

# Study on drag coefficient of rising bubble in still water

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**Abstract:** Research on the behavior of a rising bubble in still water is on the basis of Newton's theory of classical mechanics. Develop a calculation analysis and an experimental process of bubble rising behavior in order to search for an appropriate way of valuing drag coefficient, which is the key element toward this issue. Analyze the adaptability of the drag coefficient; compare the theoretical model to the real experimental model of rising bubble behavior. The result turns out that the change rate of radius could be ignored according to the analysis; the acceleration phase is transient; final velocity and the diameter of bubble do relate to the drag coefficient, but have no obvious relation with the depth of water. After series of inference analysis of the bubble behavior and experimental demonstration, a new drag coefficient and computing method is proposed.

## 1 General instructions

The bubble rising behavior appears to many applications in engineering. The kinetic regulation such as size, velocity, morphology is the key point of bubble rising behavior. To the calculating of kinetic parameters, except the physical properties of bubble gas and fluid around the bubble, the drag coefficient tends to influence conclusively to the motor process of bubble rising behavior.

## 2 Theoretical model of bubble rising behavior

According to the principle of Newton's classical mechanics, assuming that air bubbles in the process of rising is influenced by gravity, buoyancy and viscous drag force. Suppose the initial velocity is zero, in the role of these three forces, the bubble velocity is increasing; the resistance of bubble increases simultaneously, then it tends to act as an action of uniform motion in a straight line after this process.

### 2.1 Variable drag coefficient case

Depending on the assumption of variable drag coefficient, the final velocity of bubble rising behavior formula performs like below. In this formula, coefficient C is related to the drag coefficient  $C_D$ , and for the value of  $C_D$ , we can get different conclusions from many scholars.

$$v = \frac{16gr^2(\rho_l - \rho_g)}{3C\mu} \quad (1)$$

Where  $v$  = velocity of rising bubble;  $r$  = bubble radius;  $\rho_{l, g}$  = density of water and bubble gas;  $C$  = empirical constant;  $Re$  = Reynolds number;  $\mu$  = viscosity.



## 2.2 Fixed drag coefficient case

Depending on the assumption of fixed drag coefficient, the final velocity of bubble rising behavior formula performs like below.

$$v = \sqrt{\frac{8rg(\rho_l - \rho_g)}{3\rho_l C_D}} \quad (2)$$

As a consequence, it needs consolidated characteristic analyze under different conditions of different drag coefficients.

## 2.3 General analysis of drag coefficient

According to the argument above, the value of drag coefficient is the key point to the series calculation of bubble behavior. To this issue, Batchdor indicates that when a bubble is rising in the still water, the resistance formula can be

$$D = 6\pi\mu r v \quad (3)$$

Comparing to the general formula of bubble rising, it can be inferred that the value of 'C' in this model should be 24. When  $Re \leq 1$ , Lamb argue that the drag coefficient should be calculated like this.

$$C_D = \frac{24}{Re} \left[ \frac{3\mu^* + 2}{3\mu^* + 3} \right] \approx \frac{16}{Re} \quad (4)$$

When  $Re > 1$ , Levich and Moore suggest that the drag coefficient formula can be  $C_D = 48/Re$  和  $C_D = 32/Re$ ; and when  $Re > 1000$ , drag coefficient can be approximately use the fixed value  $C_D = 0.45$ .

## 3 The calculation and analysis of bubble rising behavior

In order to confirm and explore an appropriate way of valuing  $C_D$ , a series of different-radius bubble behavior is calculated. The radius comes 1mm, 5mm, 10mm. The bubble gas should be nitrogen.

### 3.1 Calculation thought and the adaptability analysis of drag coefficient

At first, it is quite hard to ensure the  $Re$  of still water around rising bubble, according to literature, in the calculation formula, there are 4 ways to confirm  $C$  relating to  $C_D$ , which correspond to different scope of application towards  $Re$ .

First, assume a scope of 'Re', then calculate respectively the bubble rising velocity according to the calculated values of different  $C$  values of the drag coefficient. There must be an  $Re$  value which is related to the exact  $C$ . Secondly, calculate back to the value of  $Re$  by this  $C$ . Thirdly,  $C$  should be chosen inside {16,24,32,48}, try to elaborate on the scope of  $Re$ , which means that it should be correspond to the assumption: when  $C=16$ ,  $Re \leq 1$ ; when  $C=32$  or  $48$ ,  $Re > 1$ . Of all those above tend to be incorrect or default, then try to use the result of fixed  $C_D$  which is 0.45 when  $Re > 1000$ , try again to confirm it as the way above.

