IOP Conf. Series: Materials Science and Engineering **226** (2017) 012011 doi:10.1088/1757-899X/226/1/012011

Combined mode I stress intensity factors of slanted cracks

A.E Ismail*[1], M.Q Abdul Rahman[2], M.Z Mohd Ghazali[3], M. Zulafif Rahim[1], M. Rasidi Ibrahim[1], Mohd Fahrul Hassan[1], Nik Hisyamudin Muhd Nor[1], A.M.T Ariffin[1], Muhamad Zaini Yunos[1]

[1]Faculty of Mechanical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia, Batu Pahat, 86400 Johor, Malaysia.
[2]Centre for Diploma Studies, Unviersiti Tun Hussein Onn Malaysia, Batu Pahat, 86400 Johor, Malaysia.
[3]Malaysia Marine and Heavy Engineering Sdn. Bhd., Pasir Gudang, 81700 Johor, Malaysia.

Corresponding author: emran@uthm..edu.my / al_emran@uthm.edu.my

Abstract. The solutions of stress intensity factors (SIFs) for slanted cracks in plain strain plate are hard to find in open literature. There are some previous solutions of SIFs available, however the studies are not completed except for the case of plain stress. The slanted cracks are modelled numerically using ANSYS finite element program. There are ten slanted angles and seven relative crack depths are used and the plate contains cracks which is assumed to fulfil the plain strain condition. The plate is then stressed under tension and bending loading and the SIFs are determined according to the displacement extrapolation method. Based on the numerical analysis, both slanted angles and relative crack length, a/L played an important role in determining the modes I and II SIFs. As expected the SIFs increased when a/L is increased. Under tension force, the introduction of slanted angles increased the SIFs. Further increment of angles reduced the SIFs however they are still higher than the SIFs obtained using normal cracks. Under bending moment, the present of slanted angles are significantly reduced the SIFs compared with the normal cracks. Under similar loading, mode II SIFs increased as function of a/L and slanted angles where increasing such parameters increasing the mode II SIFs.

1. Introduction

Stress intensity factor (SIF) is successfully used to characterize the cracks in elastic materials and structures [1]. It is also used to replace the concept of stress concentration factor leading to infinite stress especially where there is a sharp edge for example cracks. There are several other methods or techniques are used to investigate the defects [2]. On the other hand, the basic crack configurations are also available in previous study [3]. However, the oblique or slanted cracks are rarely found in open literature [3-6]. Albinmousa et al. [3] present a closed form solution for the geometrical correction factors of slanted cracks in a plain stress plate. The closed form solution of SIFs comprised of three important parameters such as inclination angles, plate widths and crack lengths. They claimed that the results from such expression are well agreed with the analytical, numerical and experimental results. Matsumto et al. [4] also investigated the SIFs of inclined cracks. However, the cracks are positioned between two different materials. Their work concentrated more on the comparison between their method and displacement extrapolation methods. Kuang and Chen [5] investigated the stress intensity

IOP Conf. Series: Materials Science and Engineering **226** (2017) 012011 doi:10.1088/1757-899X/226/1/012011

factor of two crack conditions such as slanted single edge and central cracks using displacement extrapolation method (DEM). They only focused on the 45° crack inclination and concluded that the DEM gave accurate results even course finite element model is used. Three dimensional slanted cracks in round bars under mode I tension can be found in [6-16]. The surface cracks are assumed as elliptical shapes and different slanted angles, elliptical ratios and relative crack depth are used. It is found that when the inclination angles are introduced, the SIFs increased gradually. On the other hand, slanted angles are also produces the mode II SIFs. This paper investigates the stress intensity factors of slanted edge crack in plain strain plate under mode I tension and bending loadings. The crack is modelled numerically using ANSYS finite element program. DEM is then used to calculate the SIFs and they are plotted against the relative crack depths. The effect of relative crack depths and the oblique angles on the stress intensity factors are discussed and analyzed.

