

The complementary graphical method used for profiling side mill for generation of helical surface

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Abstract. This paper presents a method developed in CATIA design environment, for profiling tools bounded by revolution peripheral surfaces — side mill tool. The graphical method is based on a complementary theorem of surface enveloping. They are presented specific algorithms and an example for profiling generating tools of helical flutes of compressors rotors with three lobes. The obtained results with graphical method are compared with those obtained by a classical method — the Nikolaev theorem. The graphical method is very intuitive and, at the same time, very rigorous. It is characterized by the simplicity of application and avoids the ambiguity case of solutions, which are frequently met in numerical methods, as profiles overlapping, generating of revolving surfaces or rotating a spatial curve around the tool's axis. Other advantage of using graphical methods is that CNC machines tools, used for generating profiled tools, allows importing the files, which directly result from graphical modeling.

1. Introduction

The Olivier fundamental theorem and the Gohman theorem [1, 2], as fundamental theorems of surface enwrapping, can be applied to the study of tools' profiling, bounded by revolution surfaces (side mill, end mill or ring tool) to generate the cylindrical helical surfaces with constant pitch [3-5].

Subsequently, specific theorems based on complementary analytical methods, as the method of the “substituting circles family” [2] or the method of “minimum distance” [6, 7] were elaborated and used for profiling side mills which generate cylindrical helical surfaces with constant pitch.

The analytical methods are rigorous and have a synthetic form of the enveloping condition but they have the disadvantage of a very complicated expression, which presupposes a form based on transcendental equations of these conditions.

The continuous development of the graphical design environment, as CATIA, allows the approach of the issue of contact between two surfaces based on the first theorem of Olivier, as enwrapping of a surfaces family, which depends on a single parameter and, in graphical shape using the capabilities of the design environment. The specific method was based on algorithms programmed in Visual Basic, to profile a side mill tool, which generates a cylindrical helical surface with constant pitch [8-10].

The methods developed based on Nikolaev specific theorem [11] allow the graphical approach of the issue of profiling side mill tools or other types of tools bounded by revolution peripheral surfaces, which generate a helical surfaces by means of enwrapping.



From the multitude of authors that have dealt with this issue, we can remark Ott and Artyukin [3], who approach in an analytical way the problem of generation of a helical surface with composed generatrix; Shalamov [5] which analyses the best shaping process for a special surface; [4] which approaches the solving strictness for the transcendental equations which emerge when the contact between a helical surface and a revolution one is determined.

Giovanni Mimmi [12] proves that the issue for profiling tools, which generates cylindrical helical surfaces with constant pitch, is not exhausted and the elaboration of new, rigorous and simple design ways can be useful when the contact between a helical surface and a revolution one is analyzed. This is the case of generation with side mill, end mill or ring tools [4].

They also approach issues of machining precision [13-15] and helical surfaces identification [16].

2. The method of “minimum distance”

The method of “minimum distance” [2, 7] is a complementary method based on a theorem specific to this method. The contact between the cylindrical helical surface with constant pitch and the future revolution surface is examined in planes perpendicular to the axis of the revolution surface.

The specific theorem is: The contact line (characteristic curve) between a cylindrical helical surface with constant pitch and a revolution surface is the geometric locus of the points belong to the helical surface where, in plane perpendicular to the revolution surface's axis, the distance to the curve which represents the intersection between the helical surface with these planes, is minimum, see figure 1.

Reference systems, see figure 1, are defined as follows:

XYZ is the reference system where it is defined the helical surface; the \vec{V} axis is overlapped with the axis Z and the own reference system of the Σ helical surface;

$X_2Y_2Z_2$ – reference system joined with the revolution surface (the primary peripheral surface of the side mill) with Z_2 axis of revolution surface – the \vec{A} axis.

The distance between axes \vec{A} and \vec{V} , measured perpendicular to the common normal, here the axis $X \equiv X_2$ – is denoted with a .

