

Computational analysis of nonlinearities within dynamics of cable-based driving systems

G D Anghelache¹ and S Nastac¹

¹”Dunarea de Jos” University of Galati, Engineering and Agronomy Faculty in Braila, Research Center for Mechanics of Machines and Technological Equipments, Calea Calarasilor 29, 810017, Braila, Romania

E-mail: danchelache@ugal.ro

Abstract. This paper deals with computational nonlinear dynamics of mechanical systems containing some flexural parts within the actuating scheme, and, especially, the situations of the cable-based driving systems were treated. It was supposed both functional nonlinearities and the real characteristic of the power supply, in order to obtain a realistically computer simulation model being able to provide very feasible results regarding the system dynamics. It was taken into account the transitory and stable regimes during a regular exploitation cycle. The authors present a particular case of a lift system, supposed to be representatively for the objective of this study. The simulations were made based on the values of the essential parameters acquired from the experimental tests and/or the regular practice in the field. The results analysis and the final discussions reveal the correlated dynamic aspects within the mechanical parts, the driving system, and the power supply, whole of these supplying potential sources of particular resonances, within some transitory phases of the working cycle, and which can affect structural and functional dynamics. In addition, it was underlines the influences of computational hypotheses on the both quantitative and qualitative behaviour of the system. Obviously, the most significant consequence of this theoretical and computational research consist by developing an unitary and feasible model, useful to dignify the nonlinear dynamic effects into the systems with cable-based driving scheme, and hereby to help an optimization of the exploitation regime including a dynamics control measures.

1. Introduction

Cables are flexible load-carrying or force transmitting tension parts, widely used within many mechanical applications. Cable-actuated systems are more dynamically responsive than rigid systems. In addition, the cable, as flexural machine part, provides better maximum payload to weight ratio and mobility compared to the rigid elements. Taking into account their highly flexibility and the dynamic exploitation regimes that characterize most of technical applications within technological machines and equipments, the necessity and opportunity of an evaluation regarding the dynamics of hoist cables obviously results. Hereby, the available literature contains various works dealing with this area, presenting both analytical/computational approaches with linear/nonlinear hypotheses, and experimental-based analyses that shown case studies results and/or computational models validation.

Benosman [1] evaluates the ropes sway dynamics in order to control the elevator systems choosing to actuate the system with a force actuator pulling on the compensation sheave. Urbano et al. [2] present different approaches useful in computational analyses of cables dynamics for weight-lifting machines. Starting from the assumption that linear spring model of cable, although it is very simple



and efficient, provides some energetically inconsistencies and spurious terms in the motion equations, the authors propose a semi-analytical method and introduce an analytical model of the cable–pulley interaction. Andrew and Kaczmarczyk [3] provide a systematic overview about the problems regarding how transient vibrations arise, why rope vibrations often occur at particular positions in the hoistway, why lateral rope vibrations may give rise to longitudinal vibration, why transient rope vibration may be very difficult to eliminate, underlining the idea that the major contribution to solve the problem of vibration mitigation within suspension ropes arises from the rope characteristics.

In the paper [4] Tarek presents some analytical investigations regarding both the static and dynamic analyses of cables under general loading, taking into account the nonlinearities and the external disturbing forces to the cables. A computational scheme for determining the dynamic stiffness coefficients of a linear, inclined, translating and viscously or hysteretically damped cable element is presented by Sarkar [5]. In this paper is also considered the coupling between inplane transverse and longitudinal modes of cable vibration. A consistent review regarding cable dynamics is provided by Starossek [6], finally providing a well justified discussion of dynamic-excitation mechanisms and also of dynamic interactions between cables and other structural elements.

Ignacio et al. [7] provides a method to simulate hoisting cables behaviour during their real time evolution. The authors propose a two-layered model composed by a model for the dynamics of a cable passing through a set of pulleys and, respectively, by an oscillation model based on the classical one-dimensional wave equation. In theirs paper, Wang et al. [8] investigate the lateral vibration of the hoisting bucket within the cable-guided hoisting system with time-varying length, in order to supply an algorithm for transitory dynamics control applicable on these kinds of systems. The work of Arrasate et al. [9] contains the results of a study to investigate the vertical vibration caused by torque ripple generated at the drive system and transmitted through the suspension ropes to the car.

Based on the real observation that long span cables are often used as main load-carrying parts in modern structures, Demšić and Raduka [10] proposed an analytical cable oscillation model for inhomogeneous boundary conditions taking into account quadratic and cubic nonlinearities of the system. Using the Galerkin method, the mathematical model of cable oscillations is reduced to a finite-degree-of-freedom model, and the asymptotic solution of the one-degree-of-freedom system is obtained using the Method of Multiple Scales (MMS) technique. It also provides analytic expressions for the resonance regions and amplitude values.

