Practical example of game theory application for production route selection

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Abstract. The opportunity which opens before manufacturers on the dynamic market, especially before those from the sector of the small and medium-sized enterprises, is associated with the use of the virtual organizations concept. The planning stage of such organizations could be based on supporting decision-making tasks using the tools and formalisms taken from the game theory. In the paper the model of the virtual manufacturing network, along with the practical example of decision-making situation as two person game and the decision strategies with an analysis of calculation results are presented.

1. Introduction

The dynamics of market changes makes looking for solutions that would support the decision-making process concerning the production planning processes [1]. The planning process is even more complicated if it concerns the functioning of several organizations at the same time and for one purpose, to produce a product with specific properties. The opportunity which opens before manufacturers, especially before those from the sector of the small and medium-sized enterprises, is associated with the use of the virtual organizations concept [2, 3, 4, 5, 6]. The virtual organization is created on the basis of a virtual network of manufacturing. The planning stage of such processes could be based on supporting decision-making tasks using the tools and formalisms taken from the game theory. The formalism of this theory could be mainly applied to precisely describe the conditions and rules of the decisional situation. In the analysed practical example, the virtual network of manufacturing co-operators is created on the base of three producers. This simplification is related with the aspiration to precisely analyse the problems considered with introducing the formalism of the game theory. Each of the co-operators is responsible for their share of production order. In this production system are realized two production processes which are further divided into two sub-variants. In the paper the model of this virtual manufacturing network, along with the flow of processes, is shown.

The decision-making situation is defined as two person game. The values of object connected with payoff function, which in proposed approach is determined as a player, according to manufacturing and transportation costs are calculated (for the first player - to minimize production cost, and for the second - to minimize transportation cost). The possible decision strategy to use together with an analysis of calculation results is also presented in the paper.

The paper is organized as follows. Section 2 is devoted to presenting the Virtual Manufacturing Network concept. Section 3 is devoted to basis of theory game in production planning problem. The practical example of production game is presented in Section 4. Finally, Section 5 presents our conclusions.

2. Virtual Manufacturing Network

Virtual manufacturing network is a linkage of several production companies, which are geographically dispersed. They temporary cooperate like one body to comply with the production order. Each of the companies spare production capacity that could be utilized for the purposes of that order. The coordinator, who is responsible for the operation of the virtual network, chooses the production capacity of all members of the network in such a way that the production was made in required time [4,5,6,7]. An example of a virtual network is shown in fig. 1.