In the $X_2Y_2Z_2$ reference system, joined with the revolution surface axis, figure 1, the helical surface's equations are reported to $X_2Y_2Z_2$ reference system of the side mill, in principle in form, see (1), with u and φ independent variable parameters:

$$X_2 = X_2(u, \varphi); Y_2 = Y_2(u, \varphi); Z_2 = Z_2(u, \varphi). \quad (1)$$

The plane perpendicular on the \vec{A} axis, in $X_2Y_2Z_2$ reference system, has the equation:

$$Z_2 = H, (H - \text{arbitrary variable}). \quad (2)$$

In this way, onto the Σ surface, a curve Σ_H is defined:

$$X_2 = X_2(\varphi); Y_2 = Y_2(\varphi); Z_2 = H. \quad (3)$$

The distance from the points from this curve to the \vec{A} axis (Z_2) is:

$$\delta = \sqrt{X_2^2(\varphi) + Y_2^2(\varphi)}. \quad (4)$$

The condition that this distance to be minimal is given by the equation:

$$X_2(\varphi) \cdot \dot{X}_{2\varphi} + Y_2(\varphi) \cdot \dot{Y}_{2\varphi} = 0. \quad (5)$$

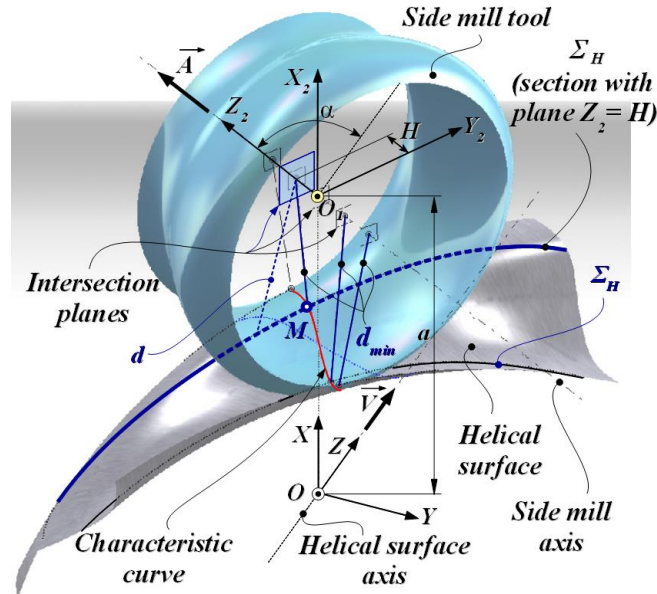


Figure 1. The “minimum distance” theorem. The plane $Z_2=H$ and curve Σ_H .

The condition (5) represents the *condition to determine a point belongs to the characteristic curve* at contact between the helical surface and a revolution one, with the axis \vec{A} .

For various values of the variable parameter H , an assembly of points is obtained which represents the characteristic curve – the contact curve of cylindrical helical surface, Σ , with the revolution surface S , the peripheral surface of side mill.

Although, the characteristic curve, in $X_2Y_2Z_2$ reference system, has equations on form:

$$C_{\Sigma} \begin{cases} X_2 = X_2(\varphi_H); \\ Y_2 = Y_2(\varphi_H); \\ Z_2 = H, \end{cases} \quad (6)$$

with H – arbitrary variable and φ_H the values of the φ parameter, which accomplish the geometric locus condition onto the helical surface, for each value of the H parameter (see equation (1) and (5)). By revolving the characteristic curve around the \vec{A} axis, the primary peripheral surface of the side mill is obtained. After this, by developing the equations of the revolution surface, the following are obtained:

$$S \begin{cases} X_2 = X_2(\varphi_H) \cos \Psi - Y_2(\varphi_H) \sin \Psi; \\ Y_2 = X_2(\varphi_H) \sin \Psi + Y_2(\varphi_H) \cos \Psi; \\ Z_2 = H, \end{cases} \quad (7)$$

with φ_H and Ψ variable parameters with H arbitrary parameter and Ψ – angular parameter in the revolving motion around \vec{A} axis.

The S_A axial section of the revolution surface is obtained by associating the condition $X_2 = 0$ with the equations (8):

$$S_A \begin{cases} S \begin{cases} X_2 = X_2(\varphi_H) \cos \Psi - Y_2(\varphi_H) \sin \Psi; \\ Y_2 = X_2(\varphi_H) \sin \Psi + Y_2(\varphi_H) \cos \Psi; \\ Z_2 = H; \end{cases} \\ -\text{Axial plane, } X_2(\varphi_H) \cos \Psi - Y_2(\varphi_H) \sin \Psi = 0. \end{cases} \quad (8)$$

In this way, by eliminating the Ψ parameter, the equations of the axial section are obtained, from (8), under the form:

$$S_A \begin{cases} R = \sqrt{X_2^2 + Y_2^2}; \\ Z_2 = H, \end{cases} \quad (9)$$

see figure 2. The side mill tool's profile is represented in figure 2, in an axial section, in coordinates R and H , see equations (9).