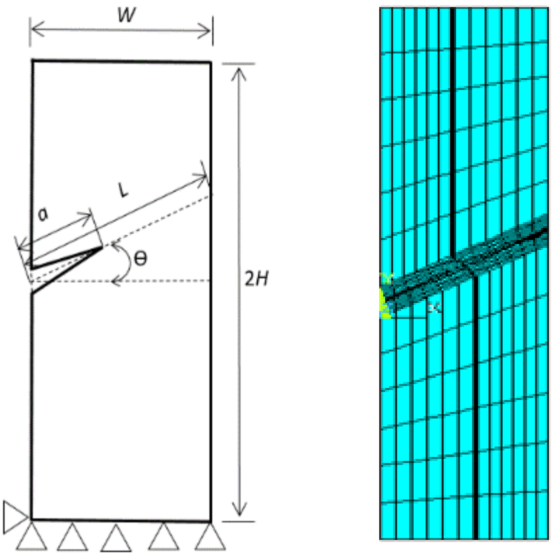
2. Methodology

ANSYS finite element program is used to model and calculate the SIFs. There are two kind of cracks are model; the first is normal cracks and the second is slanted cracks. The normal crack is used for validation purposes. The crack mouth is placed in the middle of plate height where the plate is also assumed to fulfil the plain strain condition. There are nine slanted angles used including normal cracks and they are normalized against angle of 90°. The relative angles are 0.00, 0.056, 0.111, 0.167, 0.222, 0.278, 0.333, 0.389, 0.444 and 0.500. The relative crack depth, a/L used as follows 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7. It is also assumed that the plate width is 50mm and the height, 2H = 100mm. The schematic diagram of the crack in a plate is shown in Figure 1. Displacement extrapolation method [7] is used to determine the SIFs at the crack tip. Since the crack faces are oblique, two types of SIFs are produced such as modes I and II. The SIFs are also normalized as shown in equation 1 and 2.

$$F_I = \frac{K_I}{\sigma\sqrt{\pi a}} \quad (1)$$

$$F_I = \frac{K_I}{\sigma\sqrt{\pi a}} \quad (2)$$

where K_I and K_{II} are the modes I and II SIFs respectively, while F_I and F_{II} are their corresponding geometrical correction factors or normalized SIFs, σ is an axial stress and α is crack depth.



(a) (b)

Figure 1: (a) Schematic diagram plate with slant crack and (b) Finite element model

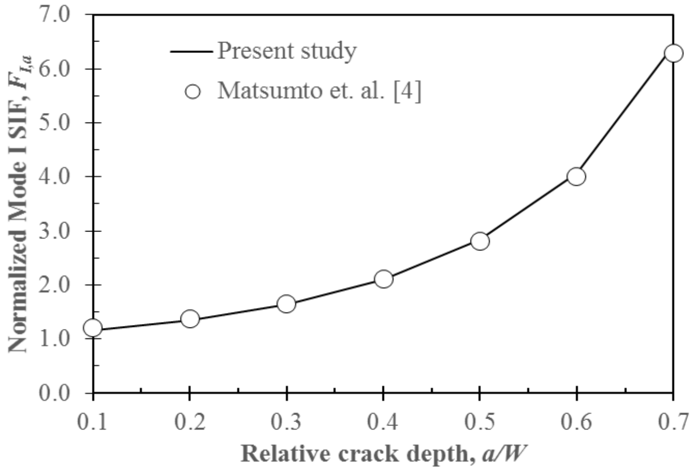


Figure 2: Finite element validations

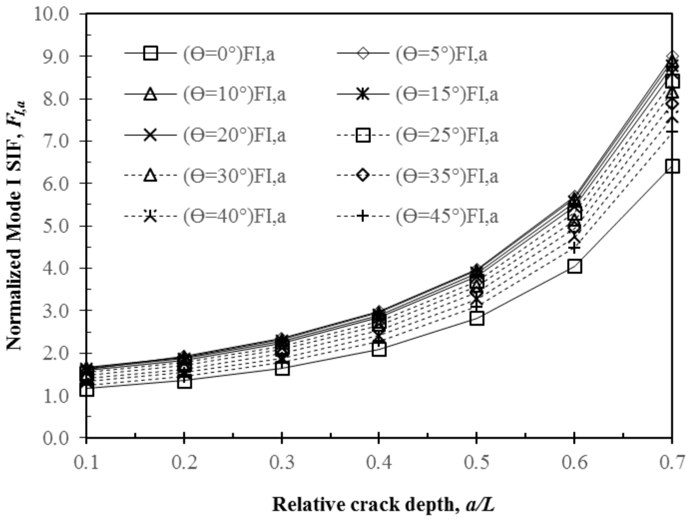
Before the model is further used, it is a compulsory work to have the validation by comparing result obtain from this study with the results obtained previously. Since there is lack of number of works on the slanted cracks then the normal cracks are used for such purposed. Based on Figure 2, it is revealed that the present model is well agreed with the existing model.