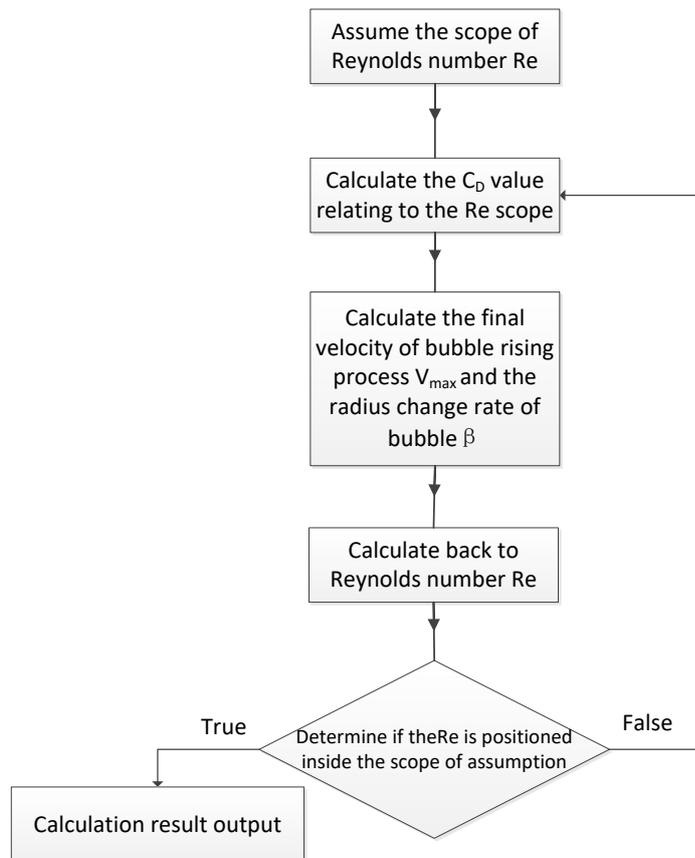


Figure 1. Flow chart for calculation of bubble rising behavior

### 3.2 Calculation analysis of bubble rising behavior

When listing facts use either the style tag List signs or the style tag List numbers.

### 3.3 Calculation analysis based on the variable drag coefficient

According to the calculation, when the bubble radius is 1mm, the final rising behavior of bubble is between 1.08-3.25, the calculation is correspond to reality compared with 5mm and 10mm bubble. Additionally, the changing rate of radius of rising bubble from start to end tends to be little, which wouldn't affect the calculation result and could be ignored according to the calculation.

Table 1. Calculation of bubble kinetic properties when radius is 1mm

C	16	24	32	48
$v_{\max}$ (m/s)	3.25	2.17	1.63	1.08
Re	6481	4320	3241	2160
$\beta_{z=2}(\times 10^{-5})$	-	5.9	4.4	2.9
$\beta_{z=0}(\times 10^{-5})$	-	7.1	5.3	3.5

Table 2. Calculation of bubble kinetic properties when radius is 5mm

C	16	24	32	48
$v_{\max}$ (m/s)	81.26	54.17	40.63	27.09
Re( $\times 10^5$ )	8.1	5.4	4.1	2.7
$\beta_{z=2}$	-	0.0074	0.0055	0.0037
$\beta_{z=0}$	-	0.0088	0.0066	0.0044

Table 3. Calculation of bubble kinetic properties when radius is 10mm

C	16	24	32	48
$v_{\max}$ (m/s)	325.0	216.7	162.5	108.3
Re ( $\times 10^6$ )	6.4	4.3	3.2	2.1
$\beta_{z=2}$	-	0.059	0.044	0.030
$\beta_{z=0}$	-	0.071	0.052	0.035



Figure 2. The morphological change during bubble rising behavior

3.3.1 *Calculation analysis based on the fixed drag coefficient.* According to the calculation above, we can definitely find that all Re number are above 1000. Thus we can use the fixed value of  $CD=0.45$ . The calculation is below:

- (1) If bubble radius is 1mm, the final velocity is 0.24 m/s.
- (2) If bubble radius is 5mm, the final velocity is 0.54m/s.
- (3) If bubble radius is 10mm, the final velocity is 0.76 m/s.

#### 4 Experimental confirmation of bubble rising behavior

This experimental design creates bubble under different pressure, monitoring by high-speed cinematography. We can monitor the whole process of bubble rising behavior and the appearance of bubble at any time of rising process. We suppose the bubble as a spheroid and calculate the volume which is equivalent to the sphere, aiming at getting the radius of bubble in any time. Finally we compare those bubble rising parameters with the former discussed calculation, trying to derive an appropriate value of  $C_D$ .