Within their thesis Mohammadshahi [11], presents some assessments regarding cable dynamics and cable internal damping coefficients. Some interesting aspects about the new trends of modern materials utilization in civil engineering applications were provided in the work [12], wherein, a comparison between the classical steel cables and the carbon fiber-reinforced plastic (CFRP) cables had been developed and presented by the authors. Taking into account that in most cases, the hoisting cable in the cable-guided hoisting system is connected to the hoisting bucket with the swivel, in the paper [13] was investigated the coupled longitudinal – torsional responses of the hoisting cable with time-varying. It has to be mentioned that the proposed model could also be used to describe the coupled vibration in the rigid rail – guided hoisting system but needs more modes than for the presented case.

Kaczmarczyk and Iwankiewicz [14] underline that one cause for elevator car vibrations was supplied by the irregularities in the guide system. In this paper, the non-stationary equations for the second-order statistical moments are formulated, and, based on case study, is shown that the weaker the correlation of the rail excitation process the higher the variance of the dynamic displacement of the car.

This study presents some theoretical aspects leading to a computational model definition for a cable–actuating system, intended for analysis of dynamic behaviour during operation. A correlative analysis between essential functional parameters was firstly provided. A dynamic evaluation of the entire system during a regular exploitation cycle was done, taking into account the nonlinear behaviour of some component parts. Finally, a spectral analysis were performed in order to dignify the

potentially correlations between the lateral (in-plane or out-of-plane) and longitudinal oscillations, and the vibration due to driving system.

2. Analysis of functional dependency between main functional parameters

Taking into account the basic assumptions regarding the cable-based load-carrying equipments available within works [15,16], the authors has considered a applicable model according with the schematization in figure 1.

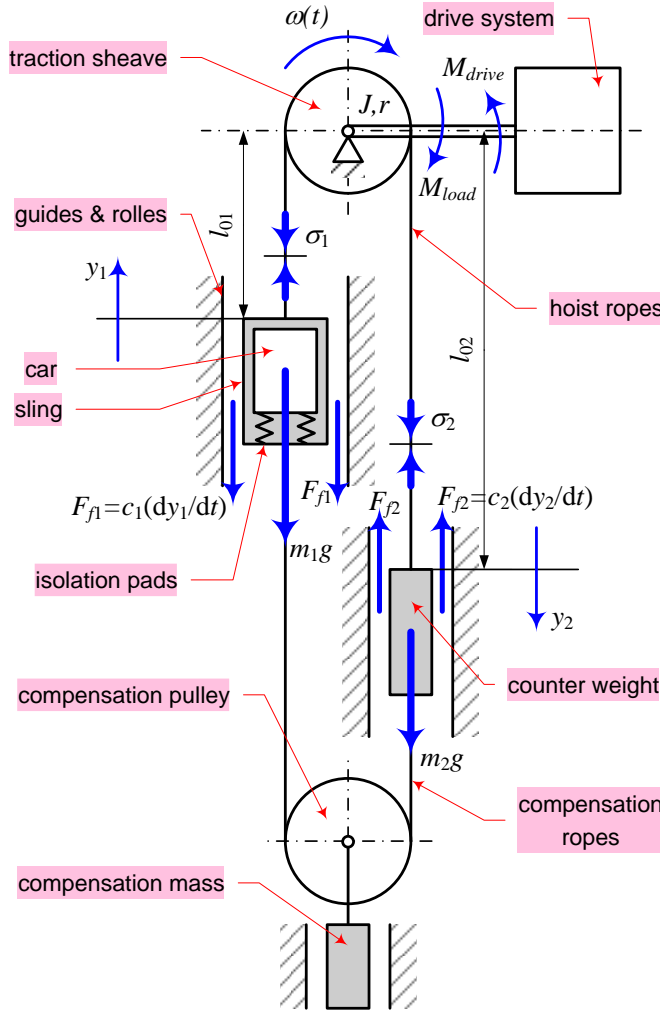


Figure 1. Schematic diagram of the model for a cable-actuating load-carrying system. Main structural parts are mentioned on diagram.

An important feature in analyzing the behaviour during operation of a cable-actuating system is given by correlation between main functional parameters – such as *load capacity*, *car weight*, and system parameters – such as *friction coefficient* of hoist cable on driving sheave, as well as cable winding angle on driving sheave. Based on schematization in figure 1, results the inequation

$$\begin{cases} G_c + Q \leq G_{cw} e^{\mu\alpha} \\ G_{cw} \leq G_c e^{\mu\alpha} \end{cases} \Rightarrow G_c \geq \frac{Q}{e^{2\mu\alpha} - 1} \quad (1)$$

where G_c denotes the car weight, G_{cw} means the counter weight, Q denotes the maximum load capacity, and the term $\mu\alpha$ provides the friction coefficient between hoist cable and traction sheave multiplied by the cable winding angle. Taking into account the four variables expression (1), the

analysis of evolution of one parameter depending on the other two can be assessed considering a constant value (conveniently chosen from current practice) for the fourth parameter.

