

# Comparison of Structural Behaviour of Laterally Loaded Pile using Pi Terms and Numerical Simulation

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**Abstract:** A set of dimensionless parameters called Pi terms are derived from a set of variables influencing the behaviour of a structural system to represent the behaviour of the system using the principle of similitude. It can be done using Buckingham's  $\pi$ -theorem or using Rayleigh method. Both the methods derive a set of pi terms which can be used to predict the behaviour of the system under consideration. A structural system of a laterally loaded pile was tested in numerical software, PLAXIS-3D and the results are compared with the predictions using the pi terms. The results are found to be comparable and hence the derived pi terms and the scale factor can be considered represent the laterally loaded pile soil system with a reasonable accuracy.

**Keywords:** pi terms, similitude, laterally loaded pile, PLAXIS 3D

## 1. INTRODUCTION

Any particular phenomenon can be described by a dimensionless group of the principal variables. Similarity between the model and the prototype is ensured when the dimensionless group has the same value in the model and prototype. A set of dimensionless parameters called Pi terms are derived from a set of variables influencing the behaviour of a structural system to represent the behaviour of the system using the principle of similitude. It can be done using Buckingham's  $\pi$ -theorem or using Rayleigh method. Both the methods derive a set of pi terms which can be used to predict the behaviour of the system under consideration.

Dimensional analysis can be done using Buckingham's  $\pi$ -theorem or using Rayleigh method [6]-[7]. Both the methods derive the same relation among the physical quantities. Researchers have adopted this method to predict the behaviour of a prototype from the experimental details available from a similar model test. [1] - [3] In the present study a set of pi terms for the analysis of a laterally loaded pile was derived from the standard methods and are used to predict the lateral deflection of the structural system. The results of prototype analysis are compared with that of a model soil-pile system modeled in PLAXIS-3D [4]-[5] and it was found that the predictions of lateral deflection of prototype pile using pi terms are in good agreement with that of the results of the prototype pile behaviour of laterally loaded soil-pile system obtained from PLAXIS-3D analysis.



## 2.DERIVATION OF PI TERMS

Pi terms are non-dimensional terms derived from the variables influencing the behaviour of a structural system. There are two different methods used to derive the Pi terms. The behaviour of a laterally loaded pile is studied and the corresponding pi terms are derived.

### 2.1 Buckingham's - theorem

The Buckingham's - theorem states that if there are  $n$  dimensional variables involved in a phenomenon which can be completely described by  $m$  fundamental quantities or dimensions (such as mass, length, time etc.) and are related by a dimensionally homogeneous equation then the relationship among  $n$  quantities can always be expressed in terms of exactly  $(n-m)$  dimensionless and independent terms. The method is now illustrated in the present case study.

The physical quantities involved in the study are,  $y$  = pile deflection (L),  $P$  = applied lateral load (F),  $D$  = outside pile diameter (L),  $L$  = pile length (L),  $EI$  = pile stiffness (FL<sup>2</sup>),  $c$  = soil cohesion (F/L<sup>2</sup>),  $h$  = height of applied load from the ground surface (L)

It can be written in general form as,

$$f_1(y, P, D, L, EI, c, h) = C \dots \dots \dots (1)$$

Thus the total number of variables is seven and these variables may be completely described by the two fundamental dimensions F and L in F-L-T system. Since there are seven variables ( $n$ ) and two fundamental dimensions ( $m$ ), there should be five ( $n-m=5$ ) dimensionless terms. Hence,

$$f_2(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = C_1 \dots \dots \dots (2)$$

In order to form these -terms, we have to choose two repeating variables since in this case  $m=2$ . As stated earlier these variables should be such that they, among them, contain all the fundamental dimensions and they themselves do not form a dimensionless parameter. Thus let us choose  $EI$  (FL<sup>2</sup>) and  $D$  (L) as repeating variables, since the above noted requirements are fulfilled by these variables.

Since the physical quantities of dissimilar dimensions can neither be added nor subtracted the terms are expressed as a product as follows.

$$\pi_1 = EI^{a_1} D^{b_1} y \dots \dots \dots (3a)$$

$$\pi_2 = EI^{a_2} D^{b_2} P \dots \dots \dots (3b)$$

$$\pi_3 = EI^{a_3} D^{b_3} L \dots \dots \dots (3c)$$

$$\pi_4 = EI^{a_4} D^{b_4} c \dots \dots \dots (3d)$$

$$\pi_5 = EI^{a_5} D^{b_5} h \dots \dots \dots (3e)$$

Expressing dimensionally in F-L-T system,  $\pi = F^0 L^0$ . Expressing equation numbers (3a) -3(e) in F-L-T system and equating the exponents of F and L of equation and solving,

