Community structure of complex networks based on continuous neural network

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Abstract. As a new subject, the research of complex networks has attracted the attention of researchers from different disciplines. Community structure is one of the key structures of complex networks, so it is a very important task to analyze the community structure of complex networks accurately. In this paper, we study the problem of extracting the community structure of complex networks, and propose a continuous neural network (*CNN*) algorithm. It is proved that for any given initial value, the continuous neural network algorithm converges to the eigenvector of the maximum eigenvalue of the network modularity matrix. Therefore, according to the stability of the evolution of the network symbol will be able to get two community structure.

1. Introduction

One is 1998 Watts and Strogats in the article published on the Nature[1-2] reveals the characteristics of small world complex network; another is 1999 Barabasi and Albert published in Scienc reveals the complex networks scale-free properties, that due to this nature of the network of the two indispensable elements of evolution. And a network model[3] of suggests that the traditional network model is based on the random graph [4].

The study of complex networks is mainly to study the structure of the community, there are many algorithms for extracting the community structure in complex networks. In 2002, Girvan and Newman[5] proposed a community structure method (GN) for hierarchical network decomposition by edge removal. This research work is considered to be a pioneering work in the study of modern community structure. GN algorithm to find the most likely to be in the community between the edges, by constantly removing these edges to get the network level. Radicchi et al. [6] has made some improvements to the GN algorithm, and proposed a new GN algorithm.

2. Related theories

2.1. Continuous neural network to solve the eigenvalues and eigenvectors of the matrix

Consider differential equation:

$$\frac{dX(t)}{dt} = AX(t) - X^T(t)AX(t)X(t) \qquad A^T = A \tag{1}$$

Which is the real symmetric matrix A, can be regarded as the right connection strength, neural network, neural network as a state, then equation (1) describes the feedback neural network for a class of continuous type. The presents a circuit simulation method [7], while for symmetric positive definite

matrices, such as [8] neural network according to the rules, the equation (1) as a kind of unsupervised neural network learning process, studies the feature extraction of the main element of the input space, prove the equation (1) characteristics of the largest the solution converges to the value of the corresponding eigenvectors from arbitrary initial value. This can be guaranteed by the following theorem.

Theorem 1: The maximum value of the formula (1) converges to the eigenvector   corresponding to the largest eigenvalue.

In this paper, a new method is presented to solve the eigenvalues and eigenvectors of a matrix.

2.2. Modularity function in community extraction

It is assumed that the network contains n nodes, and the network is divided into two groups,Let the representation $s_i = 1$ nodes belong to the first group,At that time, the node $s_i = -1$ belongs to the second group. A_{ij} (The values are 0 and 1)Represents the number of connections between i nodes and j nodes,If there is a heavy edge in the network, you can take the value of 1, A_{ij} is called the network adjacency matrix elements, if the node and the edge of the connection between the nodes is random, Mean value is $k_i k_j / 2m$, k_i means i node 's degree.($i = 1,2...n$ $i = 1,2...n$)

$m = \frac{1}{2}\sum_i k_i$ Represents   the total number of edges in a network,In the document [8], modularity function Q in the community extraction problem can be defined as

$$Q = \frac{1}{4m}(A_{ij} - \frac{k_i k_j}{2m})s_i s_j = \frac{1}{4m}s^T Bs \tag{2}$$

Here s represents vector,The i elements is s_i , $i = 1,2...n$, $\frac{1}{4m}$ is a constant,The symmetric matrix B is called a modular matrix,Its elements are:

$$B_{ij} = A_{ij} - \frac{k_i k_j}{2m} \tag{3}$$

Use (3) to simplify the matrix to find its row and column and are zero. Therefore, there is always a feature vector(1,1,...,1) of the matrix B, and the corresponding eigenvalue is 0, which is the property of the Laplasse graph matrix, which is also the basis of the graph [9-12].

3. Continuous neural network algorithm for extracting complex network community structure (CNN)

In the literature [13], the modularity function B is defined as (2),The modularity matrix B is defined as (3).If the complex network is an undirected network, the adjacency matrix A is a symmetric matrix. It can be seen from (3) that the modularity matrix B is also a symmetric matrix.Therefore. according to the literature [14], we can use the equation (1) neural network system to solve the eigenvalue matrix B of the maximum eigenvalue corresponding to the eigenvector. at the same time,Combined with the features of *Newman* value principle of community structure algorithm to extract feature vectors, we can not calculating the maximum characteristic value of matrix module B of the corresponding feature vectors, and directly through the solution of the differential equation (1), using the differential equation (1) solution of the elements in the symbol can be divided into nodes in complex networks for the two class. Accordingly, we propose the following CNN algorithm.

The problem of community structure detection in view of the complex network, we will be based on the combination of a new continuous neural network community structure detection algorithm based on feature matrix module function Q value and feature extraction of community structure in complex networks algorithm and neural network algorithm for solving matrix eigenvalues and

eigenvectors of the Newman algorithm, referred to as CNN algorithm model:

$$\frac{dX(t)}{dt} = BX(t) - X^T(t)BX(t)X(t) \quad B^T = B \tag{4}$$

Among them, the modularity matrix of complex networks defined by (2) , $X \in R^n$. According to theorem 2, the solution of the equation (3) converges to the eigenvector corresponding to the maximum eigenvalue of the modular matrix from any initial value. According to the characteristic value vector algorithm of *Newman,* we can extract the community structure in complex network according to the sign of the elements in the solution of equation (3).