3. Results and discussion

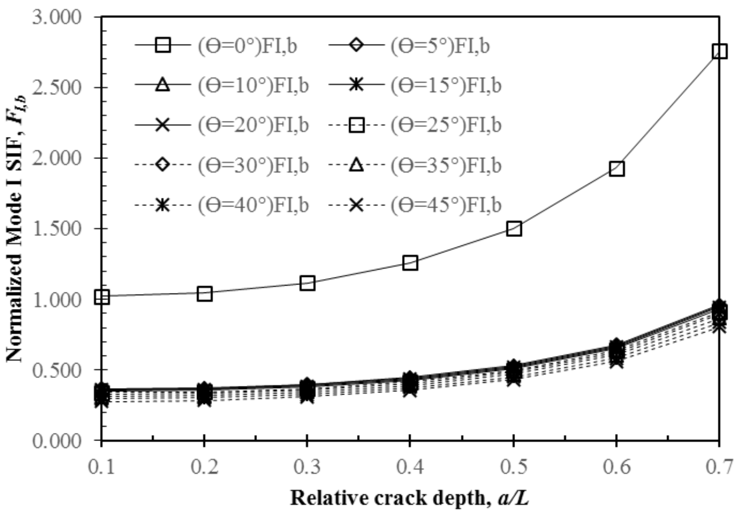
Figure 3 shows the normalized mode I SIFs under tension and bending forces. The SIFs are plotted against relative crack length, a/L for different slanted angle, θ. As expected increasing the angles increased the SIFs gradually. Figure 3(a) reveals the SIFs obtained under tension forces. Introduction the angles significantly increased the SIFs especially for 5o angle. Once the angles are increased the mode I SIFs decreased. This is due to the fact that if the cracks are slanted relative with the normal cracks, their capability to open the cracks are limited. On the other hand, increasing the slanted angles also caused the plate to fail in sliding mode and therefore producing mode II SIFs which is then discussed latter. It is also found that increasing the angles capable to reduce mode I SIFs however such values are still higher than the case of normal cracks.

Figure 3(b) reveals the effect of a/L on the mode I SIFs under bending moment when slanted angles are varied. The SIFs for normal cracks are relatively higher than the SIFs obtained from slanted cracks. It is also observed that the introduction of slanted cracks are insignificantly affected the SIFs where increasing the angles are slightly decreased the SIFs. This is due to the fact that for normal cracks, the areas between two crack faces are unbalanced. This is caused bending moment to open only the upper portion of the crack compared with the normal crack both crack faces opened almost equal in distance and therefore producing higher mode I SIFs. Figure 4 indicates the role of a/L on the mode II SIFs when slanted cracks are varied. As-expected inclining the crack faces contributed to the formation of higher mode II shear stress. Regardless of loading modes, increasing the slanted angles increasing the mode II SIFs. It is also observed that for slanted angles less than 25°, the SIFs are almost flattened where increasing a/L insignificantly increased mode II SIFs where higher slanted angles are used (> 25°) increasing a/L gradually increased mode II SIFs. This is due to the cracks are easily deformed causing by higher shear stress.

Figure 5 shows the roles of slanted angles on the SIFs when a/L is varied. There are distinct behavior of the roles of slanted angles on the SIFs even both modes of loading are similar. For mode I tension forces (Figure 5(a)), if the slanted crack is slightly increased the SIFs are also increased. However, when the angles are further increased the SIFs are seemed to decrease. Figure 5(b) shows mode I SIFs under bending moment. In general, the introduction of slanted angles reduced the SIFs significantly when compared with the SIFs of normal cracks. Further increase such angles slightly reduced the SIFs. This is due to the fact that when slanted cracks increased the tendency of crack faces to widely open have decreased since the upper portion of crack faces is displaced while the bottom face relatively not. On the other hand, the upper portion of cracks probably tend to slide along the crack plane.



(a)



(b)

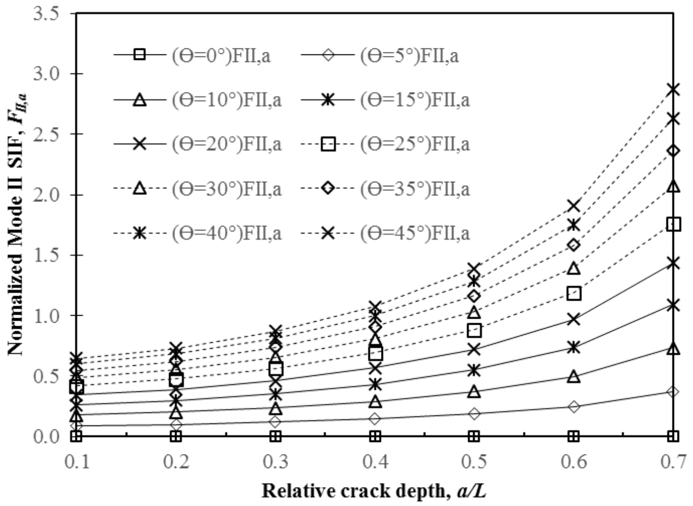
Figure 3 Normalized mode I SIFs for slant crack subjected to (a) tension loading and (b) bending moment