**Table 1 Derivation of  $\pi$ -terms**

$\pi_1 = E^{-1} L^{b1} y$	$F^0 L^0 = (FL^2)^{a1} L^{b1} L$	$a1 = 0$ $2a1 + b1 + 1 = 0$	$a1 = 0$ $b1 = -1$	$\pi_1 = E^{-1} L^{-1} y$ $\pi_1 = \frac{y}{E}$
$\pi_2 = E^{-1} L^{b2} P$	$F^0 L^0 = (FL^2)^{a2} (L)^{b2} (F)$	$a2 + 1 = 0$ $2a2 + b2 = 0$	$a2 = -1$ $b2 = 2$	$\pi_2 = E^{-1} L^2 P$ $\pi_2 = \frac{P L^2}{E}$
$\pi_3 = E^{-1} L^{b3} L$	$F^0 L^0 = (FL^2)^{a3} (L)^{b3} (L)$	$a3 = 0$ $2a3 + b3 + 1 = 0$	$a3 = 0$ $b3 = -1$	$\pi_3 = E^{-1} L^{-1} L$ $\pi_3 = \frac{y}{E}$
$\pi_4 = E^{-1} L^{b4} c$	$F^0 L^0 = (FL^2)^{a4} (L)^{b4} (\frac{F}{L^2})$	$a4 + 1 = 0$ $2a4 + b4 - 2 = 0$	$a4 = -1$ $b4 = 4$	$\pi_4 = E^{-1} L^4$ $\pi_4 = \frac{c}{E}$
$\pi_5 = E^{-1} L^{b5} h$	$F^0 L^0 = (FL^2)^{a5} (L)^{b5} (L)$	$a5 = 0$ $2a5 + b5 + 1 = 0$	$a5 = 0$ $b5 = -1$	$\pi_5 = E^{-1} L^{-1} h$ $\pi_5 = \frac{h}{E}$

Since,  $f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) = C_1$

Substituting the  $\pi$  terms in the above equation,

$$f_2\left(\frac{y}{D}, \frac{PD^2}{EI}, \frac{L}{D}, \frac{cD^4}{EI}, \frac{h}{D}\right) = C_1$$

OR

$$\frac{y}{D} = f\left(\left(\frac{PD^2}{EI}\right), \left(\frac{L}{D}\right), \left(\frac{cD^4}{EI}\right), \left(\frac{h}{D}\right)\right) \dots \dots \dots (4)$$

## 2.2 Rayleigh Method

It was proposed by Lord Rayleigh in 1899. In this method a functional relationship of some variables is expressed in the form of an exponential equation which must be dimensionally homogeneous. Thus if  $X$  is some function of variables  $x_1, x_2, \dots, x_n$  then the functional equation can be written in the following general form,

$$X = f(x_1, x_2, \dots, x_n) \dots \dots \dots (5)$$

In this equation  $X$  is a dependent variable while  $x_1, x_2, \dots, x_n$  are independent variables. A dependent variable is the one about which information is required while independent variables are those which govern the variation of dependent variables. The above equation can be expressed as,

$$X = c(x_1^a x_2^b x_3^c \dots x_n^n) \dots \dots \dots (6)$$

Where,  $c$  is a dimensionless constant which may be determined from the physical characteristics of the problem or from experimental measurements. The exponents  $a, b, c, \dots, n$  are then evaluated on the basis that the equation is dimensionally homogeneous. The

dimensionless parameters are then formed by grouping together the variables with like powers. The method is now illustrated in the present case study of a laterally loaded pile.

The physical properties involved in the study areas discussed in the above case. It can be expressed in Rayleigh method in exponent form as,

$$y = K(P^a D^b L^c (EI)^d c^e h^f) \dots\dots\dots (7)$$

Where, K is a dimensionless quantity. Substituting the proper dimensions for each variable in the exponential equation in F-L-T system,

$$(L) = F^0 L^0 \left( (F)^a (L)^b (L)^c (FL^2)^d \left( \frac{F}{L^2} \right)^e (L)^f \right) \dots\dots\dots (8)$$

For dimensional homogeneity the exponents of each dimension on the both sides of the equation must be identical. Thus, from equation no. (8)

$$\text{For F: } 0 = a + d + e \dots\dots\dots (9)$$

$$\text{For L: } 1 = b + c + 2d - 2e + f \dots\dots\dots (10)$$

Since there are seven unknown constants and two equations, two of the unknown constants must be expressed in terms of the other five unknown constants. From equation no. (9) and (10),

$$d = -(a + e) \dots\dots\dots (11)$$

$$1 = b + c + 2[-(a + e)] - 2e + f \dots\dots\dots (12)$$

Hence,

$$b = 1 + 2a + c + 4e - f \dots\dots\dots (13)$$

Substituting equation no. (11) and (13) in (7),

$$y = K(P^a D^{(1+2a+c+4e-f)} L^c (EI)^{-(a+e)} c^e h^f)$$

$$y = K \left( D^1 \left( \frac{PD^2}{EI} \right)^a \left( \frac{L}{D} \right)^c \left( \frac{cD^4}{EI} \right)^e \left( \frac{h}{D} \right)^f \right)$$

