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Calculations of flexibility module in measurements instruments

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Abstract. Piezoelectricity has found a lot of applications since it were discovered in 1880 by Pierre and Jacques Curie. There are many applications of the direct piezoelectric effect - the production of an electric potential when stress is applied to the piezoelectric material, as well as the reverse piezoelectric effect - the production of strain when an electric field is applied. This work presents a mathematical model of a new model of vibration sensor. The principle of operation of currently used sensors is based on the idea: changes in thickness of the piezoelectric plates cause the vibration of the mechanical element, so-called "fork". If the "forks" are not buried by the material deformation of the full tiles broadcasting is transmitted to receiver piezoelectric plate. As a result of vibration of receiver plates the cladding is formed on the potential difference proportional to the force. The value of this voltage is processed by an electronic circuit. In the case of backfilling "forks" the electric signal is lower. At the same time is not generated the potential for cladding tiles. Such construction have a lot of drawbacks, for example: need to use several piezoelectric plates, with the increase in number of components is increased failure of sensors, sensors have now produced two forks resonance, using these sensors in moist materials is often the case that the material remains between the forks and at the same time causes a measurement error. Mentioned disadvantages do not appear in the new proposed sensor design. The Galerkin method of the analysis of considered systems will be presented started from development of the mathematical model, to determine the graphs of flexibility and confirm two methods: exact and approximate. Analyzed beam is a part of the vibration level sensor and the results will be used to identify the electrical parameters of the generator. Designing of technical systems containing piezoelectric transducers is a complex process, due to the phenomena occurring in them. A correct description of the given device in the form of a mathematical model, already in its design phase, is a fundamental condition for its proper functioning. The presented analyzes may be used in the study of any mechanism by piezoelectric sensor, including for the steering column examination.

1. Introduction

Please note that a large group of scientists and engineers are interested in use of smart materials the designed applications. Standard materials, such as aluminum, steel, bronze etc., does not have the necessary features for the modern mechatronic systems. Smart materials are often used in many devices. Changing energy which gives this materials, allows to use when

a) electrical energy is converted to mechanical signal in transducers low-power for example in fuel injection systems in internal combustion engines etc. or in high-power transducers for example in hydraulic valves.

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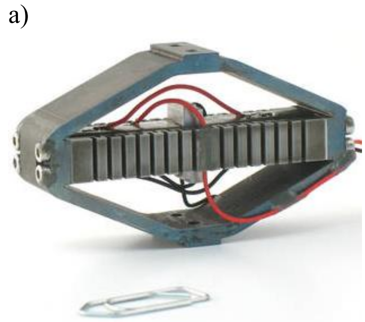
b) mechanical energy is converted to electrical signal in various types of sensors to measure the pressure, alarm sensors and protective

The most common smart materials are used is automation, robotics, nanotechnology, mechanical engineering, micromanipulation, measuring technology, very often they are used for monitoring the structural condition.

a)



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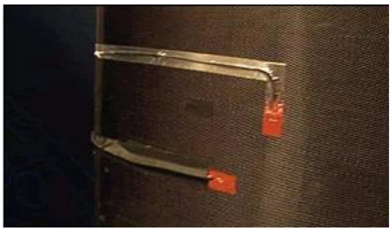
b)



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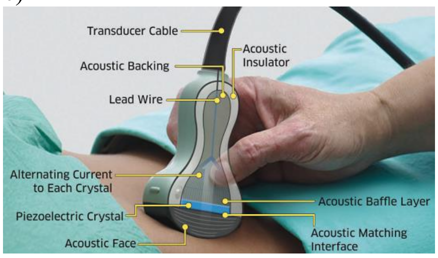

http://www.sensorsmag.com

Figure 1. a) the piezoelectric actuator; b) monitoring the structural condition by the piezoelectric foil glued on surface; c) medical imaging transducer.

Designing of application with consist of intelligent assemblies or subassemblies should not be done only to mechanical system analysis, but it should be taken under consideration also electrical part. The entity should be considered as complex system, which contains independent subsystem.