Thus, figure 2 shows the evolution of car weight depending on load capacity and friction coefficient, considering the winding angle of 180° . A pronounced increase of value needed for car weight is noticed for high loading capacities and low values of the friction coefficient (approximate three times loading capacity for friction coefficients below 0.1 values).

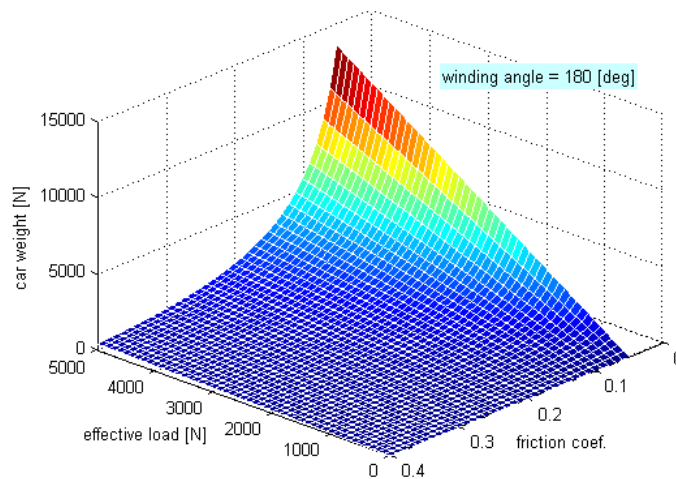


Figure 2. Car weight evolution evolution depending on load capacity and friction coefficient respectively (for a winding angle of 180°).

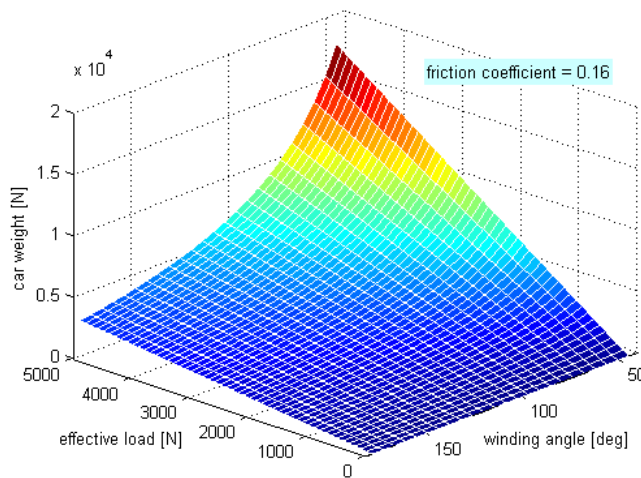


Figure 3. Car weight evolution depending on loading capacity and winding angle respectively (for a friction coefficient of 0.16).

Figure 3 shows the evolution of car weight depending on loading capacity and respectively winding angle, the friction coefficient having constant value (equal to 0.16). Analyzing the diagram in figure 3, a strong correlation between the three parameters can be noticed, the increase of loading capacity in the same time with decrease of winding angle leading to higher values needed for car weight. Within figure 4 was considered the situation when loading capacity is 5000 N (approximating 6 persons with 80 kg each) and the evolution of the car weight is shown depending on the two functional parameters: winding angle and respectively friction coefficient. Taking into account the expression (1) it is obvious that small winding angles and reduced friction coefficients require higher values for car weight, remark that also results from analyzing the diagram in figure 4.

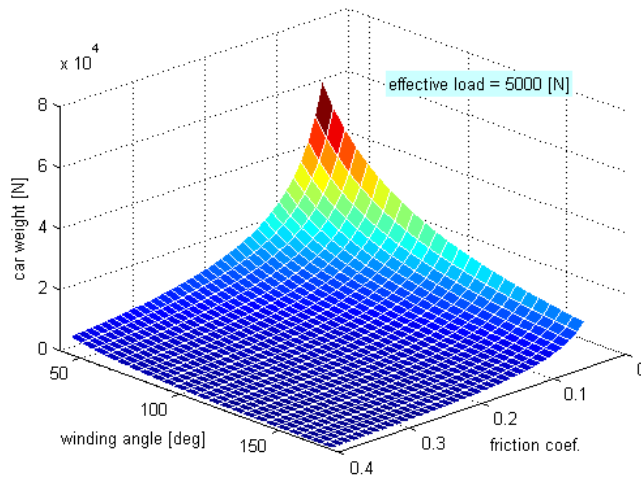


Figure 4. Car weight evolution depending on winding angle and friction coefficient respectively (for a loading capacity of 5000 N).