OR

$$\frac{y}{D} = f \left( \left( \frac{PD^2}{EI} \right), \left( \frac{L}{D} \right), \left( \frac{cD^4}{EI} \right), \left( \frac{h}{D} \right) \right) \dots\dots\dots (14)$$

### 3.SCALE FACTORS FOR THE PRESENT STUDY

Considering the Buckingham's and Rayleigh's methods of model analysis the -terms are,

$$\pi_1 = \frac{y}{D} \dots\dots\dots (15 \text{ a})$$

$$\pi_2 = \frac{PD^2}{EI} \dots\dots\dots (15 \text{ b})$$

$$\pi_3 = \frac{L}{D} \dots\dots\dots (15 \text{ c})$$

$$\pi_4 = \frac{cD^4}{EI} \dots\dots\dots (15 \text{ d})$$

$$\pi_5 = \frac{h}{D} \dots\dots\dots (15 \text{ e})$$

The scale factors ( ) are established by equating the model -terms with the prototype -terms (ie.  $= (L/D)_p / (L/D)_m$ ). Since cohesion is not a function of soil stress, it need not be scaled and hence the test soil is same as the prototype soil.

For the present study model piles are selected with two different diameters (19 mm , 25.4 mm) and three different lengths (600 mm, 700 mm, 800 mm). The -terms will be same for both model and prototype. Considering equation no. (15c),

$$\left(\frac{L}{D}\right)^m = \left(\frac{L}{D}\right)^p \dots\dots\dots (15 \text{ f})$$

Where,  $\left(\frac{L}{D}\right)$  is the length to diameter ratio of model and  $\left(\frac{L}{D}\right)^p$  is the length to diameter ratio of prototype. Assuming a prototype pile diameter of 1000 mm and a concrete mix of M30, the following dimensions of prototype pile can be modeled using the model analysis of the present study.

$$C \quad L = 600 \text{ m} \quad a \quad D = 19 \text{ m} \quad the \quad \left(\frac{L}{D}\right) = \frac{600}{19} = 31.58, \quad \left(\frac{L}{D}\right) = \left(\frac{L}{D}\right)^p \quad a \quad c, \quad a \quad f \quad p \quad w \quad h \quad d \quad D_p = 1000 \text{ m} \quad ,$$

$$L_p \text{ is } cc \quad a \quad 31580 \text{ m} = 31.58 \text{ m}$$

**Table 2 Prototype Dimensions of Pile Lengths used in the Present Model Tests**

$L_m$ (mm)	$D_m$ (mm)	$\left(\frac{L}{D}\right)^m$	$D_p$ (mm)	$L_p$ (mm) = $\left(\frac{L}{D}\right)^m * D_p$
600	19.00	31.58	1000	31.58e3
700	19.00	36.84	1000	36.84e3
800	19.00	42.10	1000	42.10e3

Keeping the length corresponding to 600 mm, 700 mm and 800 mm as 31.58 m, 36.84 m and 42 m the diameter of the field pile corresponding to 25.4 mm is calculated as 1337 mm.

**Table 3 Prototype Dimensions of Pile Diameters used in the Present Model Tests**

$L_m$ (mm)	$D_m$ (mm)	$L_p$ (mm)	$\left(\frac{L}{D}\right)^m$	$D_p$ (mm) = $L_p / \left(\frac{L}{D}\right)^m$
600	25.40	31.58e3	23.62	1337
700	25.40	36.84e3	27.56	1337
800	25.40	42.10e3	31.50	1337

Similarly, Considering equation no. (15 e),

$$\left(\frac{h}{D}\right)^m = \left(\frac{h}{D}\right)^p \dots\dots\dots(15 g)$$

**Table 4 Prototype Dimensions of Projection of Pile above the Ground used in the Present Model Tests**

$h_m$ (mm)	$D_m$ (mm)	$\left(\frac{h}{D}\right)^m = \left(\frac{h}{D}\right)^p$	$h_p$ (mm)	$D_p$ (mm)
50	19.00	2.63	2.63e3	1000
50	25.40	1.97	2.63e3	1337

Considering equation no. (15 b),

$$\left(\frac{PD^2}{EI}\right)^m = \left(\frac{PD^2}{EI}\right)^p \dots\dots\dots(15 h)$$