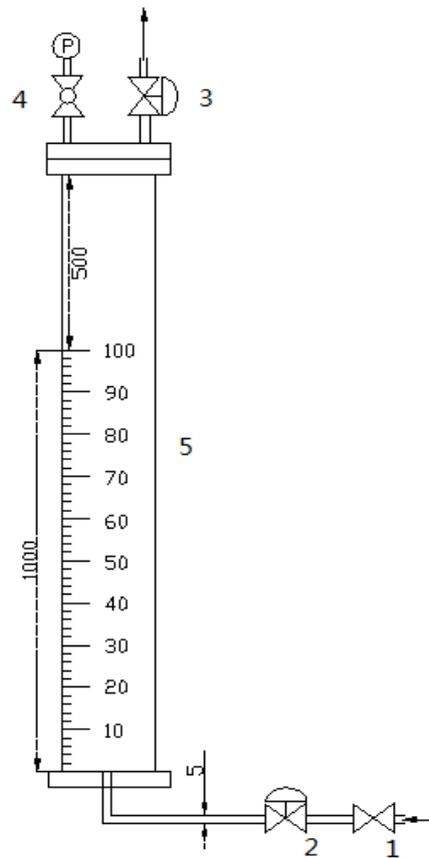


Figure 3. The experimental installation schematic diagram

The experiment is designed under the pressure of 0-1 MPa, the circular transparency tube is positioned vertically. We can check the pressure inside any time, while there is an inflow valve at the bottom of tube. The  $C_{DS}$  calculation according to the experiment result is shown below.

Table 4. Margin settings for A4 size paper and letter size paper.

Pressure drop	Bubble volume	Radius	Final	Velocity
MPa	cm <sup>3</sup>	cm		m/s
0.1	0.142	0.213	0.199	0.207
	0.196	0.250	0.201	
	0.126	0.200	0.210	
0.2	0.278	0.298	0.204	0.204
	0.179	0.259	0.211	
	0.179	0.239	0.213	
0.3	0.178	0.238	0.199	0.199
	0.153	0.221	0.209	

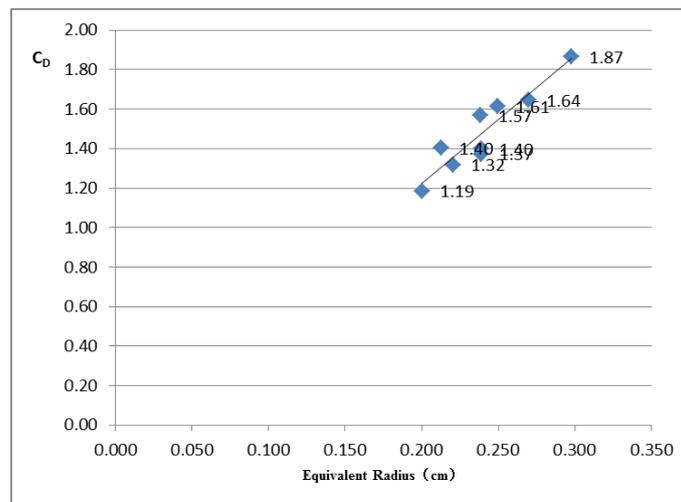


Figure 4. Calculation of drag coefficient based on experimental result

Above all, in the experiment, to a bubble with the radius of 2-3 mm, the drag coefficient is calculated between 1.3-1.7, which is different from the former study of scholars. The reason is speculated as different experimental fluid and the complex condition during bubble rising fact. By analyzing the bubble rising final velocity and the  $C_D$  calculation experimental result, the average value of  $C_D$  is 1.48, by using this fixed value of  $C_D$ , the calculation of bubble rising final velocity tends to be close to the actual value, which error could be basically controlled in 5%.

## 5 Conclusions

(1) The calculation based on the theoretical method of bubble rising behavior tends not to be reality, which is probably led by the inappropriate value of  $C_D$ .

(2) According to the calculation, during the bubble rising process, the bubble radius changing rate tends to be extremely low which could be ignored during calculation.

(3) In the case of  $Re \geq 1000$  to the bubble rising condition, the calculation of small bubble rising behavior drag coefficient can be valued a fixed 1.48, which limits the calculation error within 5% compared with real term.

(4) The process of accelerate is transient, the final velocity of bubble rising behavior is significantly related to the value of  $C_D$ , rather than the water depth.

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