Figure 6 clearly reveals the role of slanted cracks on the mode II SIFs. For both modes of loading, increasing the angles almost linearly increased the SIFs. The increments of SIFs become significant when higher a/L is used. It is also due to the effect of cracks plane on the plate stiffness. If lower a/L is used, the integrity of plate is higher compared with the plate containing higher values of a/L. Once higher a/L and slanted cracks are used, the distribution of stress around the crack tip is unbalanced where the upper portion received higher stress and therefore it is sheared along the crack plane consequently produced higher mode II SIFs are produced.

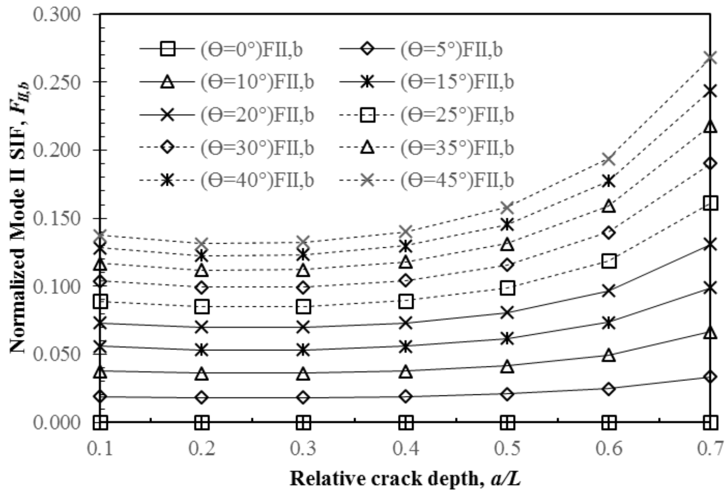
Figure 7 reveals the stress distributions around the crack tip considering different slanted angles. Figure 7(a) clearly reveals that the stress distributions of the crack tip for normal crack. It is observed that the stress distribution is almost uniform around the crack tip resulting identical crack opening mechanism for both crack faces consequently producing higher values of SIFs. While Figures 7(b) and

7(c) show the stress distribution for the slanted cracks of 25o and 45o, respectively. At the upper portion of crack face, higher stress is induced on the other hand, lower crack face experienced insignificant stress. It is then resulted lower SIFs compared with the SIFs of normal cracks. Tables 1 to 4 tabulate SIFs under tension and bending forces for modes I and II, respectively.

A regression technique is used in order to simplify the SIFs in Tables 1 to 4. For an example, mode I SIFs under tension force is used for such purpose. Equation (3) shows the developed expression to predict SIFs for different a/L and slanted angles. Table 1 lists the regression statistics for the Equation (3) and then it is replotted to shows its efficiency to predict SIFs under mode I tension force. Figure 8 shows the comparison between the SIFs predicted using Equation (3) and the results obtained numerically. It is revealed that the Equation (3) can be used to predict mode I SIFs very well.

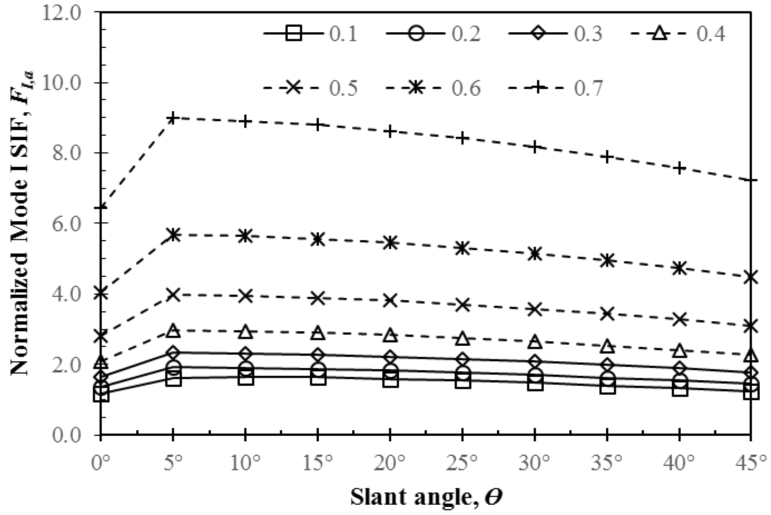


(a)

