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# Investigation on Vibration Characteristics of Fluid Conveying Single Walled Carbon Nanotube Via DTM

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**Abstract.** In this work differential transform method (DTM) is used to study the vibration behavior of fluid conveying single-walled carbon nanotube (SWCNT). Based on the theories of elasticity mechanics and nonlocal elasticity, an elastic Bernoulli-Euler beam model is developed for thermal-mechanical vibration and instability of a single-walled carbon nanotube (SWCNT) conveying fluid and resting on an elastic medium. The critical fluid velocity is being found out with different boundary conditions, i.e. Fixed-Fixed and simply supported at ends. Effects of different temperature change, nonlocal parameters on natural frequency and critical fluid velocity are being discussed.

## 1. Introduction

Nanoscale engineering materials have superior mechanical, electrical and thermal performances than the conventional structural materials. They have attracted great interest in modern science and technology after the invention of carbon nanotubes by Iijima (1991) [1]. Carbon nanotubes are allotropes of carbon with a cylindrical nanostructure. Nanotubes have a length-to-diameter ratio of up to 132,000,000:1 [2], significantly larger than any other material. The structure of a single-walled carbon nanotube can be understood by packaging a one-atom-thick layer of graphite called graphene into a seamless cylinder [2], [3], and [6]. Single-walled nanotubes are the perfect candidate for miniaturizing electronics rather than the micro- electromechanical scale currently used in electronics [9]. Carbon nanotubes are the strongest and stiffest materials in terms of tensile strength and elastic modulus respectively [7]. Eringen [8] proposed the nonlocal continuum theories for the analysis of small sized structures. In nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function not only of the strain at that point but also a function of the strains at all other points of the domain. The mechanical analyses of nanostructures, theoretical and mathematical modeling becomes an important issue when nano-engineering comes into picture. This is due to the scale effect of the nanostructures. The classical theory of elasticity being the long-wave limit of the atomic theory excludes these effects and thus this theory would fail to analyze the structure with small-scale effects accurately. Thereby size-dependent continuum-based methods [5-7] are becoming popular in modeling small sized structures as it offers much faster and accurate solutions. Sudak [9] carried out buckling analysis of multi-walled carbon nanotubes. Wang and Varadhan [10] analyzed the small scale effect of carbon nanotube and shell model. Yakobson et al. [11] introduced an atomistic model for axially compressed single walled carbon nanotube and compared it to a simple continuum shell model. Sears and Batra [12] proposed a comprehensive buckling analysis of single walled and multi-walled carbon nanotubes by molecular mechanics simulations and continuum mechanics models. Reddy reported a suitable reference concerning nonlocal theories for bending, buckling and vibration analysis of nanobeams [13]. Work related to bending, vibration and buckling analyses of carbon nanotubes and graphene sheets using nonlocal elasticity are found in R. Kumar, et al [14], B. Ravikumar [15], and Pradhan S.C., et al. [16].

In the present work, the vibration analysis of fluid conveying single-walled carbon nanotube embedded in an elastic medium is studied using the differential transformation method. Zhou [17] proposed differential transformation method to solve both linear and non-linear initial value problems in electric circuit analysis. Later Chen and Ho [18] applied this method to eigen value problems.

The Differential transform method is a semi-analytical method based on the Taylor series expansion. In this method, certain transformation rules are applied to the governing differential equations and the boundary conditions of the system. And they are transformed into a set of algebraic equations in terms of the differential transforms of the original functions. These algebraic equations give a solution which gives the desired solution of the problem.

The differential transformation of the $k^{th}$ derivative of the function u(x) is defined as follows:

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \tag{1}$$

And the differential inverse transformation of U(k) is expressed as

$$u(x) = \sum_{k=0}^{\infty} U(k)(x-x_0)^k \tag{2}$$

In real application function, u(x) is expressed as finite series and equation (2) can be written as:

$$u(x) = \sum_{k=0}^{n} U(k)(x-x_0)^k \tag{3}$$

With the use of certain transformation rules we convert the governing differential equation and associated Boundary Conditions into some algebraic equations; on solving them we get the desired results. Following transformation, table is used for this purpose.

