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Heat and Mass Transfer on MHD Free convective flow of Second grade fluid through Porous medium over an infinite vertical plate

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Abstract. In this paper, we have considered the unsteady free convective two dimensional flow of a viscous incompressible electrically conducting second grade fluid over an infinite vertical porous plate under the influence of uniform transverse magnetic field with time dependent permeability, oscillatory suction. The governing equations of the flow field are solved by a regular perturbation method for small amplitude of the permeability. The closed form solutions for the velocity, temperature and concentration have been derived analytically and also its behavior is computationally discussed with reference to different flow parameters with the help of profiles. The skin fiction on the boundary, the heat flux in terms of the Nusselt number and rate of mass transfer in terms of Sherwood number are also obtained and their behavior computationally discussed.

Keywords: Heat transfer; mass transfer; oscillatory suction; porous medium; MHD flow; second grade fluids, infinite vertical plates.

1. Introduction

Viscous incompressible fluid flow due to an impulsively started flat plate was examined by Stokes [1]. Rossow [2] examined the flow of a viscous incompressible fluid due to the impulsive motion of an infinite flat plate in the presence of a magnetic field. Sakiadis [3] analyzed analytically and by numerical scheme the boundary layer flow due to a moving flat surface. Laminar compressible boundary layer on a moving flat plate was investigated by Ackroyd [4]. Samuel and Hall [5] obtained the similarity solution for boundary layer flow on a continuous moving porous surface using a series having exponential terms. Sacheti and Bhatt [6], Bhatt and Sacheti [7] investigated Stokes and Rayleigh layers in the presence of a naturally permeable boundary. Hall effects on MHD flows over an accelerated/continuous moving plate are examined by Pop [8], Watanabe and Pop [9], Kiyanjui et al. [10]. Free convection effects on the elastico-viscous fluid flow over an accelerated plate were examined by Singh et al. [11]. Boundary layer flows in a rotating fluid system are important due to various applications in science and technology. Debnath [12] presented exact solutions of the hydrodynamic and hydromagnetic boundary layer equations in such systems. Takhar and Nath [13], Takhar et al. [14][15] investigated MHD flow over a stretching surface or moving plate in a rotating fluid. Deka et al. [16] investigated flow over an accelerated plate in a rotating system and Deka [17] examined the Hall effects in such flow in the presence of a magnetic field. Hydromagnetic channel flows in a rotating fluid system are investigated by researchers, e.g. Mandal and Mandal [18], Singh et al. [19], Singh [20], Ghosh [21], Ghosh et al. [22], Seth et al. [23], Guria et al.[24]. In some astronomical and geophysical problems and in many engineering applications the study of Coriolis force interaction with electromagnetic force in porous media is important. Hall effect is also important when the fluid is an ionized gas with low density or the applied magnetic field is very strong. Because the electrical conductivity of the fluid will then be a tensor and a current (Hall current) is induced which is likely to be important in many engineering situations (Sutton and Sherman [25]). Hydromagnetic convection flow in a rotating porous medium or in a channel partially filled by a porous medium with Hall effects are investigated by researchers such as Krishna et al. [26], Chauhan and Rastogi [27], Beg et al. [28], Chauhan and Agrawal [29 - 30]. Dileep.D.C. [31] discussed the unsteady MHD flow of viscous incompressible and electrically conducting fluid through a porous medium adjacent to an accelerated impermeable plate in a rotating system taking Hall current into account, Heat transfer is also determined. Recently Veera Krishna and Prakash [32] discussed the unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field taking hall current into account.

In view of the above studies, in this paper, we have considered the unsteady free convective two dimensional flow of a viscous incompressible electrically conducting second grade fluid over an infinite vertical porous plate under the influence of uniform transverse magnetic field with time dependent permeability, oscillatory suction taking into account.

2. Formulation and Solution of the Problem:

We considered the unsteady MHD free convection two dimensional flow of an incompressible viscous electrically conducting second grade fluid with simultaneous heat and mass transfer over an infinite vertical plate through porous medium with time dependent permeability and oscillatory suction under the influence of uniform transverse magnetic field of strength H_0. The z-axis is taken along the plate and x-axis perpendicular to it and u and w are the velocity components along the x-direction and z-directions respectively. The physical configuration of the problem is as shown in Fig. 1.

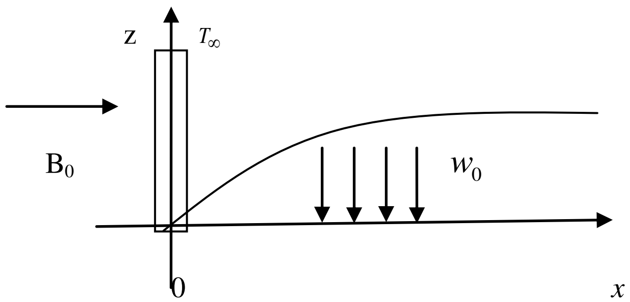

Fig. 1 Physical configuration of the problem

The basic assumptions are made as following.

1. All fluid proportions are constant.

2. The plate as well as the fluid is assumed to be at the same temperature and the concentration of species is raised or lowered.
3. The magnetic Reynolds number is small so that the induced magnetic field can be neglected in comparison to the applied magnetic field.
4. The permeability of the porous medium is assumed to be
$$K(t) = k(1+\varepsilon e^{i\omega t}) \tag{1}$$
5. The suction velocity is assumed to be
$$w(t) = -w_0(1+\varepsilon e^{i\omega t}) \tag{2}$$
Where, w_0 represents the suction or injection velocity at the plate.
6. The pressure is assumed to be constant.
7. If the plate is extended to infinite length, then all the physical variables are functions of z and t alone.