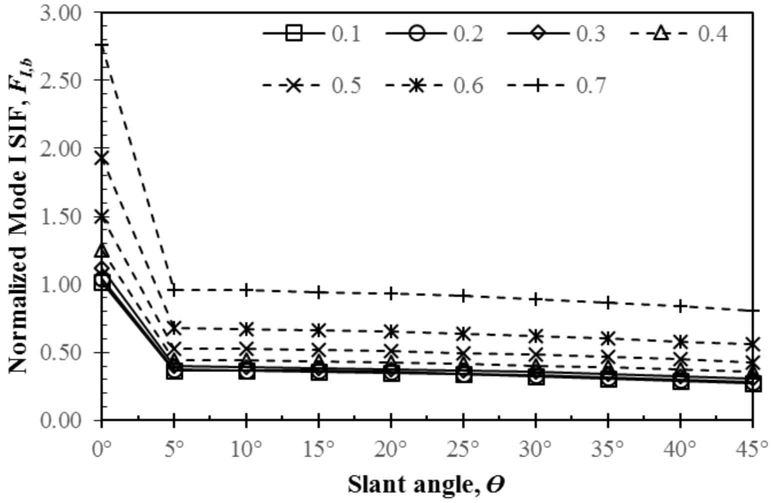


(b)

Figure 4 Normalized mode II SIFs for slant crack subjected to (a) tension loading and (b) bending moment



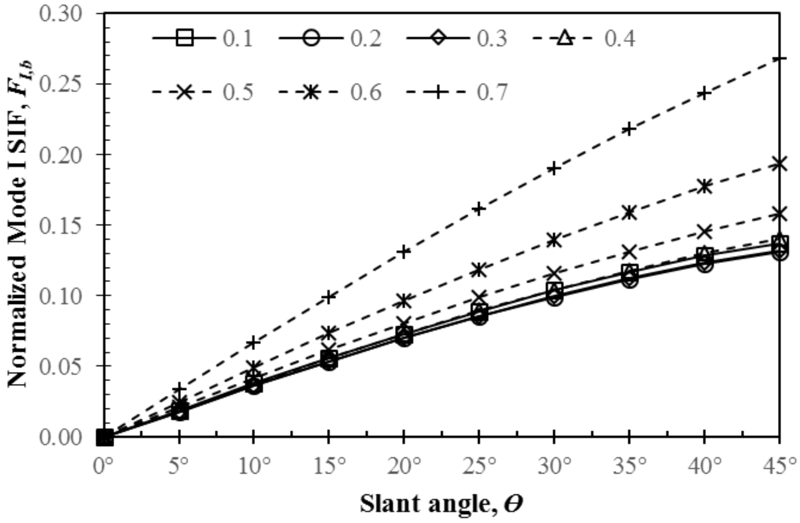
(a)



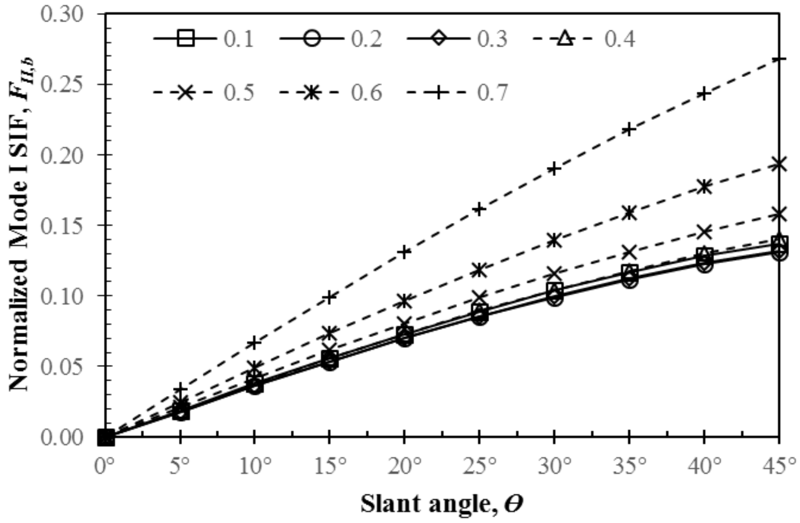
(b)