For this reason important is very precise mathematical model of analyzed piezoelectric plates. This work is an author's idea of calculations of piezoelectric systems. The base of calculation is matrix method and application of graphs method to determination characteristic parameters.

2. Principle and the structure of the piezoelectric level sensor

Piezoelectric level sensor detects the presence of powder when the vibrated sensor element, so-called fork, comes into contact with powder or liquid and the vibrational forks are covered.

Described sensor has both oscillations plates and plate work as sensors, for this reason it use both simple and inverse piezoelectric effect. Output signal is a binary signal, transmitted to the automation systems via a relay.

Figure 2. Computer model of level sensor and photo of piezoelectric bimorph with PZT plate.

Analyzed sensor manufactured by the "Nivomer" company from Gliwice were shown on fig.2. The sensors consist of receiving plates and supplying plates, connected in a bimorphic system.

Voltage, which is connected to the supply plates cause change in their thickness proportionally to the value of applied voltage. Changes of the plate thickness causing mechanical vibrations of the element. When the "forks" are not covered, full deformation of supplying plates are transferred to the receiving ones. The value of this voltage is transformed by an electronic system. In case of covered "forks", the receiving plates are no longer stretched. At the same time the potential is not generated on the facing

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of the plates. In design of level sensors the selection of parameters such as: well-chosen size plate and their number is very important.

3. The mathematical model of PZT plate

The In order to determine the matrix of piezoelectric flexibility, the piezoelectric is modelled as an element described with nine external parameters. Considering the mechanical system continuous in sections, where important parameters are: thickness, area and density of plates. Example of such system was shown on fig. 3. Basis for the analysis of piezoelectric systems are constitutive equations presented in eg.1

$$\begin{cases} \sigma = E\dfrac{\partial u}{\partial x} - \varepsilon E_p, \\ D = \varepsilon^S E_p + \varepsilon\dfrac{\partial u}{\partial x}, \end{cases} \tag{1}$$

where:

E - the modulus of longitudinal elasticity,

E_p- the intensity value of electric field,

ε - deformation,

ε^S - the electric permeability,

D - the electric induction.

After conducting transformations presented in previous work of the authors it was obtained the relations between the mechanical parameters as force and displacement taking into account the voltage generated on the plates:

$$F_1 = Z\left[\frac{u_1}{\tan kd} - \frac{u_2}{\sin kd}\right] + \frac{hi}{\omega}, \tag{2}$$

$$F_2 = Z\left[\frac{u_1}{\tan kd} - \frac{u_2}{\sin kd}\right] + \frac{hi}{\omega}, \tag{3}$$

$$U = \frac{h}{\omega}(u_2 - u_1) + \frac{1}{\omega C_0}i \tag{4}$$

which are also written in a matrix form:

$$\begin{bmatrix} F_1 \\ U \\ F_2 \end{bmatrix} = \begin{bmatrix} \dfrac{Z}{\tan kd} & \dfrac{h}{\omega} & -\dfrac{Z}{\sin kd} \\ \dfrac{h}{\omega} & \dfrac{1}{\omega C_0} & -\dfrac{h}{\omega} \\ \dfrac{Z}{\sin kd} & \dfrac{h}{\omega} & -\dfrac{Z}{\tan kd} \end{bmatrix} \begin{bmatrix} u_1 \\ i \\ u_2 \end{bmatrix}. \tag{5}$$

The matrix (5) was written as respond of system for operating extortion. The individual elements of matrix, presented as flexibility, admittance and characteristics were recorded as follows:

Mechanical dependences force-displacement presented in table 1 are indicated by red circles.

Other dependences between mechanical and electrical parameters are indicated by yellow circles given in the same table.

Electrical parameters, so dependences voltage and the current flowing in the electrical circuit were marked by blue circles in the middle of table 1.

Table 1. Matrix of flexibility, admittance and characteristics of single piezoelectric plates.