The governing equations for the unsteady MHD free convection flow of an incompressible viscous electrically conducting fluid with simultaneous heat and mass transfer over an infinite vertical plate through porous medium under the influence of uniform transverse magnetic field with respect to the frame are given by

Equation of continuity:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$
Momentum equation:
$$\frac{\partial u}{\partial t} - w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma\mu_e^{\,2} H_0^2}{\rho}u - \frac{\nu}{K(t)}u + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) \tag{4}$$
$$\frac{\partial v}{\partial t} - w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma\mu_e^{\,2} H_0^2}{\rho}v - \frac{\nu}{K(t)}v \tag{5}$$

Equation of energy:
$$\frac{\partial T}{\partial t} - w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} - S_1(T-T_\infty) \tag{6}$$
Equation of concentration
$$\frac{\partial C}{\partial t} - w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - K_1(C-C_\infty) \tag{7}$$
Combining equations (4) and (5), Let $q = u + iv$
$$\frac{\partial q}{\partial t} - w\frac{\partial q}{\partial z} = \nu\frac{\partial^2 q}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma\mu_e^{\,2} H_0^2}{\rho}q - \frac{\nu}{K(t)}q + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) \tag{8}$$

The boundary conditions are
$$\begin{aligned} q(z,t) = T(z,t) = C(z,t) &= f(t) \quad \text{at} \quad z = 0 \\ &= 0 \quad \text{at} \quad z \to \infty \end{aligned} \tag{9}$$

Where, $f(t) = 1 + \varepsilon e^{i\omega t}$ With foregoing assumptions and taking usual Boussinesq's approximation into account as well as the following non-dimensional variables.

$$q^* = \frac{q}{w_0}, z = \frac{w_0 z^*}{v}, t^* = \frac{w_0^2 t}{v}, \omega^* = \frac{v\omega}{w_0{}^2}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}$$

Making use of non-dimensional variables, the governing equations reduces to (Dropping asterisks)

$$\frac{\partial q}{\partial t} - w_o(1+\varepsilon e^{i\omega t})\frac{\partial q}{\partial z} = \frac{\partial^2 q}{\partial z^2} + \alpha\frac{\partial^3 q}{\partial z^2 \partial t} - \left(\frac{M^2}{1+m^2} + \frac{1}{K(1+\varepsilon e^{i\omega t})}\right)q + \mathrm{Gr}\,\theta + \mathrm{Gm}\,C \quad (10)$$

$$\mathrm{Pr}\frac{\partial \theta}{\partial t} - w_0(1+\varepsilon e^{i\omega t})\mathrm{Pr}\frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} - \mathrm{Pr}\, S\,\theta \quad (11)$$

$$\mathrm{Sc}\frac{\partial \phi}{\partial t} - w_0(1+\varepsilon e^{i\omega t})\mathrm{Sc}\frac{\partial \phi}{\partial z} = \frac{\partial^2 \phi}{\partial z^2} - \mathrm{KcSc}\phi, \quad (12)$$

The corresponding boundary conditions are

$$\begin{aligned} q(z,t) = \theta(z,t) = \phi(z,t) &= 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad z = 0 \\ &= 0 \quad \text{at} \quad z \to \infty \end{aligned} \quad (13)$$

Where, $M^2 = \frac{\sigma B_0^2 v}{\rho w_0^2}$ is the Hartmann number (Magnetic field parameter), $K = \frac{v^2}{k w_0^2}$ is the permeability parameter (Porosity or Darcy parameter), $\alpha = \frac{\alpha_1 w_0^2}{\rho v^2}$ is the second grade fluid parameter, $\mathrm{Pr} = \frac{v}{\alpha}$ is the Prandtl number, $\mathrm{Sc} = \frac{v}{D}$ is the Schmidt number, $\mathrm{Kc} = \frac{K_1 v}{w_0^2}$ is the chemical reaction parameter, $S = \frac{S_1 v}{w_0^2}$ is the Heat Source parameter, $\mathrm{Gr} = \frac{g\beta v(T_w - T_\infty)}{w_0^3}$ is the thermal Grashof number, $m = \tau_e \omega_e$ is Hall parameter and $\mathrm{Gm} = \frac{g\beta^* v(C_w - C_\infty)}{w_0^3}$ is the mass Grashof number.

In order to solve the equations, (10) – (12) using boundary conditions (13), we assume the solutions of the following form, because the amplitude $\varepsilon(<<1)$ of permeability is very small.

$$q(z,t) = q_0(z) + \varepsilon\, q_1(z) e^{i\omega t} \quad (14)$$

$$\theta(z,t) = \theta_0(z) + \varepsilon\, \theta_1(z) e^{i\omega t} \quad (15)$$

$$\phi(z,t) = \phi_0(z) + \varepsilon\, \phi_1(z) e^{i\omega t} \quad (16)$$

Substituting the Eqs. (14) – (16) into the Eqs. (10) – (12) respectively and equate the harmonic and non-hormonic terms to obtain the zeroth and first orders ordinary differential equations for momentum, temperature and concentration distributions.

Zeroth order:

$$\frac{\partial^2 \phi_0}{\partial z^2} + \mathrm{Sc} w_0 \frac{\partial \phi_0}{\partial z} - \mathrm{KcSc}\phi_0 = 0 \quad (17)$$

$$\frac{\partial^2 \theta_0}{\partial z^2} + Pr\, w_0 \frac{\partial \theta_0}{\partial z} - \mathrm{Pr} S\theta_0 = 0 \quad (18)$$

$$\frac{\partial^2 q_0}{\partial z^2} + w_0 \frac{\partial q_0}{\partial z} - \left(M^2 + \frac{1}{K}\right) q_0 = -\mathrm{Gr}\theta_0 - \mathrm{Gm}\phi_0 \quad (19)$$

Corresponding boundary conditions are

$$\phi_0 = 1, \theta_0 = 1, q_0 = 1 \quad \text{at} \quad z = 0 \tag{20}$$

$$\phi_0 = 0, \theta_0 = 0, q_0 = 0 \quad \text{at} \quad z \to \infty \tag{21}$$

Solving the equations (17) – (19) with relevant boundary conditions (20) and (21), we obtained the zeroth order concentration, temperature and velocity.

First order:

$$\frac{\partial^2 \phi_1}{\partial z^2} + \mathrm{Sc} w_0 \frac{\partial \phi_1}{\partial z} - \mathrm{Sc}(\mathrm{Kc} + i\omega)\phi_1 = -\mathrm{Sc} w_0 \frac{\partial \phi_0}{\partial z} \tag{22}$$

$$\frac{\partial^2 \theta_1}{\partial z^2} + Pr\, w_0 \frac{\partial \theta_1}{\partial z} - \mathrm{Pr}(S + i\omega)\theta_1 = -Pr\, w_0 \frac{\partial \theta_0}{\partial z} \tag{23}$$

$$(1 + \alpha i\omega)\frac{\partial^2 q_1}{\partial z^2} + w_0 \frac{\partial q_1}{\partial z} - \left(M^2 + \frac{1}{K} + i\omega \right) q_1 = -w_0 \frac{\partial q_0}{\partial z} - \mathrm{Gr}\,\theta_1 - \mathrm{Gm}\,\phi_1 \tag{24}$$

Corresponding boundary conditions are

$$\phi_1 = 1, \theta_1 = 1, q_1 = 1 \quad \text{at} \quad z = 0 \tag{25}$$

$$\phi_1 = 0, \theta_1 = 0, q_1 = 0 \quad \text{at} \quad z \to \infty \tag{26}$$

