vec{v}^{(S,2)}$ of *N* points in the two tooth surface is

$$\vec{v}^{(S,2)}=\begin{bmatrix} v_x^{(S,2)} \\ v_y^{(S,2)} \\ v_z^{(S,2)} \end{bmatrix}=\omega_s\begin{bmatrix} -y_s-z_s q_{2s}\cos\varphi_s \\ x_s+z_s q_{2s}\sin\varphi_s \\ q_{2s}(x_s\cos\varphi_s-y_s\sin\varphi_s) \end{bmatrix} \tag{5}$$

Where $\omega_s$ is angular velocity of tool *S*. $q_{2s}$ is the ratio of the number of orthogonal face gear teeth and the cutter tooth number.

According to the gear meshing principle, the following equations can be obtained

$$\overrightarrow{n_s}\cdot\vec{v}^{(S,2)}=0 \tag{6}$$

Taking the equation (2), (3), (5) into equation (6), the meshing equation can be obtained orthogonal face gear

$$f(\theta_s,\varphi_s,u_s)=r_{bs}-u_s q_{2s}\cos\varphi_\theta=0 \tag{7}$$

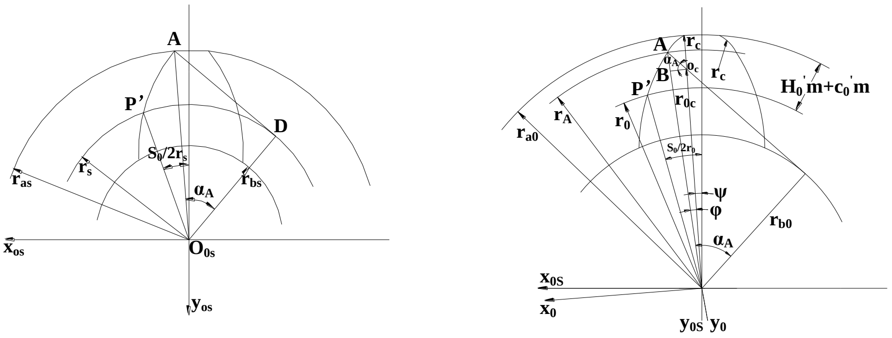
Simultaneous equations (1) and (7) can get the tooth surface equation of orthogonal face gear

$$\begin{cases} x_2=r_{bs}\left[\cos\varphi_2\left(\sin\varphi_\theta\mp\theta_s\cos\varphi_\theta)-\dfrac{\sin\varphi_2}{q_{2s}\cos\varphi_\theta}\right)\right] \\ y_2=-r_{bs}\left[\sin\varphi_2\left(\sin\varphi_\theta\mp\theta_s\cos\varphi_\theta)+\dfrac{\cos\varphi_2}{q_{2s}\cos\varphi_\theta}\right)\right] \\ z_2=-r_{bs}\left(\cos\varphi_\theta\pm\theta_s\sin\varphi_\theta\right) \end{cases} \tag{8}$$

In the above equation $\varphi_\theta=\varphi_s\pm(\theta_{s0}+\theta_s)$, $\varphi_2=q_{2s}\varphi_s$ .

*2.3 The transitional surface equation and 3D model of orthogonal face gear are established*

The structure of the gear shaper with the angle tooth top and the fillet tooth top were illustrated in figute3.(a), (b).




(a) The gear shaper with the angle tooth top

(b) The gear shaper with the fillet tooth top

**Figure 3.** Structural representation of two kinds of gear shaper cutters

In figure.3 (b), $S_0$ and $S_{os}$ of the coordinate system is constructed. $r_{a0}$ is the addendum circle radius. $r_0$ is the radius of the cutter's dividing circle. $r_{bo}$ is the tool base circle radius. The center of curvature of the tooth tip of the pinion cutter is $O_c$. Arc and involute tangent to the point $A$, Point $A$ to the diameter is $r_A$. The radius of the fillet is $r_c$. The pressure angle of involute at point $A$ is $\alpha_A$.