3. Working assumptions used for simulation of cable-actuating system dynamics

Simulation of dynamic behaviour was developed based on the model depicted in figure 1. The numerical approach of solving the non-linear mathematical model entirely complies with following working assumptions, meaning

- Analysis of the transitory mode suitable to start the system, considering the movement on lifting direction of the car and, obviously, lowering direction of the counter-weight;
- Compensation system was ignored for the approach within this paper;
- Isolation pads supposed to be rigid thus that the car mass also includes the sling mass;
- The characteristic of the driving system (electrical motor), implemented within numerical model, was obtained by linearization on two segments corresponding to working area in stable mode – nominal, and unstable mode – for starting;
- Limiting of the analysis time up to 10 seconds after movement is initiated;
- A high enough number of operating situations were simulated, some of them being extremely, as to the values that can be got by functional and structural parameters of the system (simultaneously or independently); hereinafter, only the results obtained for the one situation considered as suggestive for this study will be presented.

Taken into account the hypotheses within works [15,16], the previously assumptions and the schematization in figure 1, the mathematical model acquires the expressions as follows

$$\begin{cases} m_1 \ddot{y}_1 - c_1 \dot{y}_1 + F_{d1} + m_1 g - \sigma_1 = 0 \\ m_2 \ddot{y}_2 - c_2 \dot{y}_2 + F_{d2} - m_2 g + \sigma_2 = 0 \\ J \ddot{\theta} - c_0 \dot{\theta} + M_{load} = M_{drive} \end{cases} \quad (2)$$

where the independent variables y_1 , y_2 , θ denote car and counter-weight linear displacements, and respectively traction sheave angular displacement, $m_{1,2}$ denotes the car and counter-weight mass respectively, J means the mass moment of inertia of driving system, $c_{0,1,2}$ denotes viscous damping coefficient, g means the constant of gravitational acceleration, and M_{drive} indicates the drive torque evaluated based on torque-speed characteristic of the driving system. Based on experimental tests presented in papers [15,16], the energy dissipations within the hoisting cables $F_{d1,2}$ were supposed to be proportional with cable deformation, permanently acquiring the sign of its variation. Thus, the expression of the cable dissipation is

$$F_{d1,2} = c_d \delta_{1,2} \frac{\dot{\delta}_{1,2}}{|\dot{\delta}_{1,2}|} \quad (3)$$

wherein the numerical indexes denote the car and the counter-weight cable respectively.

An additional torque M_{load} appears at the traction sheave due to the differences between the tensions within car and counter-weight ropes respectively, thus that the expression of this torque is

$$M_{load} = r (\sigma_1 - \sigma_2) \quad (4)$$

where r denotes the effective radius of the traction sheave, and the tensions with the two ropes $\sigma_{1,2}$ are

$$\sigma_{1,2} = \delta_{1,2} k_{1,2} \quad (5)$$

with cable deformation given by the following expression

$$\delta_{1,2} = \frac{m_{1,2} g}{k_{1,2}} \pm \theta r \mp y_{1,2} \quad (6)$$

and cable stiffness approximately evaluated with the following linear assumption

$$k_{1,2} = \frac{E A}{l_{1,2}} \quad (7)$$

wherein E means the Young's modulus of the cable and A denotes the cable section area. Instantaneous length of each rope during the simulation, denotes by $l_{1,2}$ in expression (7), can be evaluated, according with the assumptions in [15,16], with the equation as follows

$$l_{1,2} = (l_{01,02} \mp y_{1,2}) \left(1 - \frac{m_{1,2} g}{E A} \right)^{-1} \quad (8)$$

with initial position lengths given by constants $l_{01,02}$. For numerical solving the nonlinear computational model, the Runge–Kutta fourth–order method was used, and a specifically application was developed within the Matlab© programming environment based on available ordinary differential equations solving facilities.

4. Analysis of dynamic behaviour of the lift under normal use mode

The analysis of behaviour under dynamic operation mode of the cable-actuating system was done by monitoring the following parameters, as follows: displacements and speeds of the car and the counter-weight respectively, angular displacement and rotating speed of the drive wheel, tensions σ_1 and σ_2 within car and counter-weight cables respectively, active torque on drive wheel shaft, the stiffness as well as the longitudinal deformation of the two hoisting ropes. The figures from 5 to 8 show the evolution of such parameters during the first 10 seconds after the motion was initiated. Thus, figure 5 contains the evolution in time of the car, counter-weight and driving sheave motion, and the forces within each hoisting rope.