Table 1: Differential Transformations for Mathematical Equations

| Original Function | Transformed Function |
|---|---|
| $y(x) = u(x) \pm v(x)$ | $Y(\mathrm{k}) = U(k) \pm V(k)$ |
| $y(x) = \lambda\, u(x)$ | $Y(k) = \lambda\, U(k)$ |
| $y(x) = \frac{d^n u(x)}{dx^n}$ | $Y(k) = (k+1)(k+2)....(k+n)U(k+n)$ |

## 2. Mathematical Formulation and Solution Procedure

### *2.1: Non-Local Formulation of SWCNT*

Eringen [8] first introduced Nonlocal elasticity theory; the Simplified constitutive relation in a differential form is given as follows:

$$(1-(e_0 a)^2 \nabla^2)\sigma = \tau \tag{4}$$

Where, $\tau$ is the classical, macroscopic stress tensor at a point, 'a' is an internal characteristic length (e.g., lattice parameter, granular size, the length of C–C bonds), $e_0a$ is a material constant, $\sigma$ is nonlocal stress tensor and $\nabla^2$ is the Laplacian operator.

For a beam type structure, in the thickness direction, the nonlocal behavior can be neglected. Thus, for a homogeneous isotropic Euler–Bernoulli beam, the nonlocal constitutive relation takes the following form: [20]

$$\sigma_{xx} - (e_0 a)^2 . \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\,\varepsilon_{xx} \tag{5}$$

Where E is the modulus of elasticity.

On the basis of the theory of thermal elasticity mechanics, the axial force $N_{t1}$can be written as,

$$N_{t1} = \frac{-EA}{1-2\nu}.\alpha_x T \tag{6}$$

Where $\alpha_x$ denotes the coefficient of thermal expansion in the direction of x-axis and ν is the Poisson’s ratio and T denotes the change in temperature.

The carbon nanotube is arranged in a manner given below in figure 1, it is embedded in the elastic medium. U is the fluid velocity, K is the elastic constant for Winkler foundation and L is the length of the nanotube.

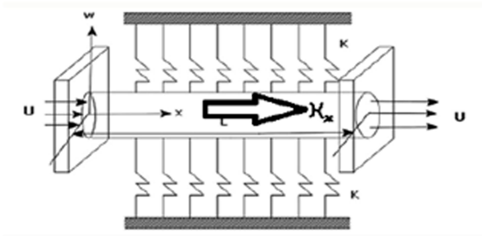


**Figure1.** Analysis model of CNT embedded in the elastic medium.

If we consider the thermal effect, the differential equation of motion related to shear force of fluid conveying SWCNT is given by (Chang, 2012),

$$\frac{\partial Q}{\partial x} = m\frac{\partial^2 w}{\partial t^2} - \frac{\partial (N_{t1}\frac{\partial w}{\partial x})}{\partial x} + Kw + M\left(\frac{\partial^2 w}{\partial t^2} + 2U\frac{\partial^2 w}{\partial x \partial t} + U^2\frac{\partial^2 w}{\partial x^2}\right) \tag{7}$$

Where Q is the shear force, w is the transverse deflection, m is the mass of nanotube per unit length and M is the mass of fluid per unit length of the beam.

Q satisfies the condition for equilibrium of Euler’s beam $Q = \frac{\partial M_1}{\partial x}$ (8)

$$M_1 - (e_0 a)^2\frac{\partial^2 M_1}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} \tag{9}$$

Where $M_1$ is the nonlocal bending moment and eq.(9) represents the nonlocal bending equation of nanotube. By combining equations (7), (8) and (9), we have

$$M_1 = -EI\frac{\partial^2 w}{\partial x^2} + (e_0 a)^2\left[m\frac{\partial^2 w}{\partial x^2} - \frac{\partial\left(N_{t1}\frac{\partial w}{\partial x}\right)}{\partial x} + Kw + M\left(\frac{\partial^2 w}{\partial t^2} + 2U\frac{\partial^2 w}{\partial x\partial t} + U^2\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{10}$$

Finally, by above equation and by eq.(9), the governing differential equation is given as

$$\begin{aligned}&\left[EI + (e_0 a)^2 N_{t1} - (e_0 a)^2 MU^2\right]\frac{\partial^4 w}{\partial x^4} - 2(e_0 a)^2 MU\frac{\partial^4 w}{\partial x^4} - (e_0 a)^2 (M+m)\frac{\partial^4 w}{\partial x^2 \partial t^2} \\ &+\left[MU^2 - N_{t1} - (e_0 a)^2 K\right]\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x\partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} + Kw = 0\end{aligned} \tag{11}$$

In this study, the Euler-Bernoulli beam model using stress gradient approach for the dynamic analysis of single-walled carbon nanotube with nonlocal effect could be considered. Where, w = w(x,t) is the transverse beam deflection, x,t are the spatial coordinates, I is the moment of inertia of carbon nanotube and $e_0$ is a constant appropriate to each material, a is an internal characteristic length.