The equation of the $\sum_{2t}$ of the surface of the transition curve of the gear surface by the enveloping principle is listed below

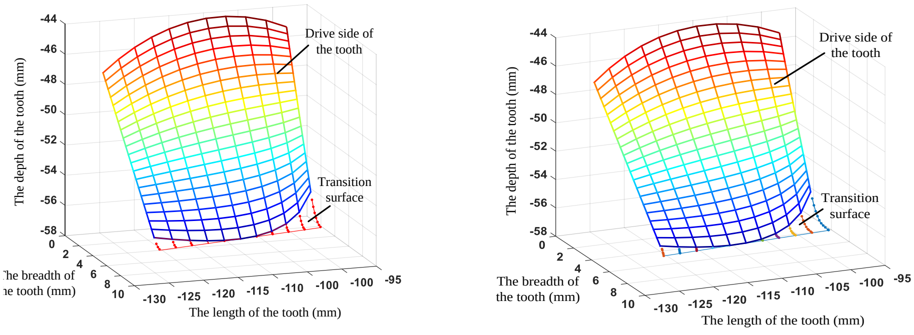
$$\begin{cases} r_{2t}(\varphi_s,\gamma_A,u_s)=\left[M_{2t,\mathrm{t}}\right]r_t(\gamma_A,u_s) \\ f(u_s,\varphi_s,\gamma_A)=0 \end{cases} \tag{9}$$

It is concluded that the transition surface equation of the orthogonal face gear cutting by the tooth tip fillet cutter is obtained

$$\begin{cases} X_{2t}=r_c\cos\varphi_2(\cos\mathrm{B}-\cos A)+r_A\cos\varphi_2\sin C \\ \qquad -\dfrac{r_A\sin\varphi_2\cos(\alpha_A+\gamma_A)-r_A\sin\varphi_2\cos\alpha_A}{q_{2s}(\cos A-\cos B)} \\ Y_{2t}=r_c\sin\varphi_2(\cos B-\cos A)-r_A\sin\varphi_2\sin C \\ \qquad -\dfrac{r_A\cos\varphi_2\cos(\alpha_A+\gamma_A)-r_A\cos\varphi_2\cos\alpha_A}{q_{2s}(\cos A-\cos B)} \\ Z_{2t}=r_c\sin A-r_c\sin B-r_A\cos C \end{cases} \tag{10}$$

In the above equation, $A=\alpha_A-\varphi-\psi-\tau-\varphi_s$ ; $B=\alpha_A+\gamma_A-\varphi-\psi-\tau-\varphi_s$ ; $C=\varphi+\psi+\tau+\varphi_s$

Based on the mathematical model of the orthogonal face gear tooth surface, the 40 tooth surface gear is taken as an example, based on the Matlab software, the theoretical tooth profile of the face gear made by two kinds of slotting cutters is obtained, as shown in figure 4. In the standard, when $m=2\sim 8\mathrm{mm}$, the tooth tip of the pinion cutter is rounded to $r_c=0.15\sim 0.4\mathrm{mm}$. In this paper, $r_c=0.3\mathrm{mm}$.




(a) The theoretical tooth profile cutted by the angle tooth top of the shaper cutter

(b) The theoretical tooth profile cutted by the rounding tooth top of the shaper cutter

**Figure 4.** Theoretical tooth profile of orthogonal face gear

The theoretical tooth profile of the two orthogonal face gears is compared, as shown in figure 5. It can be seen that the curvature of the tooth root transition surface is different. The curvature analysis shows that the radius of curvature of the orthogonal face gear cut by the rounding tooth top of shaper cutter is larger than the other.

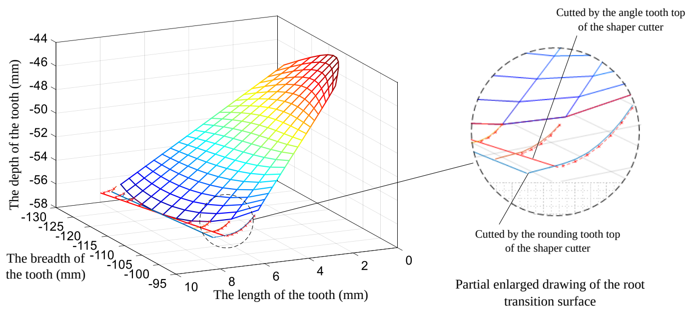



**Figure 5.** Tooth root transition surface of orthogonal face gear cutted by two kinds of gear shaper cutters

Through the numerical calculation of the gear tooth surface equation, the discrete data points can be obtained, based on Creo software, the two kinds of orthogonal gear tooth profile and 3D model can be constructed by the boundary blending, as shown in figure 6.