Solving the equations (22) – (24) with relevant boundary conditions (25) and (26), we obtained the first order concentration, temperature and velocity.

$$\phi_0 = e^{-m_1 z} \tag{27}$$

$$\theta_0 = e^{-m_3 z} \tag{28}$$

$$q_0 = A_1 e^{-m_5 z} - \frac{\mathrm{Gr}}{A_2} e^{-m_3 z} - \frac{\mathrm{Gm}}{A_3} e^{-m_1 z} \tag{29}$$

$$\phi_1 = \left(1 - \frac{\mathrm{Sc}\, w_0\, m_1}{m_1^2 + \mathrm{Sc}\, w_0\, m_1 - \mathrm{Sc}(\mathrm{Kc} + i\omega)} \right) e^{-m_2 z} + \frac{\mathrm{Sc}\, w_0\, m_1\; e^{-m_1 z}}{m_1^2 + \mathrm{Sc}\, w_0\, m_1 - \mathrm{Sc}(\mathrm{Kc} + i\omega)} \tag{30}$$

$$\theta_1 = \left(1 - \frac{\mathrm{Pr}\, w_0\, m_3}{m_3^2 + \mathrm{Pr}\, w_0\, m_3 - \mathrm{Pr}(S + i\omega)} \right) e^{-m_4 z} + \frac{\mathrm{Pr}\, w_0\, m_3\; e^{-m_3 z}}{m_3^2 + \mathrm{Pr}\, w_0\, m_3 - \mathrm{Pr}(S + i\omega)} \tag{31}$$

$$q_1 = (1 - B_1 + \mathrm{Gr} B_2 + \mathrm{Gm} B_3) e^{-m_6 z} + \frac{A_8}{A_{11}} e^{-m_5 z} + \frac{A_9}{A_{12}} e^{-m_3 z} + \frac{A_{10}}{A_{13}} e^{-m_1 z}$$
$$- \mathrm{Gr}\left(\frac{A_4}{A_{14}} e^{-m_4 z} + \frac{A_5}{A_{12}} e^{-m_3 z} \right) - \mathrm{Gm}\left(\frac{A_6}{A_{15}} e^{-m_2 z} + \frac{A_7}{A_{13}} e^{-m_1 z} \right) \tag{32}$$

The skin friction at the plate in terms of amplitude and phase is given by

$$\tau = \left(\frac{\partial q}{\partial z} \right)_{z=0} = \left(\frac{\partial q_0}{\partial z} \right)_{z=0} + \varepsilon \left(\frac{\partial q_1}{\partial z} \right)_{z=0} = F_1 + \varepsilon |F_2| \cos(\omega t + \psi)$$

Where, $F_1 = \left(\frac{\partial q_0}{\partial z} \right)_{z=0}$; $F_2 = \left(\frac{\partial q_1}{\partial z} \right)_{z=0}$ and $\tan(\psi) = \frac{\mathrm{Re}[F_2]}{\mathrm{Im}[F_2]}$.

The Nusselt number at the plate in terms of amplitude and phase is given by

$$Nu = -\left(\frac{\partial \theta}{\partial z} \right)_{z=0} = \left(\frac{\partial \theta_0}{\partial z} \right)_{z=0} + \varepsilon \left(\frac{\partial \theta_1}{\partial z} \right)_{z=0} = F_3 + \varepsilon |F_4| \cos(\omega t + \gamma)$$

Where, $F_3 = \left(\frac{\partial \theta_0}{\partial z} \right)_{z=0}$; $F_4 = \left(\frac{\partial \theta_1}{\partial z} \right)_{z=0}$ and $\tan(\gamma) = \frac{\mathrm{Re}[F_4]}{\mathrm{Im}[F_4]}$.

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The Sherwood number at the plate in terms of amplitude and phase is given by

$$Sh = -\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = \left(\frac{\partial \phi_0}{\partial z}\right)_{z=0} + \varepsilon\left(\frac{\partial \phi_1}{\partial z}\right)_{z=0} = F_5 + \varepsilon\left|F_6\right|\cos(\omega t + \zeta)$$

Where, $F_5 = \left(\frac{\partial \phi_0}{\partial z}\right)_{z=0}$; $F_6 = \left(\frac{\partial \phi_1}{\partial z}\right)_{z=0}$ and $\tan(\zeta) = \frac{\mathrm{Re}[F_6]}{\mathrm{Im}[F_6]}$.

3. Results and Discussion

We have considered the unsteady free convective flow of a viscous incompressible electrically conducting second grade fluid over an infinite vertical porous plate under the influence of uniform transverse magnate field with time dependent permeability and oscillatory suction. The governing equations of the flow field are solved by a regular perturbation method for small amplitude of the permeability. The closed form solutions for the velocity, temperature and concentration have been derived analytically and also its behavior is computationally discussed with reference to different flow parameters like M Hartmann number, K porosity parameter Pr is the Prandtl number, α the second grade fluid parameter, Sc is the Schmidt number, Kc is the chemical reaction parameter, S is the Heat Source parameter, Gr is the thermal Grashof number, Gm is the mass Grashof number, w_0 is the suction velocity and the frequency of oscillation ω. Figures (2-23) represent velocity, Figures (24-27) and Figures (28-31) represent the temperature and concentration distributions respectively. The stresses, Nusselt number and Sherwood number at the plate are evaluated numerically and discussed with governing parameters and are tabulated in the tables (1-3). Fixing the parameters M=2, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi/6$, we draw the profiles varying for each parameter while the other parameters being fixed.

From the Figures (2-13), we noticed that the magnitude of the velocity component *u* reduces with increasing the intensity of the magnetic field M, Prandtl number Pr and Heat source parameter *S*. Whereas the velocity component *u* enhance with increasing permeability parameter K, thermal Grashof number Gr or mass Grashof number Gm throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region. The magnitude of the velocity component *v* is reduces with M and K but it initially reduces and then gradually enhances with Pr and S. The reversal behaviour is observed with increasing Gr and Gm.

The Figures (14-21) depict the velocity component *u* experiences retardation in the flow field with increasing the chemical reaction parameter Kc, Schmidt number Sc, the suction velocity w_0 or the frequency of oscillation entire the fluid region. The magnitude of the velocity component *v* is reduces with Kc, Sc, w_0 but for the frequency of oscillation it enhances throughout the fluid region. We notice that the magnitude of the velocity component *u* increases and *v* enhance and then gradually retards throughout the fluid region with increasing the second grade fluid parameter α (Fig. 22-23).