**Table 5 Scale factor in the Present Model Tests**

$P_m$ (N)	$D_m$ (mm)	$E_m$ (N/mm <sup>2</sup> )	$I_m$ (mm <sup>4</sup> )	$\left(\frac{PD^2}{E}\right)^m = \left(\frac{PD^2}{E}\right)^p$
10 N	19.00	70e3	2298.21	2.24e-5
150 N	19.00	70e3	2298.21	33.66e-5
10 N	25.40	70e3	5716.54	0.90e-5
150 N	25.40	70e3	5716.54	13.53e-5

$$E_p = E_c = 5000\sqrt{f} \quad (f = 456 \text{ c } ) = 5000\sqrt{30} = 25743 = 26000$$

**Table 6 Prototype Values of Load on Pile used in the Present Model Tests**

$\left(\frac{PD^2}{E}\right)^m = \left(\frac{PD^2}{E}\right)^p$	$D_p$ (mm)	$E_p$ (N/mm <sup>2</sup> )	$I_p$ (mm <sup>4</sup> )	$P_p$ (N)
2.24e-5	1000	26000	4.91e10	28.6e3
33.66e-5	1000	26000	4.91e10	429.7e3
0.90e-5	1337	26000	15.69e10	20.5e3
13.53e-5	1337	26000	15.69e10	308.8e3

By performing a lateral load analyses of the prototype pile and correlating it with the model test, scale factors for the output deflection could be obtained[ i.e.  $\lambda_y = y/D_p / y/D$  ]. In the present study, the prototype analyses is done in PLAXIS-3D and the use of scale factor is verified.

To verify the model analysis three model tests and three prototype tests were done in PLAXIS-3D with the following specifications. Model pile is having 19 mm diameter, 600 mm

length, 50 mm height above the ground level subjected to 50N, 100 N and 200 N lateral loads. The corresponding prototype pile is having 1000 mm diameter, 31500 mm length and 2630 mm height above the ground level subjected to 143 N, 286 N and 572 N lateral loads. Dimensions in model pile are converted to prototype pile dimensions as per the similitude obtained.

The analysis was done in PLAXIS-3D and the scale factor is obtained by keeping the model pile with 100 N as the reference.

$$\text{The scale factor } \lambda_y = \left( \frac{y}{D} \right) / \left( \frac{y}{D} \right) = \left( \frac{6.8}{1} \right) / \left( \frac{3.4}{1} \right) = 0.037 \dots (15 \text{ i})$$

#### 4.COMPARISON OF STRUCTURAL BEHAVIOUR

Laterally loaded model pile and prototype pile were analysed in the numerical software PLAXIS-3D and the lateral deflections are obtained for three different loads as shown in Table 7. The properties of pile and soil are assumed suitably. The lateral deflection corresponding to 100 N for the model pile was kept as the bench mark and the deflections were predicted with the help of the pi terms derived. It was observed that the lateral deflection obtained by the numerical simulation of prototype pile and that derived using the pi terms from the model test are comparable.

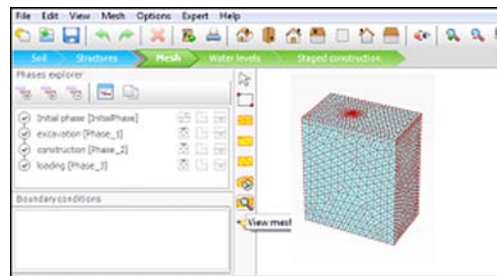


Fig. 1 PLAXIS-3D Model of the Laterally Loaded Soil-Pile System

Table 7 Comparison of Deflection of Prototype Piles used using Model Analysis and Analysis of PLAXIS-3D Model

$y_m$ , mm ( $P_m$ N)	$D_m$ (mm)	$D_p$ (mm)	$y_p$ , mm ( $P_p$ kN) (PLAXIS-3D)	Predicted $y_p$ (mm) (using PI terms)
1.5 (50 N)	19.0	1000	3.6(143 kN)	2.9
3.455 (100 N)	19.0	1000	6.8(286 kN)	-
7.763 (200 N)	19.0	1000	14.4(572 kN)	15.1

#### 5.CONCLUSIONS

A strict parallelism may not be maintained between the model pile and the prototype pile, because of the non-linear behaviour of the soil-pile system. However the model pile test results obtained are converted to prototype pile test results, using the pi theorem and compared with the prototype pile test results. It was observed that both the results are comparable. Hence it may be

concluded that a model investigation, one of the economical and easy methods used in modeling structural problems can be effectively transformed to an actual field problem with the help of a well-defined set of pi terms and the scale factors.

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