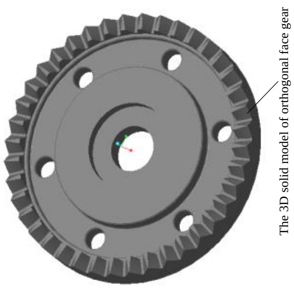



**Figure 6.** The model of all gears of orthogonal face gear

## 3. Curvature analysis of transition surface of orthogonal face gear

### *3.1 The main curvature and the main direction of transition surface*

According to the principle of differential geometry, the formula (9) can be expressed as a vector

$$\vec{r} = x_{2t}\vec{i} + y_{2t}\vec{j} + z_{2t}\vec{k} \tag{11}$$

The unit normal vector $\vec{n}$ of the transitional surface of the face gear is

$$\vec{n} = \vec{r}_{\varphi_s} \times \vec{r}_{\gamma_A} \Big/ \left| \vec{r}_{\varphi_s} \times \vec{r}_{\gamma_A} \right| \tag{12}$$

Let $L = -\vec{n}_{\varphi_s} \bullet \vec{r}_{\varphi_s}$, $M = -\vec{n}_{\varphi_s} \bullet \vec{r}_{\gamma_A}$, $N = -\vec{n}_{\gamma_A} \bullet \vec{r}_{\varphi_s}$, $E = \vec{r}_{\varphi_s} \bullet \vec{r}_{\varphi_s}$, $F = \vec{r}_{\varphi_s} \bullet \vec{r}_{\gamma_A}$, $G = \vec{r}_{\gamma_A} \bullet \vec{r}_{\gamma_A}$. In the formula, $L$、$M$、$N$ and $E$、$F$、$G$ are the first and second fundamental quantities of the surface.

The main curvature radius of the transitional surface of the face gear should be R

$$(LN - M^2)R^2 + (2FM - EN - GL)R + (EG - F^2) = 0 \tag{13}$$

The two root of the equation $R_1$ and $R_2$ of the principal curvature radius of the transitional surface of the face gear can be obtained by the formula (13), Then $1/R_1$ and $1/R_2$ are the principal curvatures of the surface of the face gear, let $K_1^{\text{II}} = 1/R_1$ and $K_2^{\text{II}} = 1/R_2$, the normal curvature of the arbitrary surface of the transitional surface of the face gear is obtained by Euler's formula

$$K^{\text{II}} = \text{H}^{\text{II}} + \text{R}^{\text{II}} \cos(2\varphi) \tag{14}$$

Where $H^{\text{II}} = (K_1^{\text{II}} + K_2^{\text{II}})/2$, $R^{\text{II}} = (K_1^{\text{II}} - K_2^{\text{II}})/2$, $\varphi$ is the angle between the vector and $e_1^{\text{II}}$ in any direction.

Assume that the main direction of the transitional surface is $\lambda = {}^{d\varphi_s}\!/_{d\gamma_A}$, the surface equation is[9]

$$(EM - FL)\lambda^2 + (EN - GL)\lambda + (EN - GM) = 0 \tag{15}$$

The formula (15) can be used to obtain the $\lambda_1^{\text{II}}$ and $\lambda_2^{\text{II}}$ of the two main directions of surface $\Sigma_{2t}$, and perpendicular to each other, unit vectors in two directions are represented by $e_1^{\text{II}}$ and $e_2^{\text{II}}$.

### *3.2 The factors analysis of affecting the main curvature of the transitional surface of face gear*

Based on the previous analysis, the main curvature of the transition surface of the tooth tip and the addendum angle cutter are obtained, as shown in figure 7. As can be seen from the graph, the main curvature of the transitional surface of the face gear produced by the tooth tip shaper cutter is larger than the main curvature of the transitional surface cut by the addendum angle cutter, it verifies the previous derivation that the transition surface produced by the gear shaper cutter is more smooth. Then, the influence of the change of the main curvature on the surface of the gear is studied from three aspects: the number of teeth, the pressure angle and the transmission ratio. The following chart is the corresponding calculation results, the calculated parameters are: face gear tooth number $N_2 = 40$, cutter tooth number $N_s = 17$, the pressure angle $\alpha_r = 20^o$, addendum radius of gear-slotting cutter $r_c = 0.3mm$.