For the dynamic analysis, the Eq.(11) can be non-dimensionalized using L ( length of CNT) and by substituting, W= w/L and X= x/L, we have;

$$\frac{A}{L^3}\cdot\frac{d^4W}{dX^4} - B(i\omega)\frac{d^3W}{dX^3} - C\frac{(i\omega)^2}{L}\frac{d^2W}{dX^2} + \frac{D}{L}\cdot\frac{d^2W}{dX^2} + 2MU(i\omega)\frac{dW}{dX} + (M+m)(i\omega)^2LW + KLW = 0 \tag{12}$$

Where,

$$A = \left[EI + (e_0a)^2N_{t1} - (e_0a)^2MU^2\right];$$
$$B = 2(e_0a)^2MU;$$
$$C = (e_0a)^2(M+m);$$
$$D = \left[MU^2 - N_{t1} - (e_0a)^2K\right].$$

*2.2: DTM Formulation:*

In order to derive DTM form of Eq. (12), we refer Table 1 and the following expression can be written easily.

$$W(k+4) = \begin{bmatrix} (B(i\omega)L.(k+1)(k+2)(k+3)W(k+3) + C(i\omega)^2L^2.(k+1)(k+2)W(k+2) - DL^2.(k+1) \\ (k+2).W(k+2) - 2MU(i\omega)L^3.(k+1)W(k+1) - (M+m)(i\omega)^2L^4.W(k) + kL^4. \\ W(k))/(\mathrm{A}.(\mathrm{k}+1)(\mathrm{k}+2)(\mathrm{k}+3)(\mathrm{k}+4)) \end{bmatrix} \tag{13}$$

*2.3: Application of Boundary conditions*

*2.3.1 SIMPLY SUPPORTED AT BOTH ENDS.*

The boundary conditions for the case of simply supported SWCNT at both the ends are defined as

$$w(0) = 0,\ w''(0) = 0,\ w(L) = 0,\ w''(L) = 0 \tag{14}$$

By using differential transformation these can be written as:

$$W(0) = 0, W(2) = \frac{MU(e_0a)^2(i\omega)L.W(1)}{EI + (e_0a)^2N_{t1}} \tag{15}$$

$$\sum_{k=0}^{N} W(k) = 0, \sum_{k=0}^{N}[F.k(k-1) + 2MU(e_0a)^2(i\omega)L.k].W(k) = 0 \tag{16}$$

Where, $F = -EI - (e_0a)^2N_{t1} + (e_0a)^2MU^2$.

We can calculate W(k), up to *n* terms from the eq.(13), by assuming W(1)= c1, W(3)=c2 and it will be substituted in eq.(15) & (16) and by solving these equations for non-trivial solution, natural frequency (ω) of the carbon nanotube can be calculated. The accuracy of natural frequency increases with increase in the value of *n*(number of iterations) and saturates at a maximum *n* value, i.e n = Nmax.

*2.3.2 FIXED AT BOTH ENDS.*

For the SWCNT supported by fixed at both the ends, the boundary conditions defined as

$$w(0) = 0,\ w'(0) = 0,\ w(L) = 0,\ w'(L) = 0 \tag{17}$$

By using differential transformation these can be written as:

$$W(0) = 0, W(1) = 0 \tag{18}$$

$$\sum_{k=0}^{N} W(k) = 0, \sum_{k=0}^{N} kW(k) = 0 \tag{19}$$

By assuming W(2) = c1, W(3) = c2 the eq.(13) can be calculated up to n terms and a similar procedure is followed as that of simply supported boundary condition.