	F_1	U	F_2
u_1	$\dfrac{\left(\frac{h^2}{\omega^2}\right)-\left(\frac{\rho VA}{\omega C_0 \tan kd}\right)}{\det Z}$	$\dfrac{\frac{h\rho VA}{\omega}\left(\frac{1}{\tan kd}-\frac{1}{\sin kd}\right)}{\det Z}$	$\dfrac{-\left(\frac{h^2}{\omega^2}\right)+\left(\frac{1}{\omega C_0}\frac{\rho VA}{\sin kd}\right)}{\det Z}$
i	$\dfrac{\frac{h\rho VA}{\omega}\left(\frac{1}{\tan kd}-\frac{1}{\sin kd}\right)}{\det Z}$	$\dfrac{-\left(\frac{\rho VA}{\tan kd}\right)^2+\left(\frac{\rho VA}{\sin kd}\right)^2}{\det Z}$	$\dfrac{\frac{h\rho VA}{\omega}\left(\frac{1}{\tan kd}-\frac{1}{\sin kd}\right)}{\det Z}$
u_2	$\dfrac{\left(\frac{h^2}{\omega^2}\right)-\left(\frac{1}{\omega C_0}\frac{\rho VA}{\sin kd}\right)}{\det Z}$	$\dfrac{-\frac{h\rho VA}{\omega}\left(\frac{1}{\tan kd}+\frac{1}{\sin kd}\right)}{\det Z}$	$\dfrac{-\left(\frac{h^2}{\omega^2}\right)+\left(\frac{\rho VA}{\omega C_0 \tan kd}\right)}{\det Z}$

4. Bimorph system

According to the marks presented in [8] it was assigned values from table 1 to the edges of the graph presented in fig. 3. where ${}_1x_1^{(i)'}$, ${}_1x_2^{(i)'}$ are the system answers for excitations ${}_2x_1^{(i)'}, {}_2x_2^{(i)'}$, as shown in Figure 3.

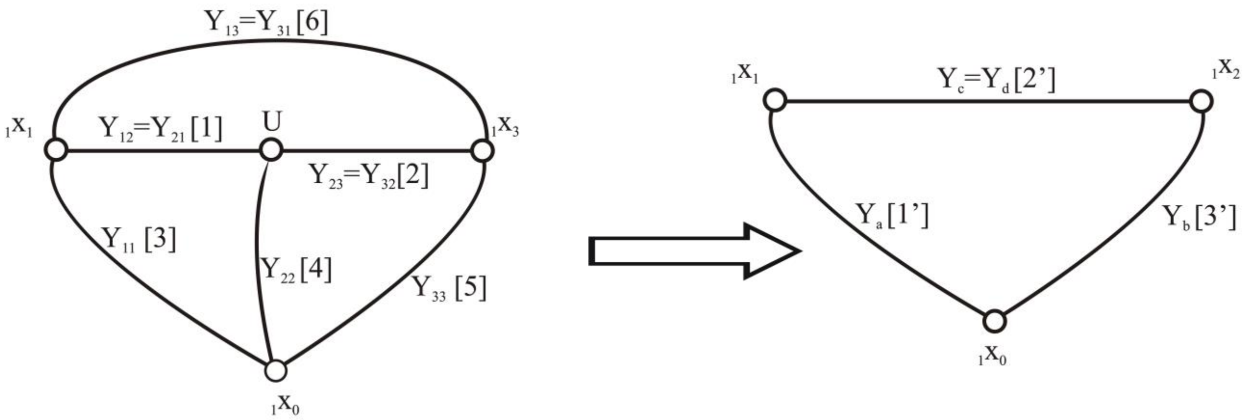

Figure 3. Assigned values from table 1 to the edges of the graph.