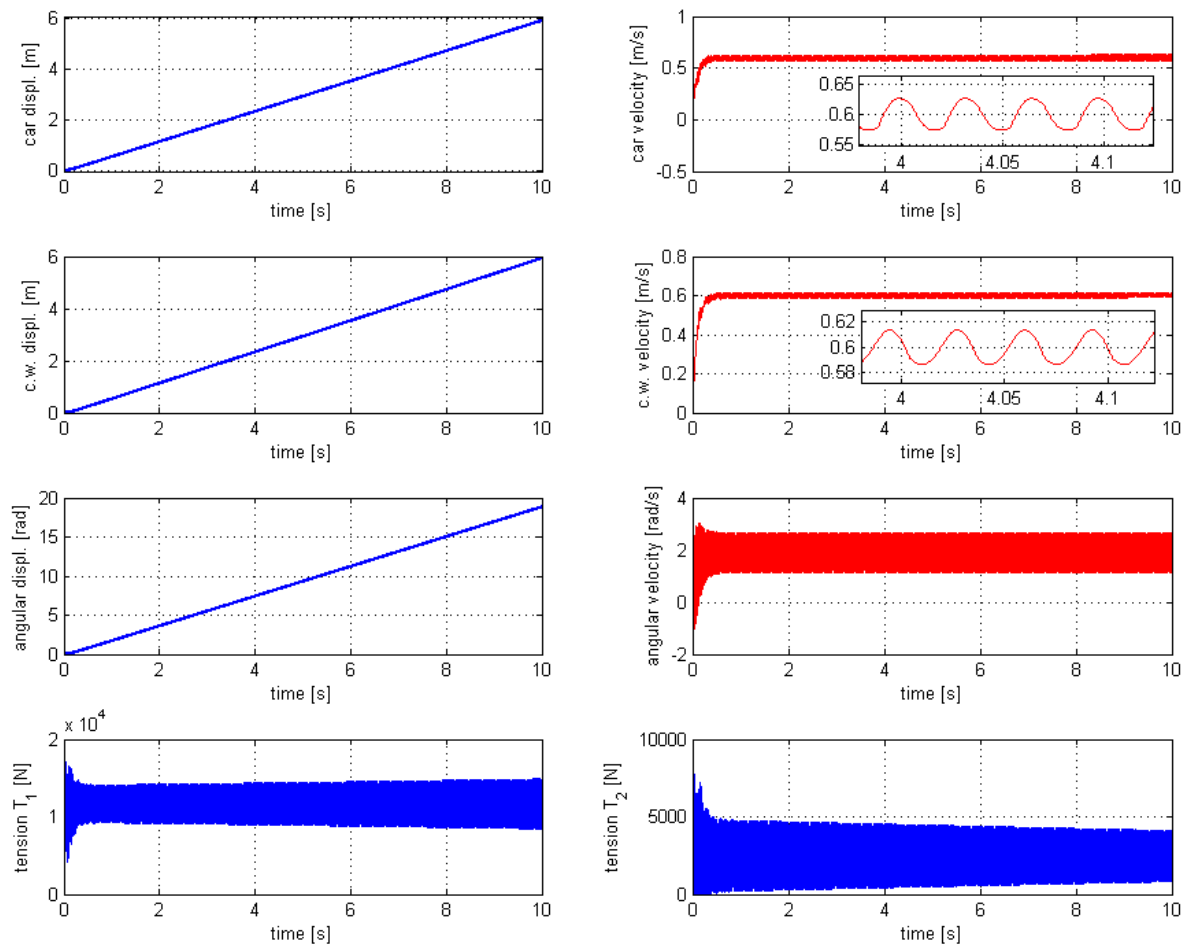


Figure 5. Timed evolutions of the displacements and the velocities respectively of the car, counter-weight, and drive sheave, as well as of the tensions σ_1 , σ_2 within the hoist ropes. Detail views for car and counter-weight speeds were provided.

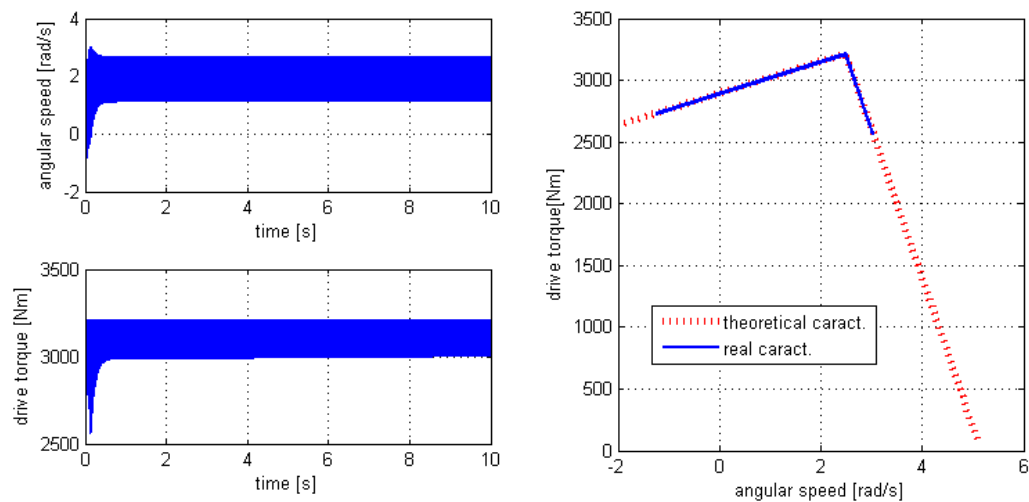


Figure 6. The evolution of angular speed and torque on drive sheave – *left side diagrams*, and the theoretical/real characteristics of driving system (evaluated with reduced parameters on drive shaft) – *right side graph*.