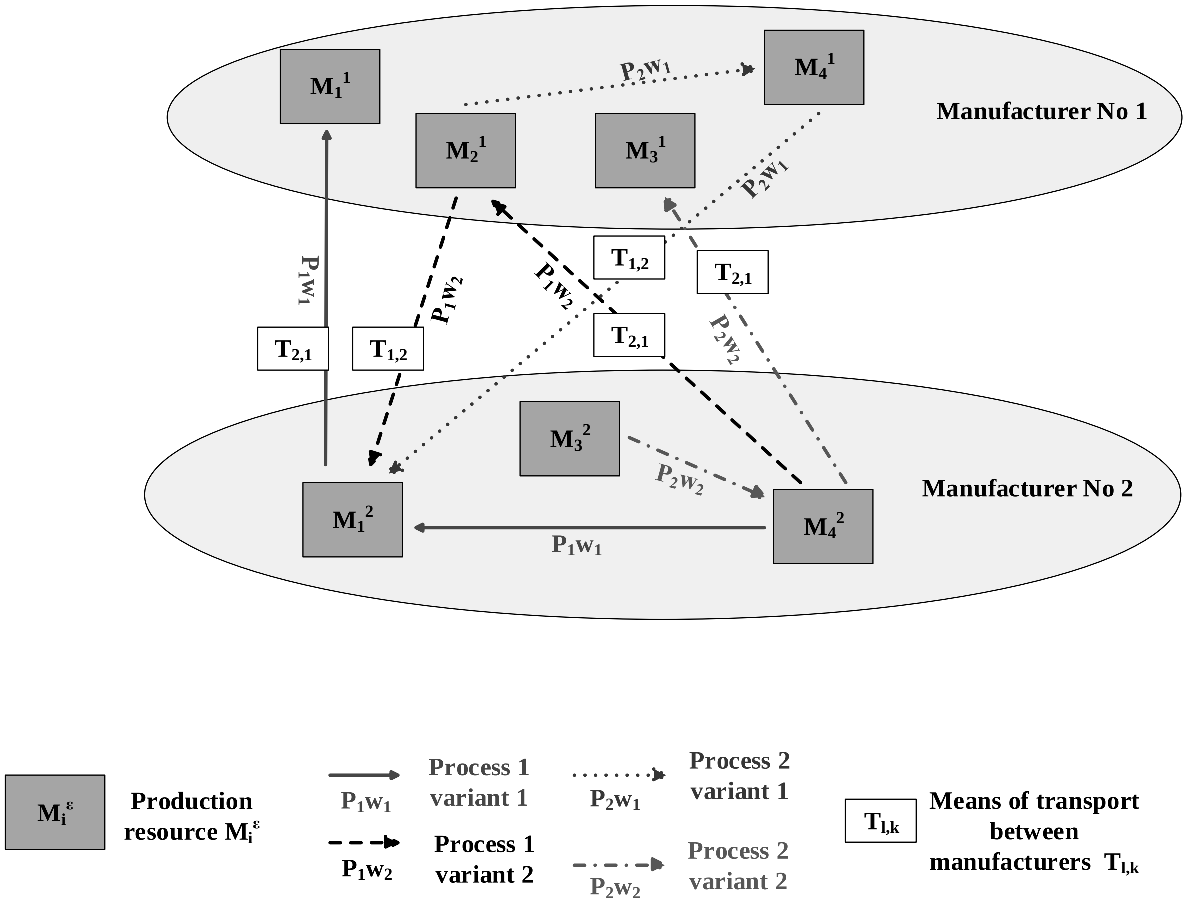

Figure 1. Example of virtual manufacturing network.

The exemplar virtual network was created by two manufacturers. The cooperation in this area would not be possible, if the information technology would not be properly developed. Nowadays there are a lot of methods to contact between manufacturers, for example through e-mails, contact platforms, teleconferencing. They all allow undertakings so specific cooperation even if cooperating manufacturers are quite distant.

The coordinator evaluates the capabilities of each participant in the network and developing feasible production route. If only there is a configuration of production resources of manufacturers that fulfills requirements, the coordinator chooses the solution corresponding to the assumed criteria and requirements. If there is a situation, on which coordinator can't choose a solution based on available possibilities of the chosen manufacturers then he introduces changes to the cooperation net. For example he can add new manufacturer or replace someone who exist yet, to look for new opportunities and relationships between manufacturers and generates new solutions [8, 9, 10]

In this paper, the decision-making problem is the choice a route in the production planning process, based on the implementation of formalisms taken from the game theory. The formalisms are used in the decision-making situation, where conflict exists.

3. The key concepts of game theory in production planning problem

Formalisms derived from the game theory allow to describe the decision problem by means of decision-making components: the players, strategies, players' actions and the payoffs. Defining the decision problem discussed in the paper, regarding the route selection from a set of alternative routes, in the form of production game, it was necessary to adapt the definitions of the basic game theory concepts to this type of problem.

It was decided to adopt the criteria for evaluating solutions corresponding to the concept of players. Each of them has actions available in the set of strategies related to the set of route variants for the planned production processes. Objects representing goal functions correspond to the concepts of the players. Function of the objective, in turn, are related to production costs and transport costs between manufacturers in the network. Each of them strives to minimize their costs [11, 12, 13]. A summary of the key concepts describing the decision situation using game theory is presented in table 1.