Figures (24-27) showed the effect of Heat source parameter *S*, the Prandtl number Pr, suction velocity v_0 and the frequency of oscillation ω on the temperature of the flow field. We noted that the temperature of the flow field diminishes as the Prandtl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number. With increasing heat source parameter reduces the temperature of the flow field. This may happen due the elastic property of the fluid. It is observed that temperature of the flow field diminishes as the suction parameter or the frequency of oscillation increases.

Figures (28-31) depict the effect of the Schmidt number Sc and the frequency of oscillation ω on concentration distribution. The concentration distribution decreases at all points of the flow field with the increase in the Schmidt number Sc or chemical reaction parameter Kc. This shows that the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field. Also, it is observed that presence of the frequency of oscillation ω or increasing the suction velocity reduces the concentration distribution.

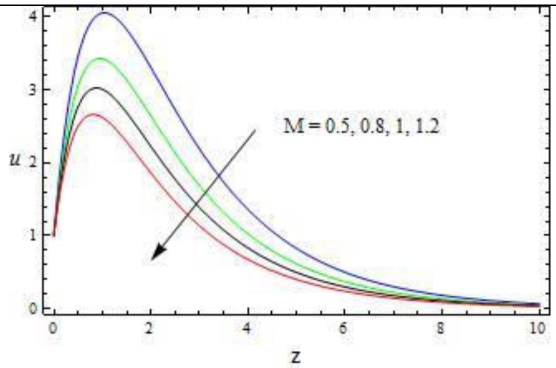

Fig. 2. The velocity Profiles for u against M with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi/6$

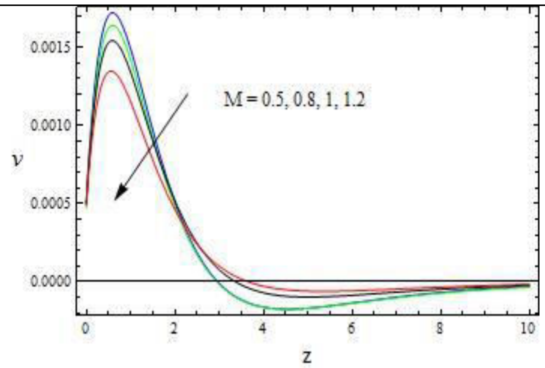

Fig. 3. The velocity Profiles for v against M with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi/6$

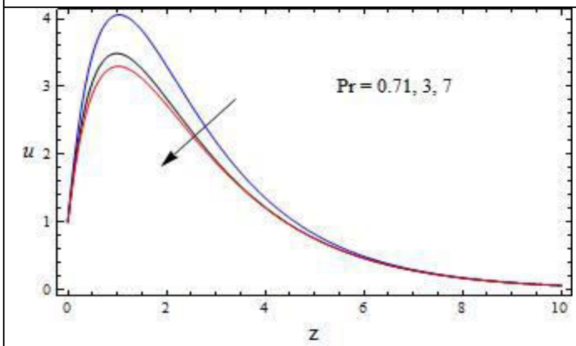

Fig. 4. The velocity Profiles for u against Pr with M=2, α =1, K=1, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2and $\omega = \pi/6$

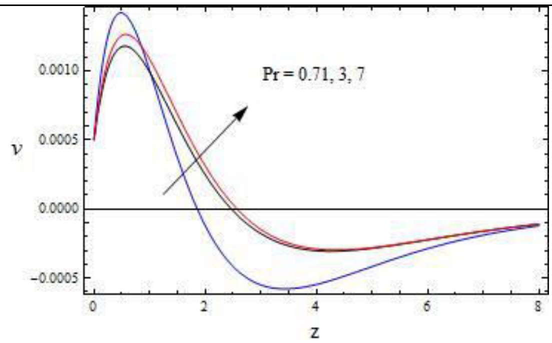

Fig. 5. The velocity Profiles for v against Pr with M=2, α =1, K=1, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2and $\omega = \pi/6$

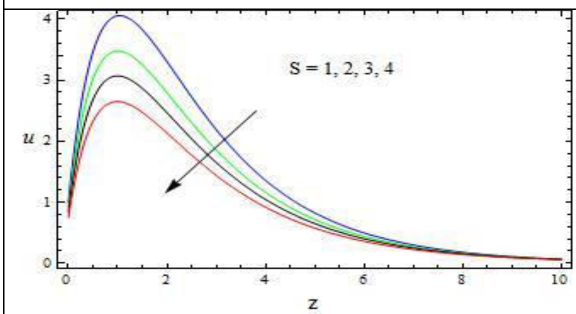

Fig. 6. The velocity Profiles for u against S with M=2, α =1, K=1, Pr=0.71, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi/6$

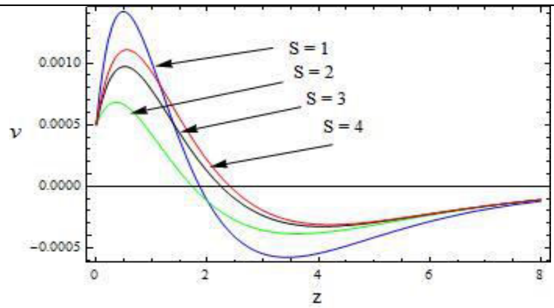

Fig. 7. The velocity Profiles for v against S with M=2, α =1, K=1, Pr=0.71, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi/6$

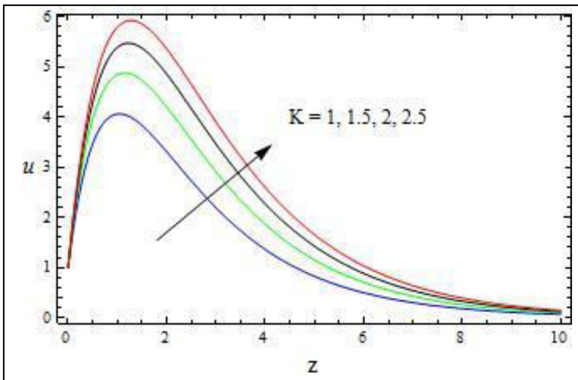

Fig. 8. The velocity Profiles for *u* against *K* with
M=2, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

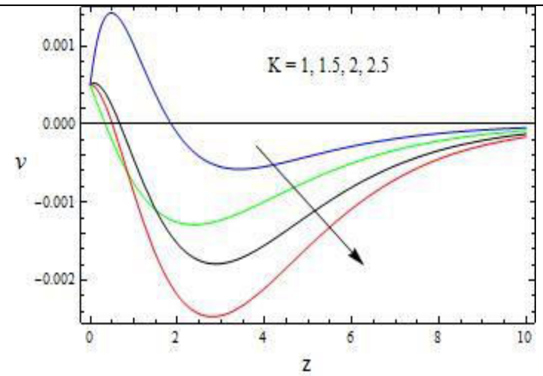

Fig. 9. The velocity Profiles for *v* against *K* with
M=2, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