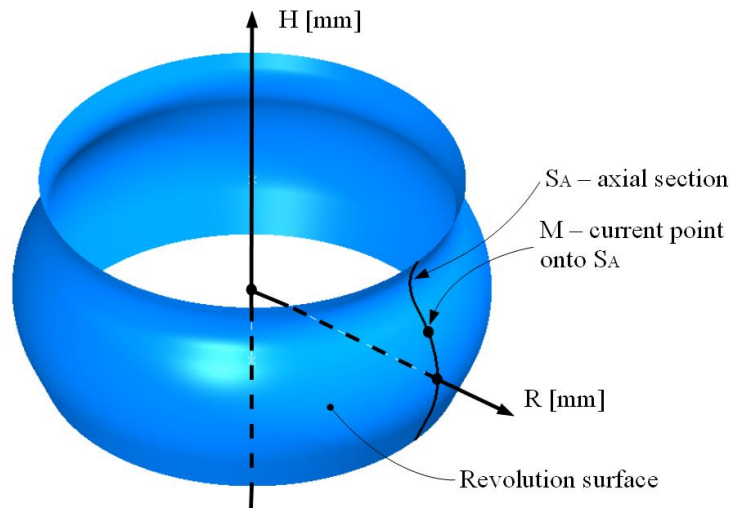


Figure 2. Axial section of the side mill (in principle); coordinates R and H defined in (9).

3. The graphical method of the “minimum distance” in CATIA for profiling the side mill for generating the roots compressor worm

3.1. Numerical application for a compressor rotor

The graphical method, developed in the CATIA design environment, initiates the methodology with the design of the rotor's frontal profiles. As example, in figure 3, it is presented the model of the crossing profile of the rotor: AC - circle arc with radius r , epicycloids arc generated by the singular point B onto the conjugated rotor; BD - circle arc with radius r .

The BC circle arc is a passing curve determined by the B singular point on the conjugated worm. The two conjugated worms of the Roots compressor are identically in crossing section but have helix in opposite senses.

With the command “*HELIX*”, the helix enveloped on the imaginary cylinder with radius R_r and pitch $P_E = 2\pi p$ is generated. This helix is needed to obtain the 3D model of the worm.

It is defined the reference system XYZ (the Z axis is overlapped with the \vec{V} axis of the helical surface). With the command “*SWEEP*”, the 3D model of the worm is generated by using the

generating profile and the previously constructed helix. This is the case of clockwise worm with p helical parameter, see figure 3, *a* and *b*.