Table 1. The key concepts of game theory for production game.

concept	traditional meaning	production game
Game	conflict or cooperation between decision-makers; wide range of behavioural relations	selecting production route from the set of available alternative routes for production flow planning
Players	groups of people, consumers, producers, decision makers, etc.	an agents/objects representing functions of the objective (related to production costs and transport costs)
Strategies	set of all possible actions, that define the behaviour of a players	set of routes' variants of production processes possible to use in planning production flow
Players' actions	choice available to the players depending on the circumstances	choice of routes selection from the set of permissible routes
Solution concept	set of effects for making earlier decision by all players	the costs of production and transport calculated for the chosen routes' variant
Payoffs for players	a value received from arriving at a particular outcome to a player	the value of the objective function for a set of strategies

Considering a non-zero sum game in which participants make rational decisions through which they maximize their payoffs, a method of searching for dominated strategies is used In this class of games, dominated strategies for players should be sought for the corresponding values of the payoff matrix - for one of the players it is looking for dominated row, for the second one- column. Knowing the payout values, the player is able to determine which strategy will dominate the other (provided that there is one) and follow the solution in the game (discard dominated strategies). The following example assumes that there is a set of strategies in the game from which the players will choose a solution concept.

4. Example of production game

The virtual manufacturing network which consists of two producers is being considered. In the production order there are two production processes P_1 and P_2. For each production process there are two different versions of the alternative routes. Process flow data is written in MP_j^w matrices (respectively *j-th* process and *w-th* variant):

$$\mathrm{MP}_1^1 = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 3 & 7 \end{bmatrix} \mathrm{MP}_1^2 = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 1 \\ 4 & 5 & 3 \end{bmatrix} \begin{matrix} \text{manufacturer } No \text{ in network} \\ \text{production resource } No \text{ for } h^{-th} \text{ operation} \\ \text{processing times for each operation} \end{matrix}$$

$$\mathrm{MP}_2^1 = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 3 \end{bmatrix} \mathrm{MP}_2^2 = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 4 & 3 \\ 4 & 4 & 5 \end{bmatrix} \begin{matrix} \text{manufacturer } No \text{ in network} \\ \text{production resource } No \text{ for } h^{-th} \text{ operation} \\ \text{processing times for each operation} \end{matrix}$$

The unit production cost (C^P) and the unit costs of transport (C^T):

$$C^P = [c^P_{\varepsilon,k}] = \begin{matrix} & 1 & \cdots & K \\ 1 & C_{1,1} & \cdots & C_{1,K} \\ \vdots & \vdots & \cdots & \vdots \\ R & C_{R,1} & \cdots & C_{R,K} \end{matrix} = \begin{bmatrix} 6 & 4 & 3 & 5 \\ 3 & 0 & 2 & 2 \end{bmatrix} \qquad C^T = [C^T_{\varepsilon,\varepsilon}] = \begin{matrix} & 1 & \cdots & R \\ 1 & C_{1,1} & \cdots & C_{1,R} \\ \vdots & \vdots & \cdots & \vdots \\ R & C_{R,1} & \cdots & C_{R,R} \end{matrix} = \begin{bmatrix} 0 & 118 \\ 118 & 0 \end{bmatrix}$$

where:
$K = max\ (m_\varepsilon)$ – maximum number of resources available in the network members,
R – the number of manufacturer in the network,
$C^P_{\varepsilon,k}$ - the unit cost of technological operation execution on k^{th} resource belonging to the ε^{th} manufacturer,
$C^T_{\varepsilon,\varepsilon}$ - the unit cost of transportation between manufacturers belonging to the network.

The batch size for P_1 is 120 parts and for P_2 is 180 parts.