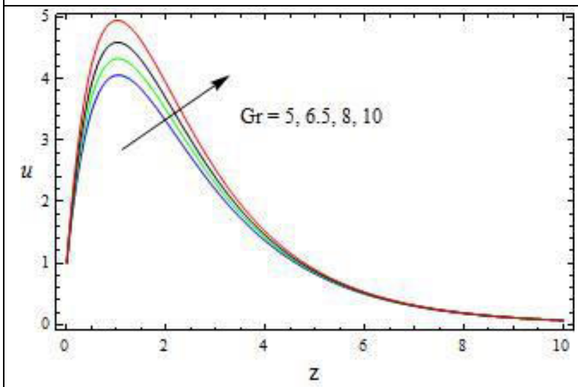

Fig. 10. The velocity Profiles for *u* against Gr with
M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gm=10, w_0=0.2 and $\omega = \pi / 6$

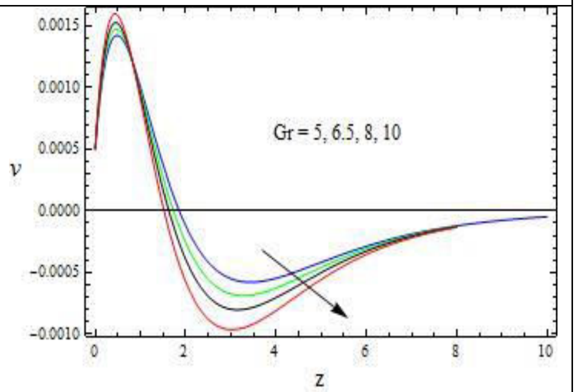

Fig. 11. The velocity Profiles for *v* against Gr with
M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gm=10, w_0=0.2 and $\omega = \pi / 6$

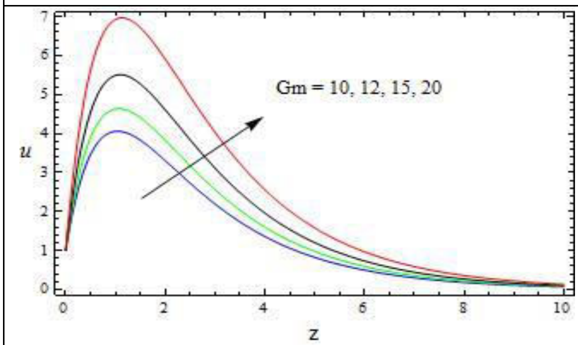

Fig. 12. The velocity Profiles for *u* against Gm with
M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, w_0=0.2 and $\omega = \pi / 6$

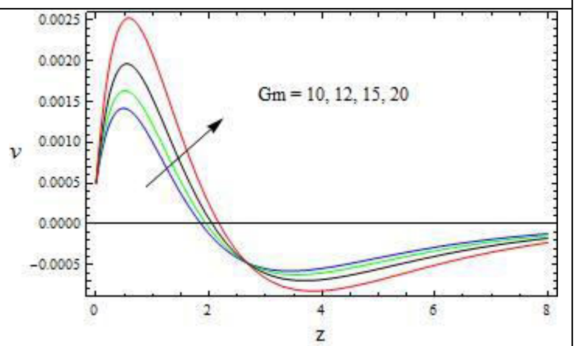

Fig. 13. The velocity Profiles for *v* against Gm with
M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, w_0=0.2 and $\omega = \pi / 6$

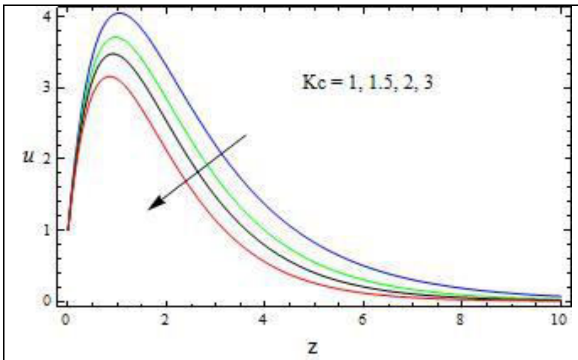

Fig. 14. The velocity Profiles for u against Kc with M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

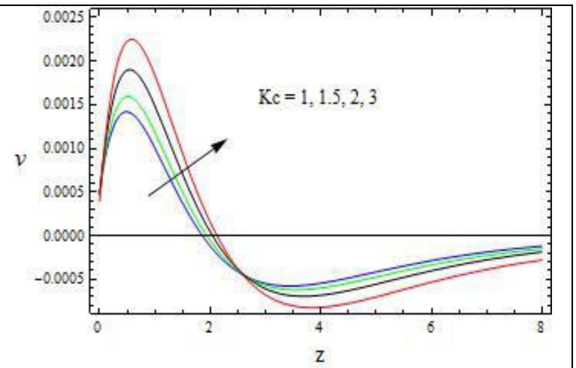

Fig. 15. The velocity Profiles for v against Kc with M=2, α =1, K=1, Pr=0.71, S=1, Sc=0.22, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

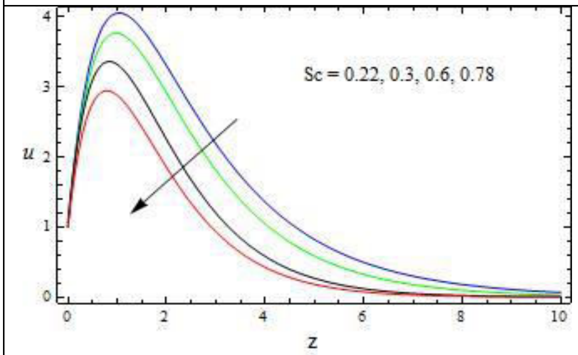

Fig. 16. The velocity Profiles for u against Sc with M=2, α =1, K=1, Pr=0.71, S=1, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

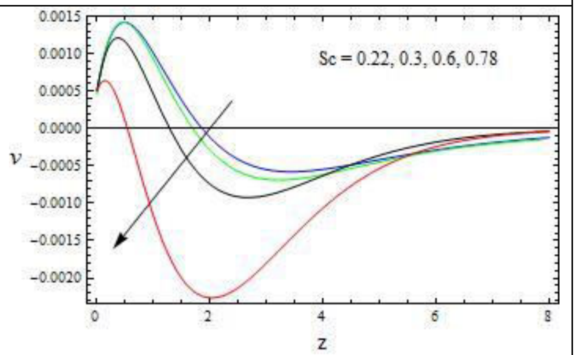

Fig. 17. The velocity Profiles for v against Sc with M=2, α =1, K=1, Pr=0.71, S=1, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