Usage of a structured 4-vertex graph in a detailed analysis of piezoelectric can be time-consuming activity. Therefore, modelling was carried out by a replacement graph.
In order to use the well-known graph method [6, 7] was converted into a three vertex graph in the graph with two vertices according to formulas:

$$Y_b = Y_{3'} = \frac{\det\limits_Z (A_{1'} \cap A_{2'})}{\det\limits_Z A_{1'2'}} = \frac{Y_4(Y_2+Y_5)+Y_5(Y_1+Y_3)}{Y_1+Y_2+Y_4}. \tag{6}$$

$$Y_c = -Y_d = Y_{2'} = \frac{\det\limits_Z (A_{1'} \cap A_{3'})}{\det\limits_Z A_{1'3'}} = \frac{Y_1(Y_2+Y_6)+Y_6(Y_2+Y_4)}{Y_1+Y_2+Y_4}. \tag{7}$$

$$Y_a = Y_{1'} = \frac{\det\limits_Z (A_{2'} \cap A_{3'})}{\det\limits_Z A_{2'3'}} = \frac{Y_1(Y_3+Y_4)+Y_3(Y_2+Y_4)}{Y_1+Y_2+Y_4}. \tag{8}$$

As already mentioned in this article, piezoelectric plate located in the level sensor creates so-called bimorph structure. This is nothing else as a few plates are connected together in a manner as shown in fig. 3 or one by one. So three vertex reduced graph (fig.3) can be represented as shown on fig.4, and in this form it is possible to apply aggregation method to analysed bimorph system.

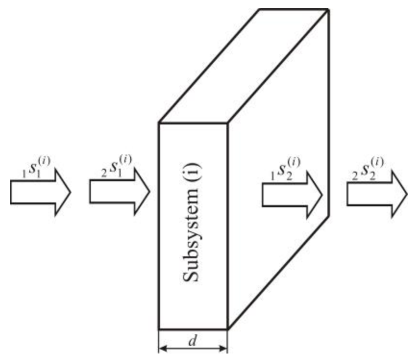

Figure 4. Model of replacement piezoelectric plate.

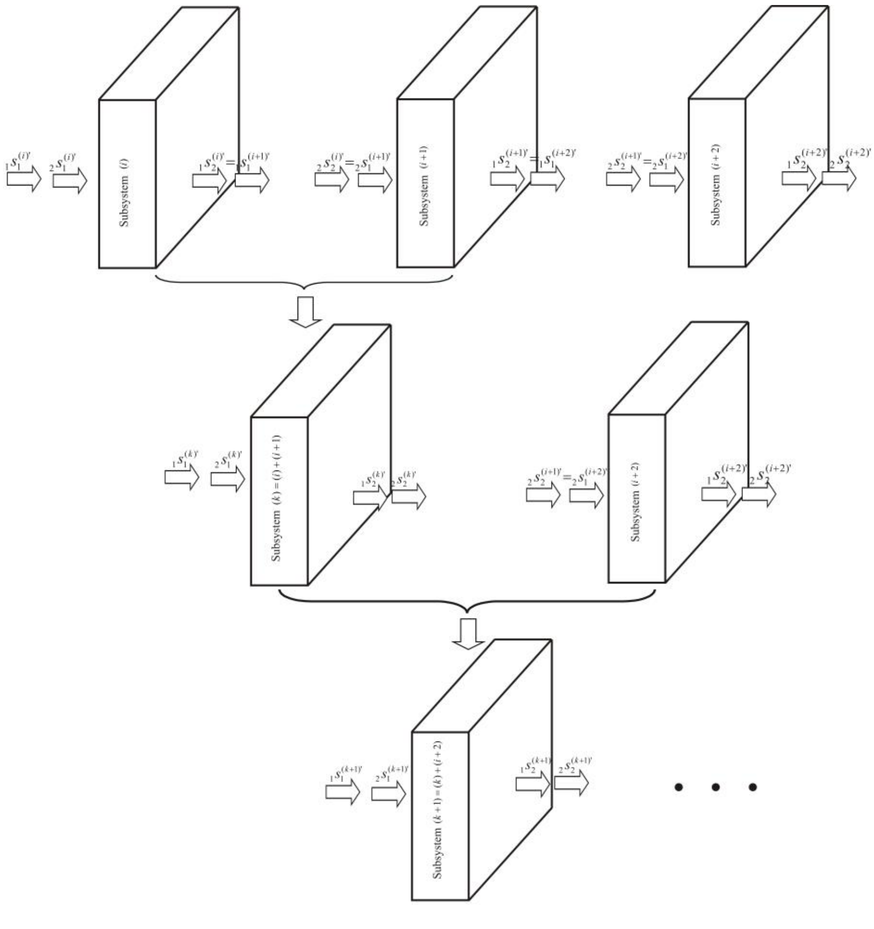

Figure 5. Diagram of plates connection into a bimorph system.