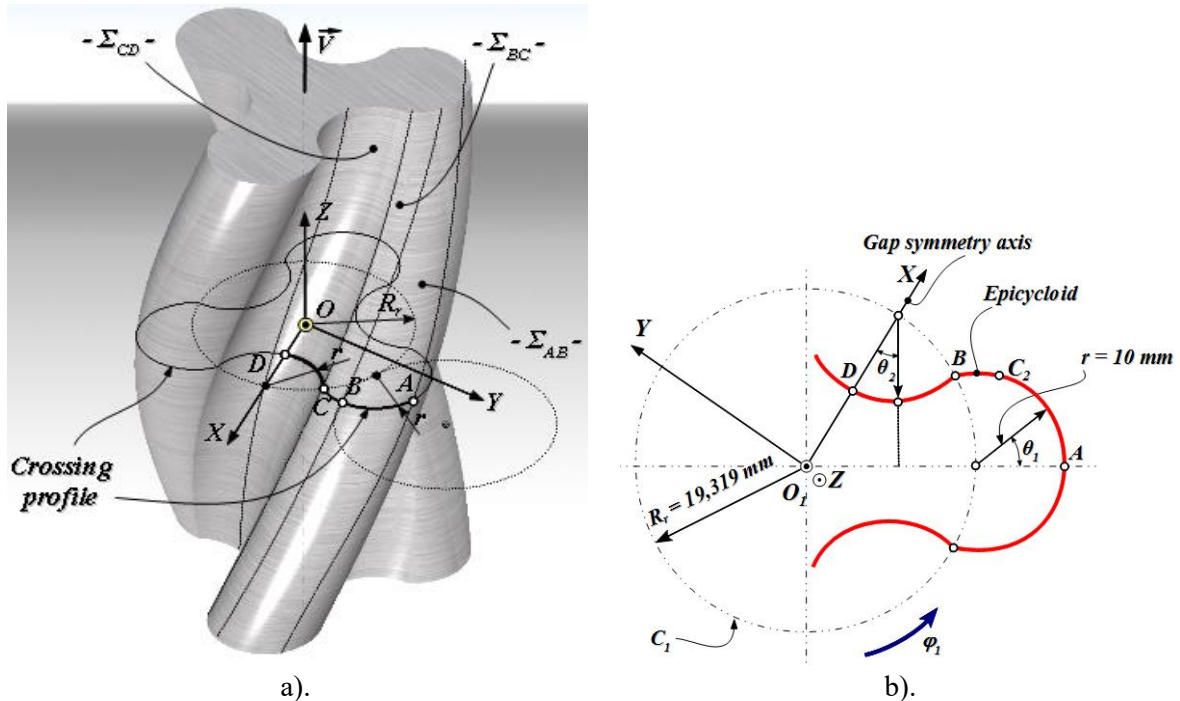


Figure 3. The Roots compressor rotor: solid model of the Roots compressor rotor (a); the frontal profile, rolling centrodes (b); C_1 - centrode associated with the rotor.

It is defined the solid model of the worm, with \vec{V} axis and p helical parameter, linked with the reference system XYZ , see figure 4. Regarding this system they are defined:

- the \vec{A} axis, at distance a (a – input data) onto the common normal of the \vec{A} and \vec{V} axes and inclined with angle α regarding the \vec{V} axis, see figure 4 and relation (10)

$$\alpha = \arctg\left(\frac{p}{R_e}\right) \quad (10)$$

- in planes $Z_2=H$ (see figure 4) perpendicular planes on the axis \vec{A} (Z_2), from the equidistant points is determined the Σ_H curves, as in-planes sections with the 3D model of rotor (using “INTERSECTION” command) – see also figure 1.

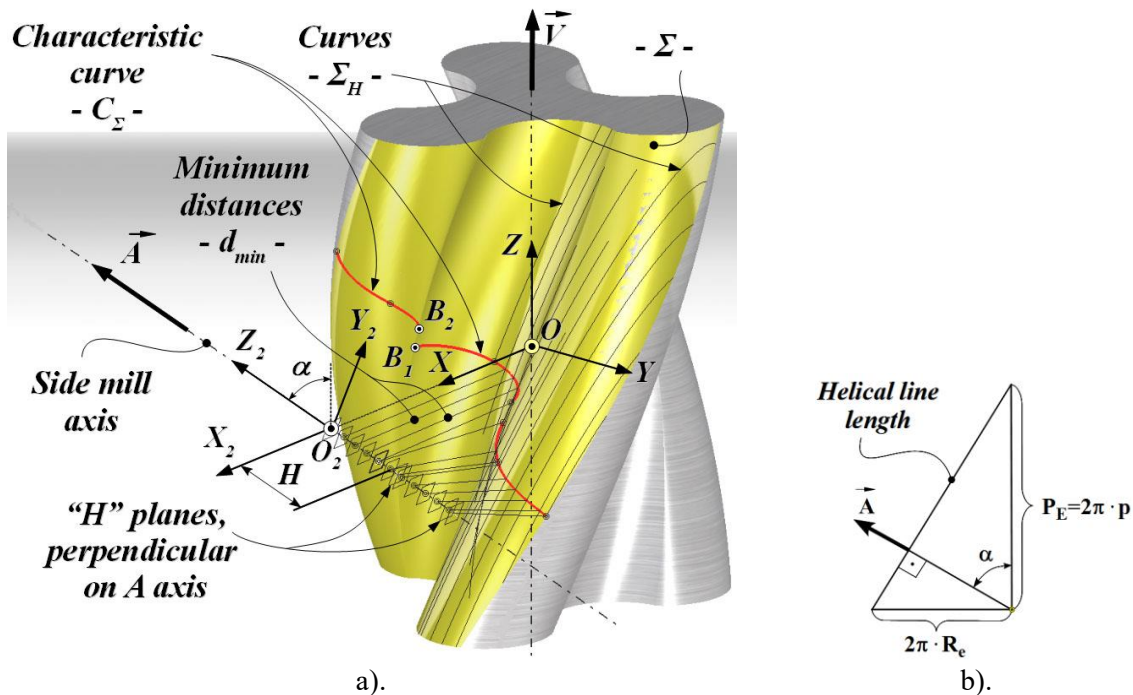


Figure 4. Relative position of \vec{A} and \vec{V} axis; the planes $Z_2 = H$, normal to the \vec{A} axis; profiles Σ_H of the composed helical surface Σ ; minimal distances in planes $Z_2 = H$; discontinuity points B', B'' (a); unfold of the helix with radius R_e and P_E helical pitch (b).