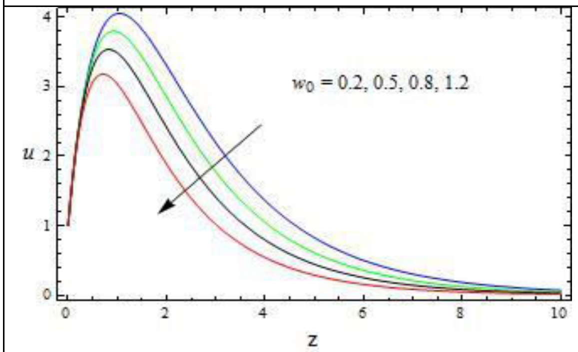

Fig. 18. The velocity Profiles for u against w_0 with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, M=2 and $\omega = \pi / 6$

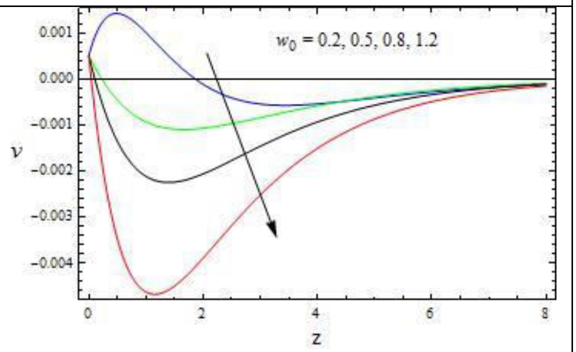

Fig. 19. The velocity Profiles for v against w_0 with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, M=2 and $\omega = \pi / 6$

IOP Conf. Series: Materials Science and Engineering **225** (2017) 012267 doi:10.1088/1757-899X/225/1/012267

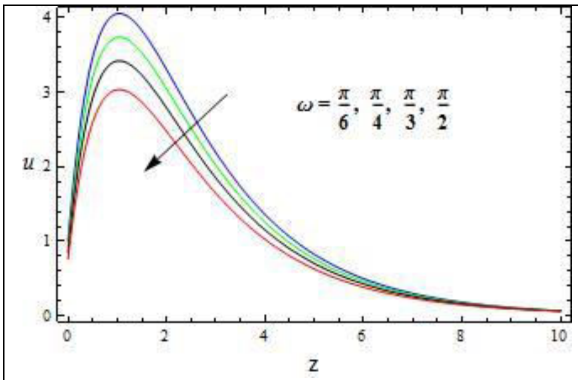

Fig. 20. The velocity Profiles for u against ω with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and M=2

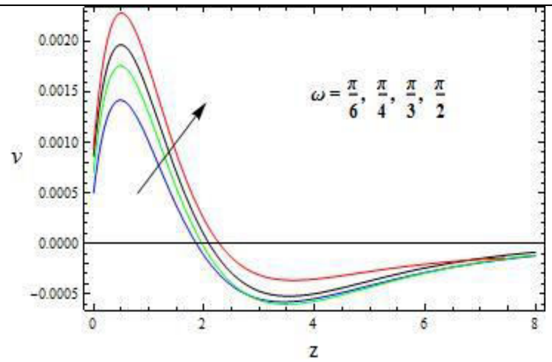

Fig. 21. The velocity Profiles for v against ω with K=1, α =1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and M=2

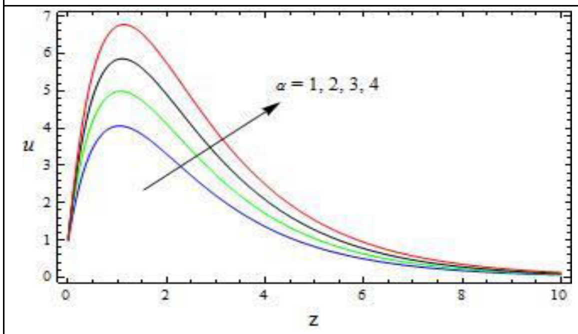

Fig. 22. The velocity Profiles for u against α with M=2, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

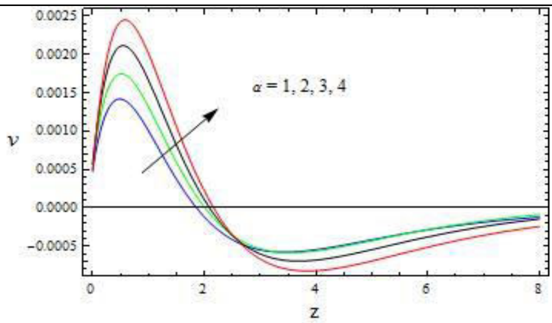

Fig. 23. The velocity Profiles for v against α with M=2, K=1, Pr=0.71, S=1, Sc=0.22, Kc=1, Gr=5, Gm=10, w_0=0.2 and $\omega = \pi / 6$

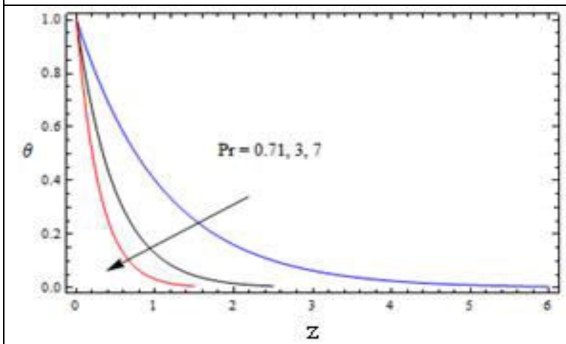

Fig 24: The temperature profile against Pr with $\varepsilon = 0.001,\ t = 0.2,\ S = 1, \omega = \pi / 6, w_0 = 0.2$

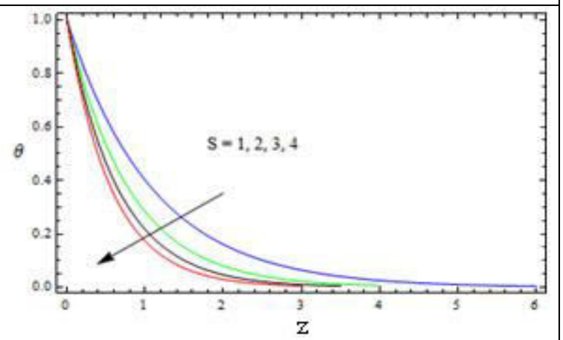

Fig 25: The temperature profile against S with $\varepsilon = 0.001,\ t = 0.2, \omega = \pi / 6, w_0 = 0.2, \Pr = 0.71$