To sum up, set up the adjacency matrix of complex network, this paper gives the CNN community structure extraction algorithm steps are as follows:

Step **1**: Calculate Modularity matrix *B* of complex networks,The elements are $B_{ij} = A_{ij} - \frac{k_i k_j}{2m}$, k_i Represents the degree of i node, A_{ij} represents the adjacency matrix of a complex network.

Step 2:Given any initial value $X(0)$,Solve $\frac{dX(t)}{dt} = BX(t) - X^T BX(t)X(t)$ solution X^*

Step 3:According to the solution X^*, we get the label vector s of the community structure.The elements in the vector are as follows

$$s_i = \begin{cases} +1, assume, x_i \geq 0 \\ -1, assume, x_i \leq 0 \end{cases} \qquad i = 1,2...n$$

Step 4: Extract $+1$ the corresponding node of s .

4. Simulation experiment analysis

4.1 . experimental data
In this paper, we used the karate club network (Zachary 's Karate Club) data to do the experiment.The network data download URL is http://www-personal.umich.edu/~mejn/netdata/ [15].

4.1.1 (Zachary's Karate Club) [16]. The problem of community extraction is one of the most widely used examples. Karate club network consists of 34 nodes, each node represents a member of the network, the 78 sides expressed among the members of the social intercourse, because of the conflict between the supervisor and the club president, the club members were divided into two groups respectively by the supervisor and the principal as the center, see detailed literature [16].

4.2. Experimental results and analysis
In the experimental part, we will analyze the stability and classification effect of continuous neural network algorithm on the club network and dolphin network. Select all the nodes in the network and a number of nodes to analyze the stability of the CNN algorithm, as shown in Figure 1, figure 2 shows.
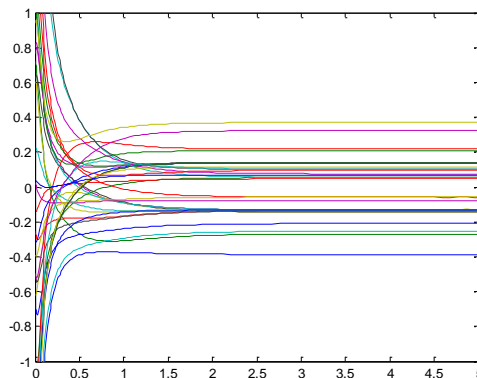
**Figure 1.** stability analysis of the CNN algorithm on the karate club network
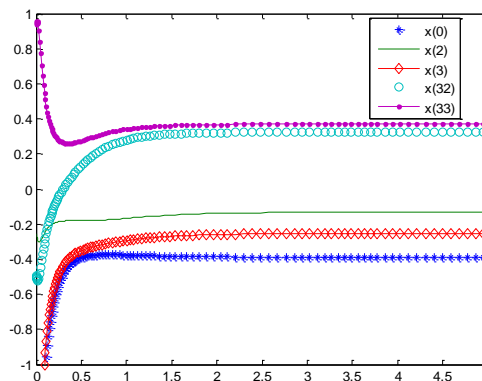
Figure 2. stability analysis of some points on the karate club network based on CNN algorithm

From Figure 1 we can see that when t=0, the arbitrary initial value$X(0)$, with the passage of time, we find that the final convergence network state corresponding to each node in the network of the club to a stable state, i.e. the characteristics of convergence to the module matrix *B* club network value of the corresponding feature vector.

Therefore, the community structure in the club network is extracted according to the symbol of the stable state of the network. From Figure 2 we can see clearly the convergence of some state of the CNN network, can be found by any given initial value, after only 1.5, network evolution network 0, 2, 3, 32, 33 of the five nodes reached a steady state. It shows that the CNN algorithm has good stability.

The continuous neural network algorithm is used to extract the community structure of the karate club network, as shown in figure 3.
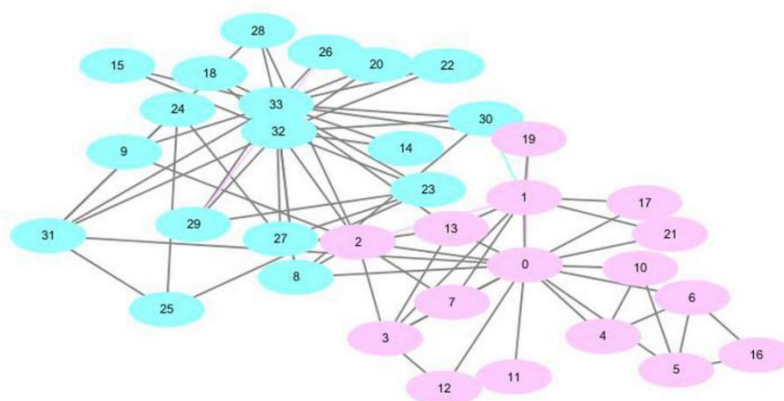
Figure 3. the classification of community structure in karate club network

Figure 3 shows the simulation results of the continuous neural network algorithm on the karate club network. The experimental results show that the classification algorithm of continuous feature vector algorithm, the karate club network results with feature vector algorithm (Q=0.3715), showed that the text of the algorithm is reasonable.

5. Conclusion

Complex network is a powerful tool to study complex systems, many of the real systems can be abstracted into complex networks, the nodes of the network correspond to the entities in the system. The research shows that there are different degrees of community structure in many real networks, and the community structure determines some functional properties of the network, so the description in the network and detection of these community structure has important practical significance.This paper is based on the detection of community structure in complex networks.The algorithm of continuous neural network extraction algorithm of community structure based on the function module Q of the index, and the related algorithm of our algorithm and have made comparative, show the effectiveness of our proposed algorithm.

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