Normals on the Σ_H curves are constructed from the intersection points between the \vec{A} axis (overlapped with Z axis) and the H planes. The command used is “LINE” with option “NORMAL TO CURVE”. The lengths to these normals are the “minimal distances”.

The intersection point of a Σ_H curve with the corresponding normal (the normal foot onto Σ_H) is a point of the characteristic curve C_Σ .

The assembly of these points (see table 1) represents *the characteristic curve* at contact between the Σ (helical surface) with S — the primary peripheral surface of the side mill tool, figure 5.

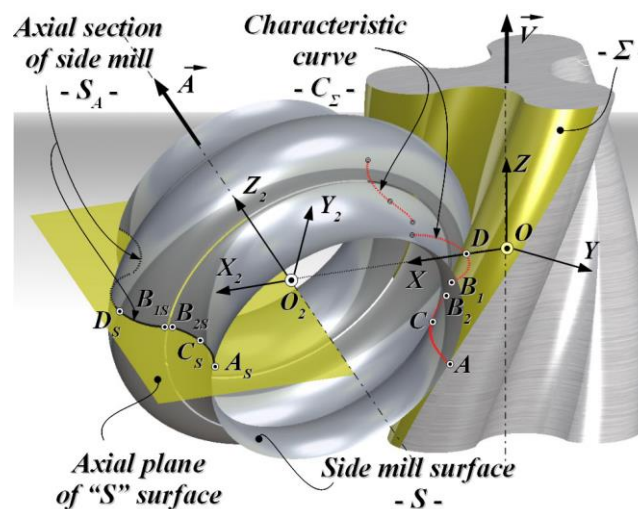


Figure 5. 3D model of the side mill primary peripheral surface – S ; axial section S_A .

Table 1. Characteristic curve on Σ in XYZ reference system (measured coordinates in CATIA).

Zone	X [mm]	Y [mm]	Z [mm]
AC	22.530	18.761	-16.846
	23.106	17.922	-16.012
	23.534	16.965	-15.216

	22.596	10.555	-11.035
	21.912	9.647	-10.370
CB'	21.912	9.647	-10.370
	21.471	9.184	-9.954
	21.006	8.752	-9.528

	18.032	7.391	-6.466
	17.774	7.570	-5.775
B''D	17.246	8.704	-2.683
	15.842	8.344	-2.579
	14.503	7.805	-2.404

	9.439	1.387	-0.418
	9.319	0.000	0.000

Table 2. The coordinates of side mill's axial section (measured coordinates in CATIA).

Zone	R [mm]	H[mm]
A _S C _S	28.7790	-23.7100
	28.0960	-22.6230
	27.5800	-21.4470

	28.0970	-13.9550
	28.7200	-12.8320
C _S B _S '	28.7200	-12.8320
	29.0370	-12.3510
	29.3720	-11.8810

	31.8200	-9.4610
	32.3240	-9.1830
B _S ''D _S	32.7630	-9.0750
	34.1640	-8.7030
	35.5000	-8.1390

	40.5600	-1.4440
	40.6810	0.0000

The characteristic curve shows a discontinuity in the B point, see B' and B'' , due to the fact that in this point the normal to the Σ surface is not unique. So, the B point is a singular point onto the helical surface.

The S_A axial section represents the profile of the “check template” of tool, the required solution for this type of problem.