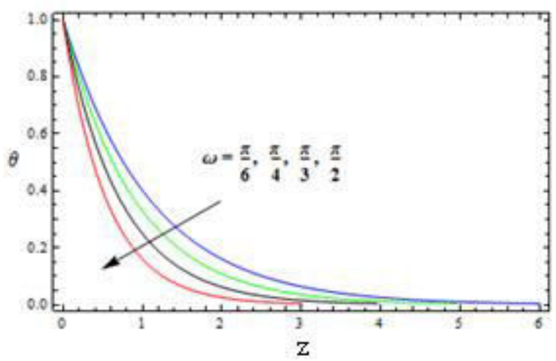

Fig 26: The temperature profile against ω with $\varepsilon = 0.001,\ t = 0.2,\ S = 1, w_0 = 0.2,\ \Pr = 0.71$

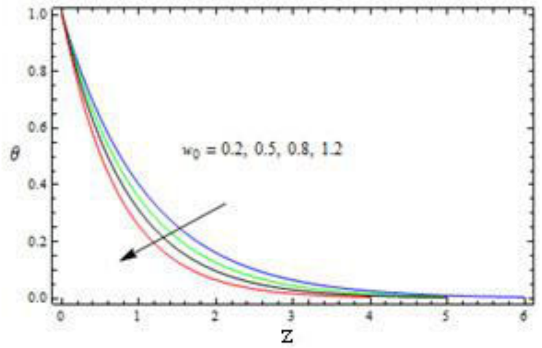

Fig 27: The temperature profiles against with $\varepsilon = 0.001,\ t = 0.2,\ S = 1, \omega = \pi / 6,\ \Pr = 0.71$

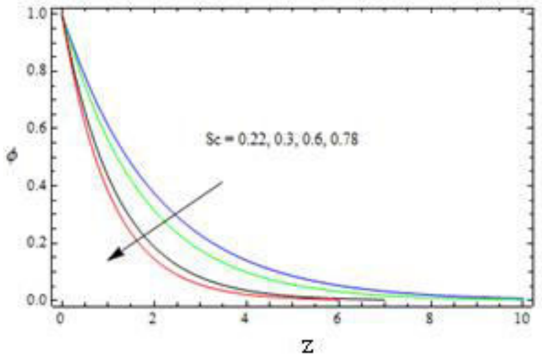

Fig 28: The Concentration profile against Sc with $\varepsilon = 0.001,\ t = 0.2, \omega = \pi / 6,\ \mathrm{Kc} = 1, w_0 = 0.2$

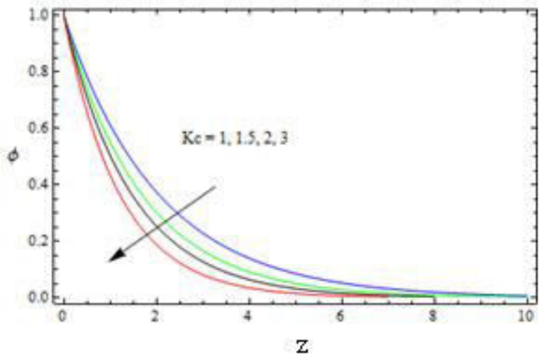

Fig 29: The Concentration profiles against Kc with $\varepsilon = 0.001,\ t = 0.2, \omega = \pi / 6,\ w_0 = 0.2, \mathrm{Sc} = 0.22$

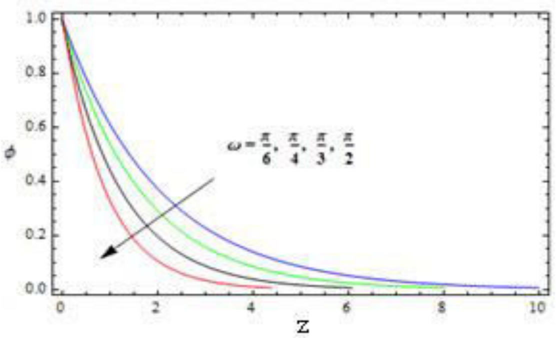

Fig 30: The Concentration profile against ω with $\varepsilon = 0.001,\ t = 0.2,\ \mathrm{Kc} = 1, w_0 = 0.2, \mathrm{Sc} = 0.22$

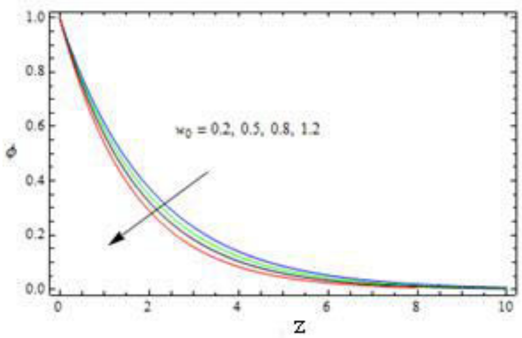

Fig 31: The Concentration profiles against w_0 with $\varepsilon = 0.001,\ t = 0.2, \omega = \pi / 6,\ \mathrm{Kc} = 1, \mathrm{Sc} = 0.22$

The skin friction, Nusselt number and Sherwood number are calculated numerically and tabulated in the tables (1-3). The skin friction increases with the increase in *K* Gr, Gm, α and w_0 and decreases with the increase in *M*, Pr, Sc, Kc, *S* and ω (Table. 1). Nusselt number (Nu) at the surface of the plate and amplitude increase with increase *S*, Pr and w_0. Also enhance the amplitude, the rate of heat transfer decrease the phase angle with increase the frequency of oscillations ω (Table. 2). Schmidt number Sc, chemical reaction parameter Kc, the frequency of oscillations ω and suction parameter w_0 increase the amplitude, the phase angle the rate of mass transfer at the surface of the plate (Table. 3).