Diagrams in figure 6 depict both the evolution in time of driving system – torque and angular speed respectively, and the effective characteristic of drive process. Comparatively, within diagram on right-side in figure 6, was presented the theoretical characteristic assumed to be provided by the actuating system. A three-dimensional representation of the effective characteristic (gained by computer simulation) compared to the theoretical one, taking into account the additional variable time, is shown in figure 7, wherein the theoretic characteristic was depicted as a compact surface, to reveal the compatibility between this and the real one according with the model timed evolution.

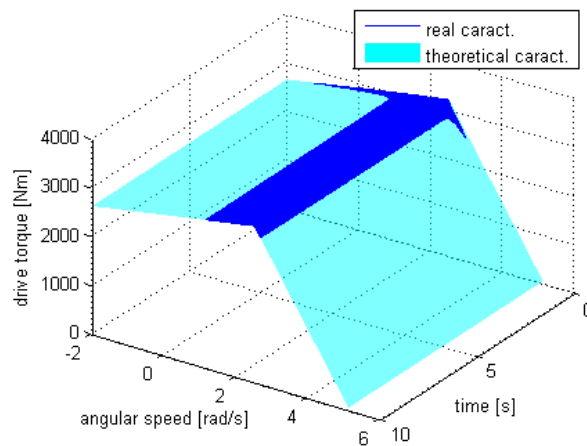


Figure 7. Evolution in time of the effective characteristic of actuating system, evaluated with reduced parameters on drive shaft, compared to the theoretical one.

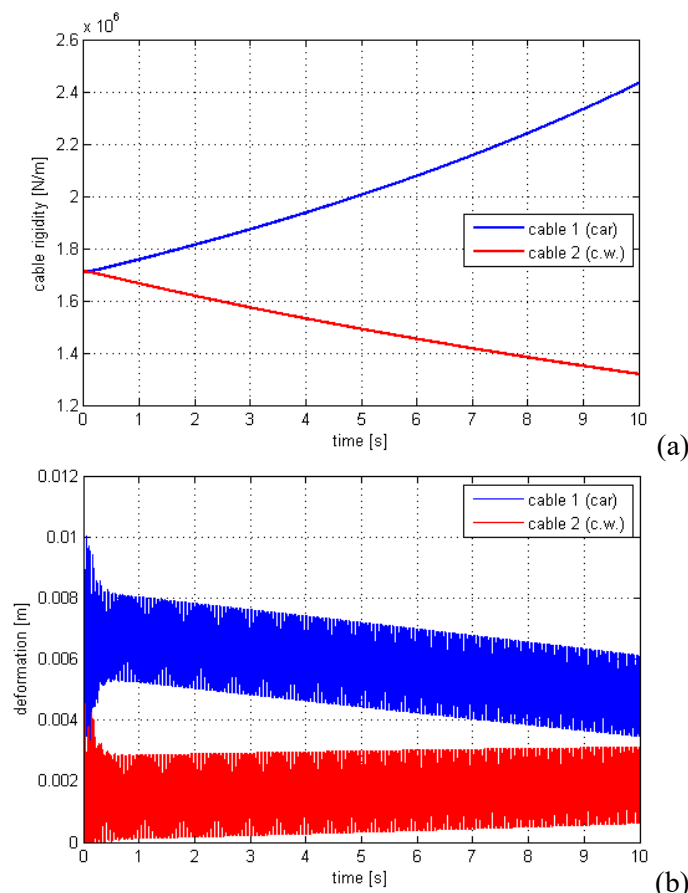


Figure 8. Timed evolutions of the hoisting cable rigidity (a) and deformation (b).

Figure 8 contain two sets of diagrams as follows: the timed evolution of the rigidity of car and counter-weight ropes – see figure 8(a), and respectively, the evolution of the global deformation within these ropes – see figure 8(b).