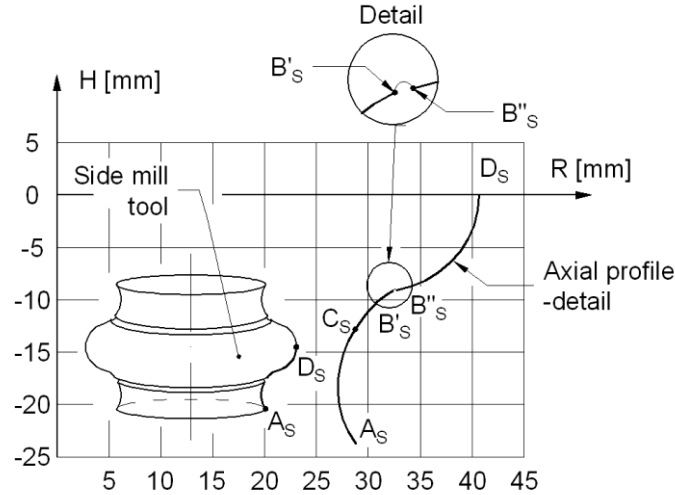


Figure 6. Axial section of side mill.

3.2. Discontinuities of the composed peripheral surface of side mill

The existence of the B singular point onto the composed frontal profile of rotors, leads to discontinuities of the characteristic curve and, as a consequence, discontinuities of the side mill's primary peripheral surface, see figure 6.

The technological solution to this problem is the physical breaking of the tool's profile or a virtual surface which are not involved in the generating process, see figure 6, B' and B'' points.

A technological form of the Roots compressor worms should avoid the singular points on its profile.

4. Method validation

4.1. The composed profile of the three-lobed rotor

The Roots compressor includes two three-lobed rotors, with geometry presented in figure 3b.

The XYZ reference system is defined joined with the C_l centred, with the X axis overlapped to the symmetry axis of the lobe. The C_l centred is a circle with radius R_r .

Is presented an example for the arc:

$$BD \begin{cases} X = R_r \cdot \cos\left(\frac{\pi}{3}\right) - r \cdot \cos\left[\frac{2\pi}{3} - \theta_2\right]; \\ Y = R_r \cdot \sin\left(\frac{\pi}{3}\right) - r \cdot \sin\left[\frac{2\pi}{3} - \theta_2\right]; \end{cases} \quad (11)$$

$$\theta_{\min} = 0; \theta_{\max} = \arccos\left[\frac{r}{2R_r}\right].$$

The helical flank of the worm is described by transformation:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{\omega}_3^T(\Psi) \cdot \begin{pmatrix} X_{BD} \\ Y_{BD} \\ 0 \end{pmatrix} + p \cdot \Psi \cdot \vec{k}, \quad (12)$$

where $\begin{pmatrix} X_{BD} \\ Y_{BD} \\ 0 \end{pmatrix}$ is the matrix formed with the equations (11) of the worm's crossing section.

As result, the helical flank has the equations:

$$\Sigma_{BD} \begin{cases} X = \left[R_r \cos\left(\frac{\pi}{3}\right) - r \cos\left(\frac{2\pi}{3} - \theta_2\right) \right] \cos \Psi - \left[R_r \sin\left(\frac{\pi}{3}\right) - r \sin\left(\frac{2\pi}{3} - \theta_2\right) \right] \sin \Psi; \\ Y = \left[R_r \cos\left(\frac{\pi}{3}\right) - r \cos\left(\frac{2\pi}{3} - \theta_2\right) \right] \sin \Psi + \left[R_r \sin\left(\frac{\pi}{3}\right) - r \sin\left(\frac{2\pi}{3} - \theta_2\right) \right] \cos \Psi; \\ Z = p\Psi. \end{cases} \quad (13)$$

with ψ angular parameter and p – helical parameter.

Similarly, the others two surfaces of the rotor are determined.

4.2. The Nikolaev condition

As a validation criterion of the graphical method we accept that the profiling method of the side mill, an analytical method, based on the fundamental method – the method of helical decomposition – known as the Nikolaev theorem [11], figure 7, see figure 1.

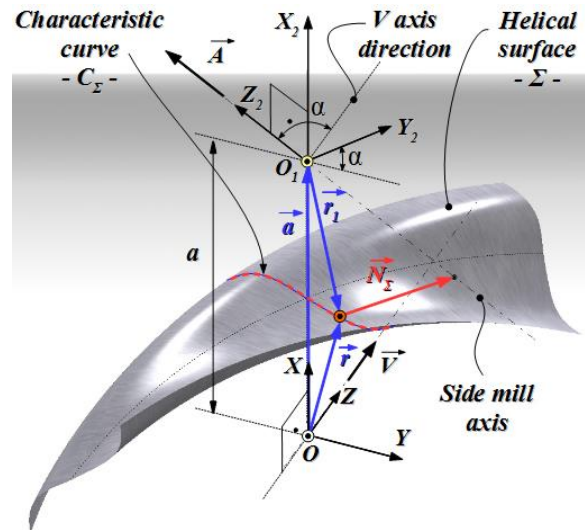


Figure 7. The position of the side mill tool regarding the reference system associated with the helical surface of the rotor lobe for the compressor (the Nikolaev theorem).