Table 1. Shear stresses with $\varepsilon = 0.001,\, t = 0.2$

M	K	Pr	Sc	Kc	S	Gr	Gm	w_0	ω	α	Amplitude $\lvert F_2 \rvert$	Phase Angle (ψ)	τ
0.5	**2**	**0.71**	**0.22**	**1**	**1**	**5**	**10**	**0.2**	$\pi/6$	1	11.7485	-1.08748	9.94785
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/6$	1	4.25447	-1.55825	3.44514
3	2	0.71	0.22	1	1	5	10	0.2	$\pi/6$	1	1.28477	-1.41963	0.97856
2	**3**	0.71	0.22	1	1	5	10	0.2	$\pi/6$	1	12.9475	-1.00415	10.8452
2	**4**	0.71	0.22	1	1	5	10	0.2	$\pi/6$	1	13.1854	-0.92652	11.5958
2	2	**3**	0.22	1	1	5	10	0.2	$\pi/6$	1	10.9414	-1.24525	8.74785
2	2	**7**	0.22	1	1	5	10	0.2	$\pi/6$	1	10.8955	-1.30859	8.13748
2	2	0.71	**0.3**	1	1	5	10	0.2	$\pi/6$	1	13.2411	-1.01985	9.40855
2	2	0.71	**0.6**	1	1	5	10	0.2	$\pi/6$	1	13.5784	-0.09415	8.21658
2	2	0.71	0.22	**1.5**	1	5	10	0.2	$\pi/6$	1	13.0471	-0.99958	9.30452
2	2	0.71	0.22	**2**	1	5	10	0.2	$\pi/6$	1	14.0748	-0.84774	8.86748
2	2	0.71	0.22	1	**2**	5	10	0.2	$\pi/6$	1	11.6411	-1.13896	9.61744
2	2	0.71	0.22	1	**3**	5	10	0.2	$\pi/6$	1	11.3477	-1.18785	9.40965
2	2	0.71	0.22	1	1	**8**	10	0.2	$\pi/6$	1	12.8411	-0.98965	11.6632
2	2	0.71	0.22	1	1	**10**	10	0.2	$\pi/6$	1	13.7744	-0.92748	12.8784
2	2	0.71	0.22	1	1	5	**15**	0.2	$\pi/6$	1	16.7895	-1.17858	13.8763
2	2	0.71	0.22	1	1	5	**20**	0.2	$\pi/6$	1	21.9477	-1.22748	17.8566
2	2	0.71	0.22	1	1	5	10	**0.5**	$\pi/6$	1	23.0841	-0.60881	10.0366
2	2	0.71	0.22	1	1	5	10	**0.8**	$\pi/6$	1	33.0974	-0.02078	10.5748
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/4$	1	8.03784	-0.88785	9.91520
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/3$	1	6.43295	-0.63128	9.91152
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/2$	1	5.48012	-0.27362	9.91012
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/6$	2	6.25647	-1.00258	10.2241
2	2	0.71	0.22	1	1	5	10	0.2	$\pi/6$	3	3.22274	-0.22963	15.2601

Table 2. Nusselt number (Nu) with $\varepsilon = 0.001$, $t = 0.2$

Pr	S	w_0	ω	Amplitude $\lvert F_4 \rvert$	Phase Angle γ	Nu
0.71	**1**	**0.2**	$\pi/6$	1.00365	1.39869	0.91666
3	1	0.2	$\pi/6$	2.20629	1.44928	2.05788
7	1	0.2	$\pi/6$	3.60564	1.47239	3.43676
0.71	**2**	0.2	$\pi/6$	1.31588	1.48355	1.26473
0.71	**3**	0.2	$\pi/6$	1.56988	1.51423	1.53210
0.71	1	**0.5**	$\pi/6$	1.10256	1.46078	1.03861
0.71	1	**0.8**	$\pi/6$	1.21656	1.48335	1.17317
0.71	1	0.2	$\pi/4$	1.06122	1.31218	0.91670
0.71	1	0.2	$\pi/3$	1.12272	1.24229	0.91673

Table 3. Sherwood number (Sh) with $\varepsilon = 0.001$, $t = 0.2$

Sc	Kc	w_0	ω	Amplitude $\lvert F_6 \rvert$	Phase Angle (ζ)	Sh
0.22	**1**	**0.2**	$\pi/6$	0.536249	1.365880	0.491611
0.3	1	0.2	$\pi/6$	0.632410	1.373015	0.578602
0.6	1	0.2	$\pi/6$	0.916662	1.393062	0.836984
0.22	**1.5**	0.2	$\pi/6$	0.629483	1.429490	0.596922
0.22	**2**	0.2	$\pi/6$	0.712364	1.464392	0.685691
0.22	1	**0.5**	$\pi/6$	0.574058	1.422405	0.527289
0.22	1	**0.8**	$\pi/6$	0.603609	1.454160	0.565233
0.22	1	0.2	$\pi/4$	0.566008	1.276890	0.491634
0.22	1	0.2	$\pi/3$	0.599008	1.206869	0.491650

Table 4. Comparison of Results

Sc	w_0	ω	Sh	
			Ashraf et al.[33]	Present work [Kc=0]
0.22	**0.2**	$\pi/6$	0.045262	0.044020
0.3	0.2	$\pi/6$	0.061854	0.060238
0.6	0.2	$\pi/6$	0.120552	0.120325
0.22	**0.5**	$\pi/6$	0.115245	0.110179
0.22	**0.8**	$\pi/6$	0.176334	0.176133
0.22	0.2	$\pi/4$	0.046625	0.044236
0.22	0.2	$\pi/3$	0.046246	0.044253

4. Conclusions

The unsteady free convective flow of a viscous incompressible electrically conducting fluid over an infinite vertical porous plate under the influence of uniform transverse magnetic field with time dependent permeability and oscillatory suction has been discussed. The results are very good agreement with Ashaf [33] and shown in Table. 4. The conclusions are made as the following.

1. The velocity reduces with increasing the intensity of the magnetic field or Prandtl number Pr or Heat source parameter *S*.
2. The velocity enhance with increasing thermal Grashof number Gr or mass Grahof number or second grade fluid parameter.

3. The resultant velocity enhances with increasing the permeability parameter K throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region.
4. The reversal behaviour is observed with increasing Schmidt number Sc, chemical reaction parameter Kc, suction parameter w_0 or the frequency of oscillation ω.
5. The magnitude of the temperature of the flow field diminishes as the Prandtl number, Heat source parameter S or suction parameter w_0 or the frequency of oscillation.
6. The concentration reduces at all points of the flow field with the increase in the Schmidt number Sc, chemical reaction parameter Kc, suction parameter w_0 and presence of the frequency of oscillation ω.
7. The skin friction increases with the increase in K Gr, Gm, α and w_0 and decreases with the increase in M, Pr, Sc, Kc, S and ω.
8. Nusselt number (Nu) at the surface of the plate and amplitude increase with increase S, Pr and w_0. Also enhance the amplitude, the rate of heat transfer decrease the phase angle with increase the frequency of oscillations ω.
9. Schmidt number Sc, chemical reaction parameter Kc, the frequency of oscillations ω and suction parameter w_0 increase the amplitude, the phase angle the rate of mass transfer at the surface of the plate.

Increase the frequency of oscillations ω, enhance the amplitude, the rate of mass transfer decrease the phase angle.

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