As shown in the diagram is $K_{12}$, one of the principal curvatures of the transitional surface of the face gear, the other principal curvature $K_{22}$ is negative value, but its absolute value is similar to $K_{12}$, so it is not given in the paper.

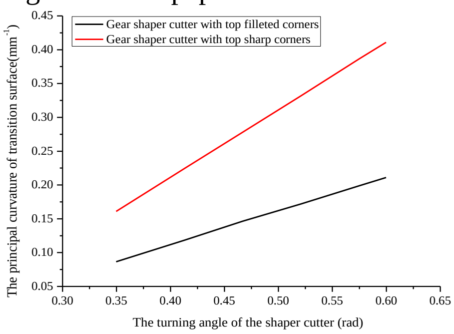



**Figure 7.** Main curvature of transition surfaces of two kinds of cutting tools

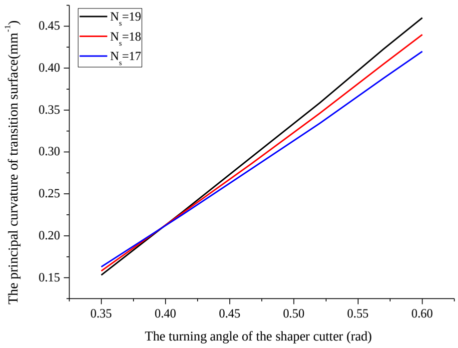



**Figure 8.** Influence of the number of teeth on the main curvature

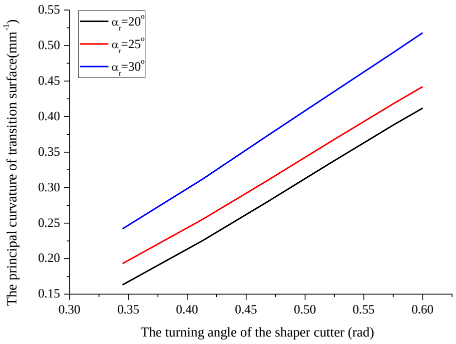



**Figure 9.** Influence of pressure angle $\alpha_\gamma$on the main curvature

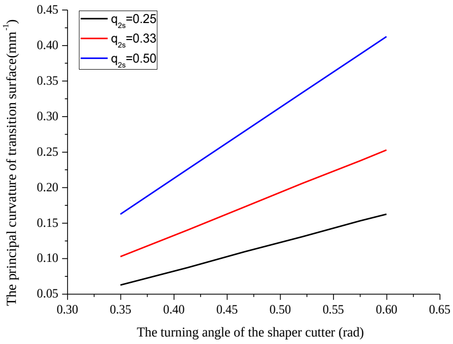



**Figure 10.** Influence of tooth number ratio $q_{2s}$ on the principal curvature

According to the analysis, it can be seen that for the main curvature of the transitional surface of the face gear, the small variation of the number of teeth of the gear shaper cutter has little effect on the main curvature of the transitional surface of the gear, furthermore, the changes near the work surface and away from the work surface at the main curvature of the opposite situation; The increase of the pressure angle of the gear shaper cutter will make the main curvature of the surface of the face gear become larger; The increase of the transmission ratio will make the main curvature of the transitional surface of the face gear become larger.

## 4. Conclusion

(1) Using Matlab and Creo software to carry out the 3D modeling of the gear, the 3D model of two kinds of face gears made by slotting cutters are obtained, the accurate modeling and visualization of the whole profile of the gears are realized.

(2) The main curvatures of the transitional surfaces of two kinds of gear cutters are analyzed, it is known that the main curvature of the transitional surface of the face gear produced by the tooth tip shaper cutter is larger than the main curvature of the transitional surface cut by the adthedendum angle cutter, it verifies the correctness of the previous derivation.

(3) The influence of the number of gear teeth, the pressure angle and the transmission ratio on the main curvature of the transitional surface of the gear is analyzed.

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