Figure 5 Normalized mode I SIFs for against slant angles for (a) tension loading and (b) bending moment

$$F_{I,a} = \begin{bmatrix} 2.4 - 16.246\left(\frac{a}{L}\right) + 3.965\left(\frac{2\theta}{\pi}\right) + 74.982\left(\frac{a}{L}\right)^2 - 140.036\left(\frac{a}{L}\right)^3 \\ -23.698\left(\frac{2\theta}{\pi}\right)^2 + 32.846\left(\frac{2\theta}{\pi}\right)^3 + 18.876\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right) - 72.159\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^2 \\ +122.353\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^3 + 114.546\left(\frac{a}{L}\right)^4 - 14.628\left(\frac{2\theta}{\pi}\right)^4 - 76.882\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^4 \end{bmatrix} \quad (3)$$



(a)

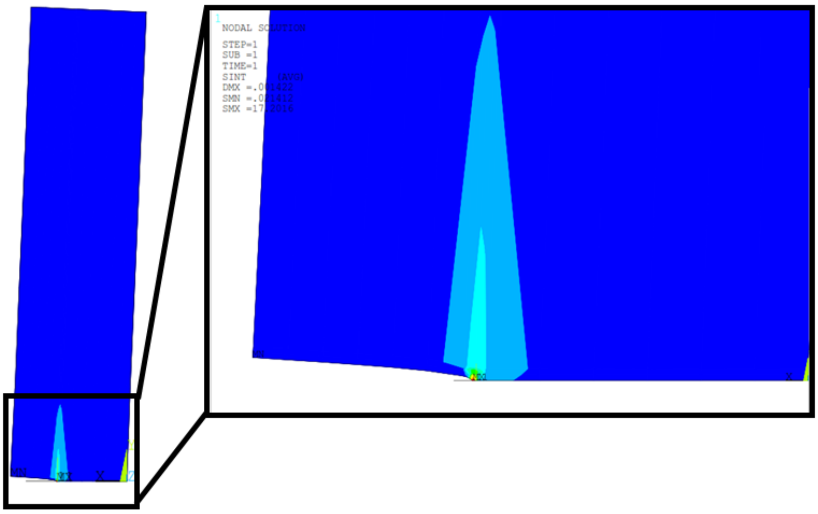


(b)

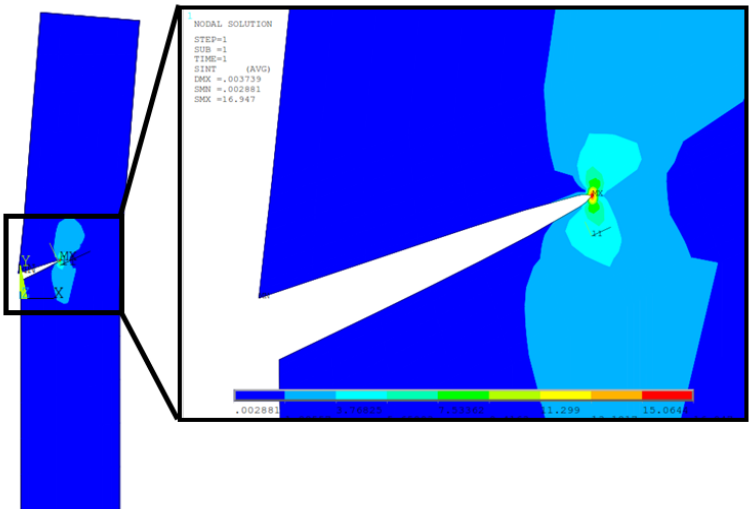
Figure 6 Normalized mode II SIFs for against slant angles for (a) tension loading and (b) bending moment

Table 1 Regression statistics

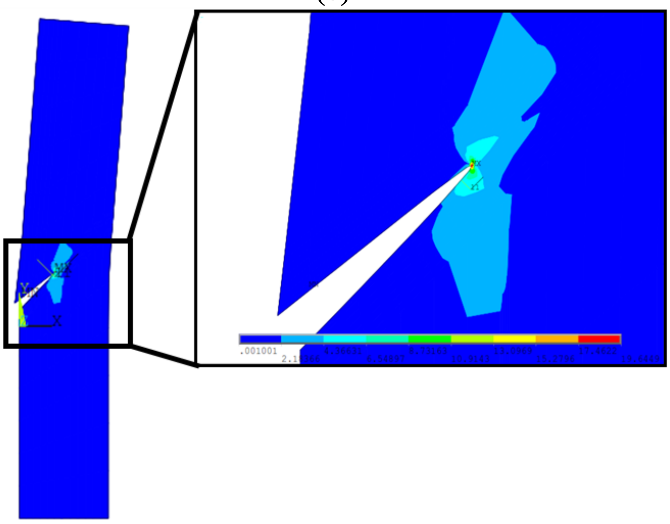
Type of loading	Multiple R	R square	Standard error
Tension	0.997	0.995	0.184