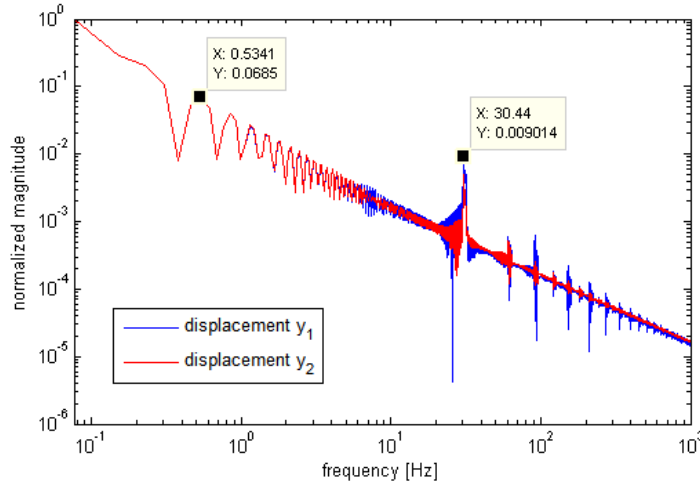


Figure 9. Spectral composition of the car and the counter-weight motion respectively.

It was evaluated the spectral compositions of the car and the counter-weight motion respectively, and the afferent diagrams was comparatively depicted in figure 9. Normalized magnitudes were assumed and full logarithmic representation scale was used in order to provide clearly the prevailing frequency ranges appearing within these signals.

During the regular exploitation cycle, the hoisting ropes, due to their structural configuration, provide a sway dynamic behaviour characterized by both longitudinal and transversal oscillations. One of the objectives of this study consists by a comparative analysis of frequencies ranges due to cable oscillations and the actuating process respectively, with underlining the potential superposition imminences.

Hereby, it was firstly assumed the beam transversal vibration hypothesis, thus that the pulsation of n^{th} mode (for pinned end constraints) could be evaluated with following expression

$$p_n = \frac{(n \pi)^2}{l^2} \sqrt{\frac{E I_z}{\rho A}} \quad (9)$$

wherein I_z denotes the area moment of inertial, ρA means the specific mass on unit length, and l denotes the total length of the beam.

According with the string model, the pulsation of n^{th} mode of transversal vibration, taking into account the tension σ within the string, could be evaluated with following expression

$$p_n = \frac{n \pi}{l} \sqrt{\frac{\sigma}{\rho A}} \quad (10)$$

For the longitudinal vibration hypothesis, both models – string or beam – provide the same equation for the pulsation of n^{th} mode, as follows