The decision problem of production flow planning regarding the route selection from a set of alternative routes, using the formalisms derived from the game theory is defined for following decision-making components:

- players agents/objects representing functions of the objective[11, 12]:
    - First player – minimize of production costs:

$$u_1(s) = \sum_{j=1}^{n} \sum_{h=1}^{H_j^{S\alpha[j]}} \left(c^P{}_{(mp_{1h}),(mp_{2h})} \cdot mp_{3h} \cdot N_j \right)$$

    - Second player – minimize of transport costs:

$$u_2(s) = \sum_{j=1}^{n} \sum_{h=1}^{\left(H_j^{S\alpha[j]}-1\right)} \left(c^T{}_{(mp_{1h}),(mp_{1(h+1)})} \cdot N_j \right)$$

where:
s – set of strategies, corresponding to the execution of selected routes versions $s = \{s_1, s_2, \ldots s_\alpha \ldots s_A\}$,

$A = \prod_{j=1}^{n} v_j$ – number of strategies for players,
v_j – number of routes variants for *j-th* production process,
H_j^w - number of technologies operations *w-th* routes variants for *j-th* production process,
$s_\alpha = (\sigma_\alpha^1, \sigma_\alpha^2, \ldots, \sigma_\alpha^n)$,
σ_α^j – number of variant for *j-th* production process for $\alpha - th$ strategy,
$s_\alpha[j] = \sigma_\alpha^j$,
mp_{1h} – manufacturer No,
mp_{2h} – production resource No for h-th technological operation,
mp_{3h} – processing times for each operation,
N_j – batch size.

Correspondingly, for the first and second player - the amount of costs generated depending on the route variant of the production processes are given in table 2 and 3.

Table 2. Production costs.

Variants of route	Cost
for MP_1^1, N_1=120	7080
for MP_1^2, N_1=120	4440
for MP_2^1, N_2=180	5760
for MP_2^2, N_2=180	5580

Table 3. Transport costs.

Variants of routes	Costs
for MP_1^1, N_1=120	14160
for MP_1^2, N_1=120	28320
for MP_2^1, N_2=180	42480
for MP_2^2, N_2=180	21240

- Strategies (players' actions):

Players have a set of strategies that are sets of corresponding route numbers for production processes:

$$s = \{s_1, s_2, s_3, s_4\}$$

where:
$s_1 = (1,1)$, $s_2 = (1,2)$,
$s_3 = (2,1)$, $s_4 = (2,2)$.

For example for strategy s_1: the first variant of the first production process, and the first variant of the second production process.

- Payoffs for the players:

Payoffs for players correspond to the cost functions of both players associated with the player strategies:

$$u(g) = \{u_1(s), u_2(s)\}$$

$$u_1(s_1) = 12840 \qquad u_2(s_1) = 56640$$
$$u_1(s_2) = 12660 \qquad u_2(s_2) = 35400$$
$$u_1(s_3) = 10200 \qquad u_3(s_3) = 70800$$
$$u_1(s_4) = 10020 \qquad u_4(s_4) = 49560$$

For the calculated payoffs function values, a payoffs matrix for players has been created (table 4).

Table 4. Payoffs matrix for the players.

		Player G_2			
		s_1	s_2	s_3	s_4
Player G_1	s_1	12840 / 56640	12840 / 35400	12840 / 70800	12840 / 49560
	s_2	12660 / 56640	12660 / 35400	12660 / 70800	12660 / 49560
	s_3	10200 / 5664	10200 / 35400	10200 / 70800	10200 / 49560
	s_4	10020 / 56640	10020 / 35400	10020 / 70800	10020 / 49560

For the first player the strategies s_1, s_2, s_3, respectively, are dominated by the s_4 strategy. For the second player, strategies s_1, s_3 and s_4 are dominated by s_2 strategies. The solution concept of the game in the form of equilibrium points are the following pairs of strategies: s_1/s_1, s_2/s_3, s_3/s_4 and s_4/s_2. Rejecting strategies that not allowed, the only solution is a pair of routes: s_1/s_1.

5. Summary

Considered in this paper decision problem concerns the route selection method from the allowed route set in virtual manufacturing network. The formalism of the game theory was used to describe the decision situation. The basic rule for considered decision problem is a limitation in the final choice of strategy for defined game production solution. Therefore, that for each strategy the specific sets of production route variants for all processes are assigned, players must seek solutions for the same set of strategies. Consequently, the solution of the game should result in a single set of routing variant. The subject of further research will be related to study of the potential for more widespread application of the presented methodology in the operational level of production management, especially for virtual organizations.

6. References

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