(a)



(b)



(c)

Figure 7 Crack tip deformation under bending moment (a) normal, (b) 25^o and (c) 45^o slanted cracks

Table 2 Normalized mode I SIFs for slanted cracks subjected to tension loading

Θ	*a/L*						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0	1.16	1.34	1.64	2.09	2.82	4.05	6.42
5	1.60	1.92	2.34	2.97	3.98	5.69	9.00
10	1.65	1.90	2.31	2.95	3.95	5.64	8.90
15	1.62	1.87	2.27	2.90	3.88	5.56	8.78
20	1.58	1.82	2.22	2.83	3.80	5.45	8.62
25	1.53	1.77	2.15	2.75	3.70	5.31	8.42
30	1.47	1.70	2.07	2.65	3.57	5.14	8.17
35	1.40	1.62	1.98	2.54	3.43	4.95	7.89
40	1.32	1.53	1.88	2.41	3.27	4.73	7.57
45	1.23	1.44	1.76	2.27	3.09	4.49	7.21

Table 3 Normalized mode II SIFs for slanted cracks subjected to tension loading

Θ	*a/L*						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.09	0.10	0.12	0.15	0.19	0.25	0.37
10	0.18	0.20	0.24	0.29	0.37	0.50	0.73
15	0.26	0.30	0.35	0.43	0.55	0.74	1.09
20	0.34	0.39	0.46	0.57	0.72	0.97	1.43
25	0.42	0.48	0.56	0.69	0.88	1.19	1.76
30	0.49	0.55	0.66	0.81	1.03	1.40	2.07
35	0.55	0.62	0.74	0.91	1.17	1.59	2.36
40	0.60	0.68	0.81	1.00	1.29	1.76	2.63
45	0.65	0.73	0.87	1.08	1.39	1.91	2.87

Table 3 Normalized mode I SIFs for slanted cracks subjected to bending moment

Θ	*a/L*						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0	1.02	1.021	1.021	1.021	1.021	1.021	1.021
5	0.36	0.368	0.368	0.368	0.368	0.368	0.368
10	0.36	0.365	0.365	0.365	0.365	0.365	0.365
15	0.35	0.359	0.359	0.359	0.359	0.359	0.359
20	0.35	0.350	0.350	0.350	0.350	0.350	0.350
25	0.33	0.339	0.339	0.339	0.339	0.339	0.339
30	0.32	0.326	0.326	0.326	0.326	0.326	0.326
35	0.31	0.311	0.311	0.311	0.311	0.311	0.311
40	0.29	0.294	0.294	0.294	0.294	0.294	0.294
45	0.27	0.275	0.275	0.275	0.275	0.275	0.275

Table 4 Normalized mode II SIFs for slanted cracks subjected to bending moment

Θ	*a/L*						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	0.04	0.04	0.04	0.04	0.04	0.05	0.07
15	0.06	0.05	0.05	0.06	0.06	0.07	0.10
20	0.07	0.07	0.07	0.07	0.08	0.10	0.13
25	0.09	0.09	0.09	0.09	0.10	0.12	0.16
30	0.10	0.10	0.10	0.10	0.12	0.14	0.19
35	0.12	0.11	0.11	0.12	0.13	0.16	0.22
40	0.13	0.12	0.12	0.13	0.15	0.18	0.24
45	0.14	0.13	0.13	0.14	0.16	0.19	0.27