$$p_n = \frac{n \pi}{l} \sqrt{\frac{E}{\rho}} \quad (11)$$

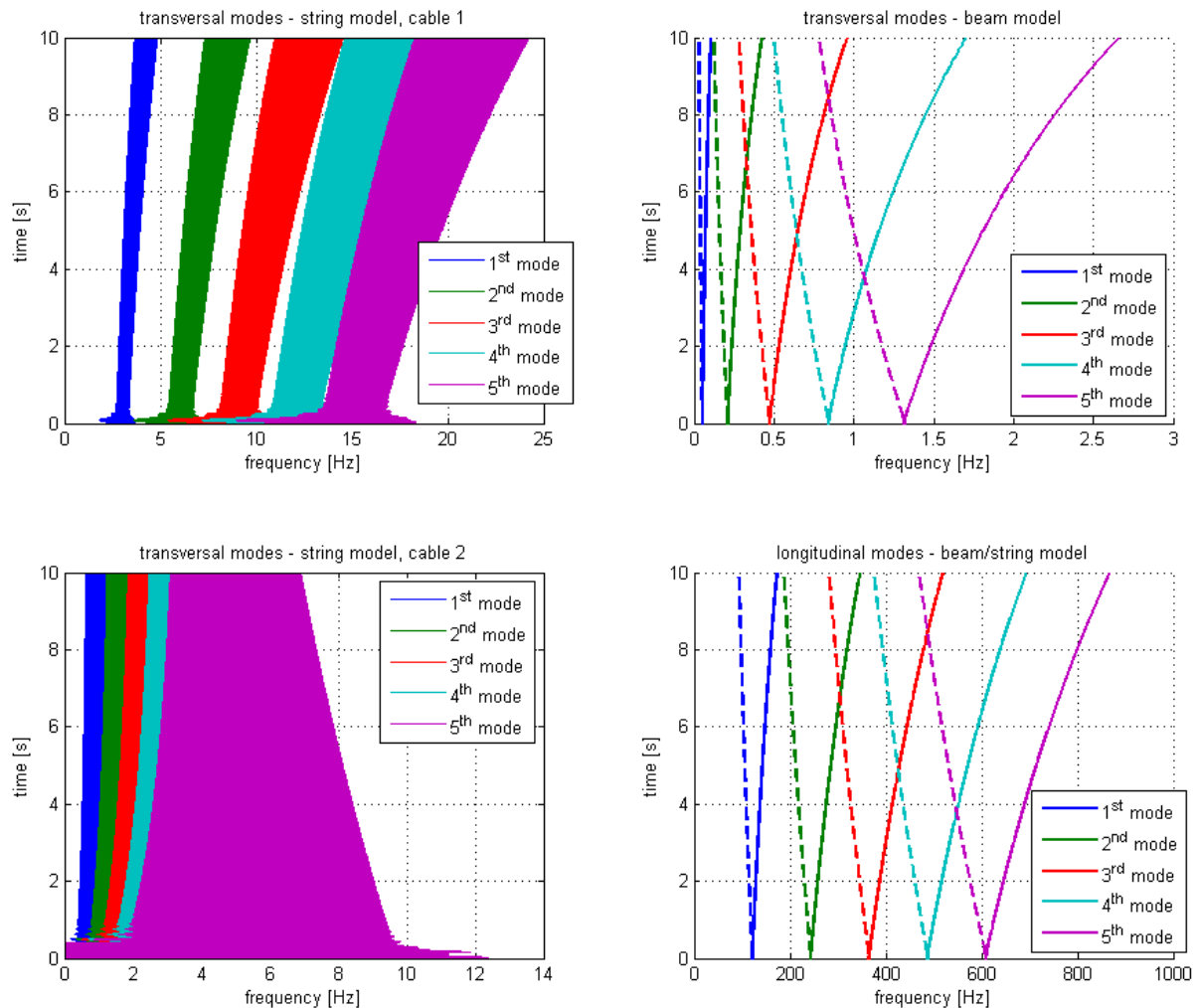


Figure 10. Timed evolutions of the five representative modal frequencies according with beam/string theory for transversal and longitudinal vibration respectively. Car hoisting rope with continuous line and counter-weight rope with dashed line within right-side diagrams were supposed.

Taking into account the expressions (9...11) and the timed evolutions of the ropes length during the simulation, the authors was evaluated the changes of specific frequencies for the first five modes. Gained diagrams were provided in figure 10, for each beam/string hypothesis and, respectively for the car and counter-weight rope.

5. Discussions on the results

Correlative analysis of the main parameters involved with inequation (1) and respectively the correspondent diagrams in figures 2 to 4 dignify the importance of the motion stability during the exploitation regime. Such as example, the variation of the winding angle implies changes of the modifications of the actual working point within those diagrams. Sway dynamics of hoisting ropes leads to continuously variations of the winding angle and, hereby, the ropes dynamics needs to be deeply analyzed in order to reduce dynamic effects within the whole cable-driven system. Occasionally correlation between cables sway and car/counter-weight dynamics supplies transitory evolutions due to temporary/locally resonances.

Analyzing diagrams in figures from 5 to 7 result a stable dynamics of the car and counter-weight, because of the rapidly stabilization of the working point within the actuating system characteristic. Instantaneous changes of deformations in ropes, leading the continuously oscillations of instant length

of each rope – see figure 8 – also provide stable dynamics within the cable-based actuating system. Hereby, the frequencies distribution on spectral diagram in figure 9 provides some peaks according to each part of the system. A joint time-frequency analysis also dignifies stable prevailing frequencies during the entire simulated cycle. It has to be mentioned that this analysis was performed supposing the actuating system working within the range close to the maximum motor torque – see diagrams within figures 6 and 7. Hereby, the transitions between stable and unstable operational mode of driving system provides additional vibratory states into the actuated system.

Taking into account the previous paragraph considerations, results that the transitory dynamics results by sway motion of the hoisting ropes and the occasionally overlaps with driving system dynamics. Diagrams in figure 10 present the evolution of modal frequencies, for the first five modes, during the simulation cycle. A preponderant transitory evolution was provided by the string hypothesis, for transversal vibration, for each hoisting cable. In addition, whatever the hypothesis, an increasing tendency for frequency shifting was observed for upper modes, even if those weight within global motion decrease with the modal number growing up. Lower modes provide reduced frequency shifting, thus that is was relative easily to design and implement vibration isolation solutions according with these frequencies, supposing that those modes supply largest magnitudes within the global motion.

Comparative analysis of spectral ranges from diagrams in figures 9 and 10 respectively, shows the superposition (continuously variable in time) of the characteristic frequency ranges due to specific motion of component parts, and underlines the usefulness of such as dynamic study in order to evaluate potential sources of transitory dynamics within the whole system.

6. Conclusions

This paperwork shows a specific case of a cable-based actuating system and the results presented and analyzed within previous sections were based upon real data for the entire range of parameters involved in the computational model. It has to be mentioned that those aspects that were neglected or removed from initial physical model did not affect the generalization character of this analysis. Some irrelevant nonlinear aspects regarding damping energy within car, counter-weight and traction sheave evolution was neglected mainly because of missing realistic information about additional parameters that will must involved in computational model.

For the studied system, a good correlation was found between the values obtained by computerized simulation and the further data available from the experimental observations.

The final conclusion is that the special usefulness of such computational model is supported by the entire set of input data which assure consistency, appropriateness, relevance and also, the lack of (minimizing) estimating type approximations within the evaluations of focused parameters. Even when using some simplifying assumptions, the validation of final results obtained based on some real data is a performance factor for the proposed dynamic model.

7. References

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