Subsequently the matter under consideration is the application of the transition matrix in the modelling of individual plates and complex piezoelectric systems. Figure 4 presents the model of piezoelectric plate with parameters distributed continuously. The left and the right end of plate is free. Such model, with replacement graph, was shown in Figure 4 and marked with the symbol (*i*). It was assumed that the index prim ${}_1s_1^{(i)'}$, ${}_1s_2^{(i)'}$, ${}_2s_1^{(i)'}$, ${}_2s_2^{(i)'}$ reduced graph, where: ${}_1s_1^{(i)'}$, ${}_1s_2^{(i)'}$ are the system answers for excitations ${}_2s_1^{(i)'}, {}_2s_2^{(i)'}$, as shown in Figure 3. The longitudinal vibrations of piezoelectric plates are considerate, so the changes its thickness due to the applied voltage or power. The relations between values ${}_1s_1^{(i)'}$, ${}_1s_2^{(i)}$ and ${}_2s_1^{(i)'}$, ${}_2s_2^{(i)}$ were presented as follows:

$$ {}_1S^{(i)'} = Y^{(i)}\ {}_2S^{(i)'}, \tag{9}$$

Method of joining successive piezoelectric plate is shown in Figure 5.

According to the formula (10) was derived which describing the flexibility at the end of the last plate of the complex system.

$$Y_b^{(k+1)} = \frac{Y_b^{(i+2)}\left(\frac{Y_b^{(i)}\ Y_b^{(i+1)}}{Y_d^{(i)}\ Y_d^{(i+1)}} + \frac{Y_a^{(i+1)}\ Y_b^{(i+1)} + Y_c^{(i+1)}\ Y_d^{(i+1)}}{Y_d^{(i)}\ Y_d^{(i+1)}}\right) - \left(\frac{Y_a^{(i+1)} - Y_b^{(i)}}{Y_d^{(i)}\ Y_d^{(i+1)}}\right)\left(Y_a^{(i+2)}\ Y_b^{(i+2)} + Y_c^{(i+2)}\ Y_d^{(i+2)}\right)}{Y_b^{(i+2)}\left(\frac{Y_a^{(i+1)} - Y_b^{(i)}}{Y_d^{(i)}\ Y_d^{(i+1)}}\right) + \left(\frac{Y_a^{(i+1)}\ Y_b^{(i+1)} + Y_c^{(i+1)}\ Y_d^{(i+1)}}{Y_d^{(i)}\ Y_d^{(i+1)}}\right)}. \tag{10}$$

The components of equation (10), such as: Y_a, Y_b, Y_c, Y_d were derived in the previous work of the author.

5. Influence of piezoelectric plates number in stack on characteristics of mechatronic system.

In this paragraph, graphical charts of characteristics of piezoelectric plates were shown. The base for generating charts was equation (10).