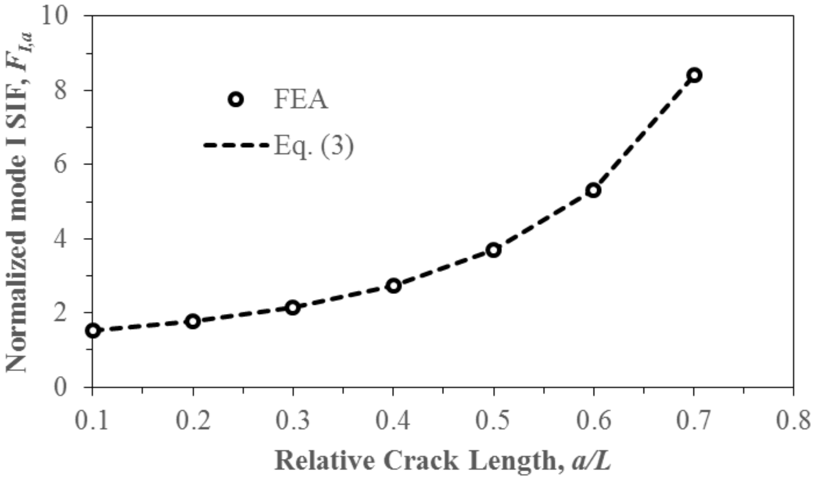


Figure 8 Comparison mode I SIFs obtained using finite element method and Eq. (3)

4. Conclusion

It was noted that the most convenient method to predict crack deformation in a plate having slant cracks by evaluating the SIFs obtained using finite element analysis. The SIFs are much depends on four main variables namely slant crack angle ϴ, relative crack depth, α/L and type of load applied to the surface crack. As expected, SIFs increased when α/L increased however the SIFs decreased when ϴ increased for mode I crack subjected to tension loading. SIFs value decreased for crack subjected to bending moment but the decrement is insignificant.

References

[1] Ismail AE, Ariffin AK, Abdullah S and Ghazali, MJ 2012 Off-set crack propagation analysis under mixed mode loadings *International Journal of Automotive Technology* **12** (2) 225-23.
[2] Newman Jr JC, James MA, and Zerbst U 2003 A review of the STOA/CTOD fracture criterion *Engineering Fracture Mechanics* **70** 371-385.
[3] Albinmousa J, Merah N and Khan SMA 2011 A model for calculating geometrical factors for a mixed-mode I-II single edge notched tension specimen *Engineering Fracture Mechanics* **78** 3300-3307.

[4] Matsumto T, Tanaka M, and Obara R 2000 Computation of stress intensity factors of interface cracks based on interaction energy release rates and BEM sensitivity analysis *Engineering Fracture Mechanics* **65** 683-702.
[5] Kuang JH and Chen LS 1993 A displacement extrapolation method for two dimensional mixed mode crack problems *Engineering Fracture Mechanics* **46** (5)735-741.
[6] Ismail AE, Mohd Tobi AL and NH Mohd Nor 2015 Stress intensity factors of slanted cracks in round bars subjected to mode I tension loading *AIP Conference Proceedings* **1660** 070027.
[7] Gustavo VG, Jaime P and Manuel E 2000 K_I evaluation by the displacement extrapolation method *Engineering Fracture Mechanics* **66** (1) 243-255.
[8] Hammouda MMI, Fayed AS and Sallam HEM 2002 Mode II stress intensity factors for central slant cracks with frictional surfaces in uniaxially compressed plates *International Journal of Fatigue* **24** (2) 1213-1222.
[9] Hedayati E and Vahedi M 2014 Using extended finite element method for computation of the stress intensity factor, crack growth simulation and predicting fatigue crack growth in a slant-cracked plate of 6061-T651 aluminum *World Journal of Mechanics* **4** (3) 24-30.
[10] Ismail AE 2009 Mode I stress intensity factors for slanted cracks in round bars *International Review of Mechanical Engineering* **8** (3) 186-266.
[11] Ismail AE, Ariffin AK, Abdullah and Ghazali MJ 2011 Off-set crack propagation analysis under mixed loadings *International Journal of Automotive Technology* **12** (2) 225-232.
[12] Ismail AE, Ariffin AK and Abdullah S 2011 Stress intensity factors for surface cracks in round bar under single and combined loadings *Meccanica* **47** (5) 1141-1156.
[13] Ismail AE, Ariffin AK, Abdullah S, Ghazali MJ, Abdulrazzaq M and Daud, R 2011 Stress intensity factors under combined bending and torsion moments *Journal of Zheijang University-Science A* **13** (1) 1-8.
[14] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2012 Stress intensity factors under combined tension and torsion loadings *Indian Journal of Engineering & Materials Science* **19** (2) 5-16.
[15] Ismail AE, Ariffin AK, Abdullah S, Ghazali MJ and Daud R 2011 Mode III stress intensity factors of surface cracks in round bar *Advanced Materials Research* **214** 192-196.
[16] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2010 Stress intensity factors for surface cracks in round bars under combined bending and torsion loadings *Int. Review of Mechanical Engineering* **4** 827-832.