The reference system are defined, see figure 7:

The XYZ reference system is associated with the helical flanks of the rotor; the axis Z is overlapped to the \vec{V} axis of the helix.

$X_2Y_2Z_2$ – is the reference system associated with the future side mill, axis \vec{A} , the revolution axis is perpendicularly to the helix with the maximum diameter of the helical surface.

The helix unfolds on the cylinder with radius R_e is presented in figure 4b.

The Nikolaev condition assume the co planarity of vectors

$$(\vec{N}_\Sigma, \vec{A}, \vec{r}_1) = 0, \quad (14)$$

where: \vec{N}_Σ is the normal to the Σ surface of the three-lobed rotor flute;

\vec{A} - the versor of the side mill tool axis;

\vec{r}_1 - the position vector of the point from the helical surface regarding the O_I origin of the $X_2Y_2Z_2$ reference system, see figure 7 and 4 for graphical solution.

For the characteristic curve's coordinates measured on the graphical construction, figure 4 and table 3, the θ_2 and Ψ parameters are identified. Also, on this base is identified the vector

$$\vec{r}_1 = \vec{r} - \vec{a} \quad (15)$$

$$\vec{r} = X \cdot \vec{i} + Y \cdot \vec{j} + Z \cdot \vec{k}, \text{ see (13)} \quad (16)$$

$$\vec{a} = a \cdot \vec{i} \quad (17)$$

with X , Y and Z coordinates from table 3, on the surface Σ_{BD} .

The normal to Σ_{BD} is calculated, for θ_2 and Ψ identified from the coordinates equality from table 3 and with equations (14)-(15) and it is calculated the Nikolaev condition for $\alpha = 67.971^\circ$ and vector form A .

The value of the Nikolaev condition should be zero. In table 3, it is indicated the error of the Nikolaev condition accomplishing, representing the graphical method error regarding the Nikolaev fundamental method at determination of the characteristic curve onto Σ_{BD} surface.

Table 3. Value of Nikolaev theorem in points of characteristic curve.

X [mm]	Y [mm]	Z [mm]	Error
17.246	8.704	-2.683	-0.0010209
15.842	8.344	-2.579	-0.0011718
14.503	7.805	-2.404	-0.0009053
13.259	7.091	-2.174	-0.0009863
12.138	6.209	-1.894	-0.0006438
11.169	5.175	-1.572	-0.0010194
10.381	4.008	-1.213	-0.0009091
9.798	2.734	-0.825	-0.0005352
9.439	1.387	-0.418	-0.0005222
9.319	0.000	0.000	3.119e-023

It is obvious, from table 3, that the error of the Nikolaev theorem accomplishing, for points belong to the characteristic curve is acceptable from the technical point of view, being at level $1 \cdot 10^{-3}$ mm.

In order to make the verification of the results obtained by graphical method, an own design algorithm was applied. The verification was made only for zone DB starting from the assumption that, if the results are correct for this zone, then they will be correct for the other zones as well.

In order to achieve this, the points from the characteristic curve were considered and, in each point, the normal to the helical surface was calculated. Next, in these points the values of the product for the determination of the Nikolaev theorem value (14) were calculated.

The results are presented in table 3.

It is obvious that the error level is small enough to be insignificant from the technical point of view.

5. Conclusions

This paper proposes a new method, expressed in graphical form, in CATIA design environment, for profiling tools bounded by revolution surfaces, which generate helical flutes of with constant pitch.

The method starts from 3D models of the surface to be generated, based on the capabilities of CATIA graphical environment, and determines the characteristic curves at contact between a cylindrical surface with a revolution surface, based on a complementary theorem of the surface enveloping, the method of “minimum distance”. The results are compared with those obtained using an analytical method.

The proposed graphical method is simple, rigorous and very intuitive, constituting a real alternative in this domain for the classical analytical methods.

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