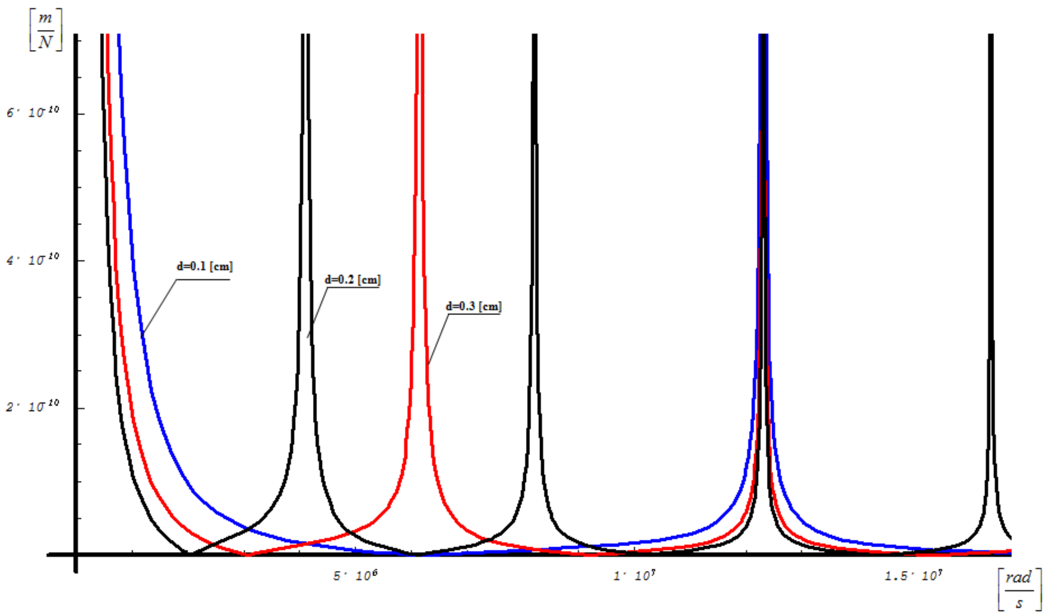

Figure 6. Characteristics of piezoelectric plate in domain of the frequency depending on the thickness of the plate.

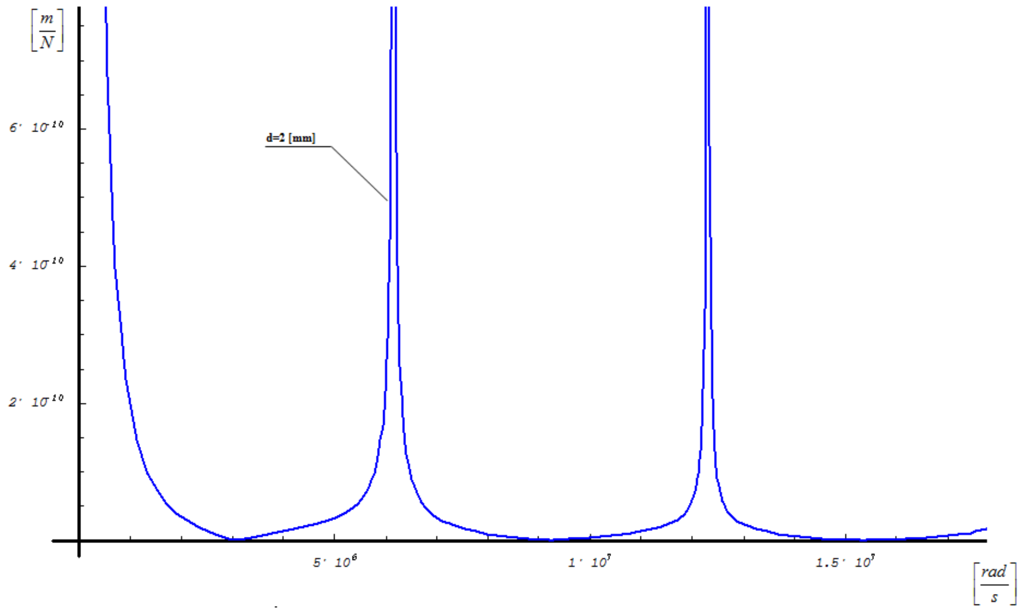

Figure 7. Characteristics of piezoelectric stack with two plates with thickness $d = 1\ [mm]$

It was noted that the characteristics of a single plate with thickness $d = 1\ [mm]$ are the same as two connected piezoelectric plates with thickness $d = 0.5\ [mm]$ each. This statement is correct only by assuming ideal connection of plates without for example taking the glue thickness layer into examination.

6. Acknowledgements

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