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## 3. Results and Discussion

*3.1 Validation:*

In Fig. 2, and Fig.3 validation of the results has been done with the results available in the literature [20]. The outer radii Rout = 3.5 nm and thickness of the nanotube h = 0.34 nm. The mass density of single-walled carbon nanotube is 2.3 $g/cm^3$ with Young's modulus E of 1 TPa, the density of water is 1 $g/cm^3$, aspect ratio L/(2Rout) = 100, nonlocal parameter $e_0a/L$ is taken from 0 to 0.05 and Winkler constant K from 0 to 0.1 MPa. In the present study, consider two cases of temperature region, low and high. The coefficient of thermal expansion $\propto_x=-1.6\times10^{-6}$ $K^{-1}$ and $1.1\times10^{-6}$ $K^{-1}$ for low or room temperature and high temperature region respectively is considered. The Poisson's ratio is considered as 0.3.

The nanotube becomes more flexible and the natural frequencies get reduce with increase in flow velocity. The fundamental natural frequency becomes zero and the nanotube becomes unstable when the flow velocity exceeds a certain value, this corresponds to the instability of the single walled carbon nanotube and the flow velocity producing the zero natural frequency is classified as the critical flow velocity of the system.

*3.1.1 Fixed-Fixed boundary condition.* Figure.2 is drawn between the natural frequency (y-axis) of the beam and fluid flow velocity (x-axis) entering through it in a high-temperature region. The change in temperature is taken as 25 K, the nonlocal parameter is 0.05 and Winkler elastic constant K is taken as 0 MPa.

We can observe that the points or data obtained here are very close to the available results of Chang [20].

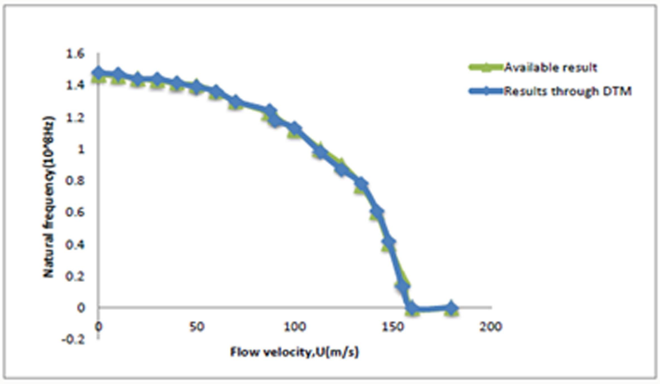


**Figure 2.** Natural frequency v/s flow velocity (At T=25K, $e_0a/L$=0.05, K=0 MPa)

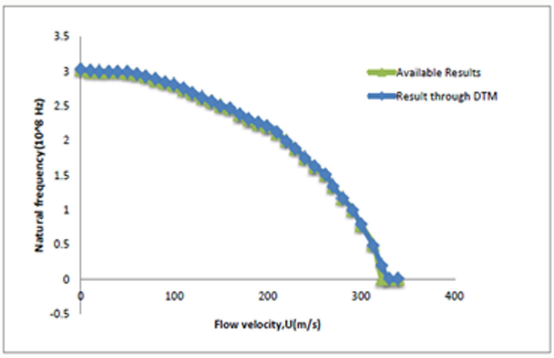


**Figure 3.** Natural frequency v/s flow velocity (At T=15K, $e_0a/L$=0.05, K=0.1 MPa)

In Figure 3, the comparison has been carried out between the results through DTM and available results (Chang, 2012) with a temperature change of 15 K, ea/L at 0.05 and K at 0.1 MPa in low-temperature region. We have

clearly seen a very good agreement between both the methods a very good. We observe that natural frequency increases with the increase of Winkler constant.
After the validation of the results, we have observed that the application of the differential transformation method (DTM) gives the values very close to the available results in the literature [20].

*3.2 NEW RESULTS AND DISCUSSION: Simply Supported boundary condition*

*3.2.1 Effect of temperature.* In Figure 4, a variation of fundamental frequency is drawn in the low-temperature region by keeping the value of the nonlocal parameter and elastic force constant at 0.05 and 0 MPa respectively, where the coefficient of thermal expansion has negative values. There is an increase in the temperature change (from 0K to 35K) which tends to increase the natural frequencies of the single-walled carbon nanotube as well as critical flow velocity in the low-temperature region when the flow velocity is lower than the critical flow velocity. Moreover, the natural frequency and critical flow velocity are $1.2\text{x}10^8$ Hz and 215 m/s which are much lower than the fixed-fixed boundary condition (Figure 2) at T=15K.

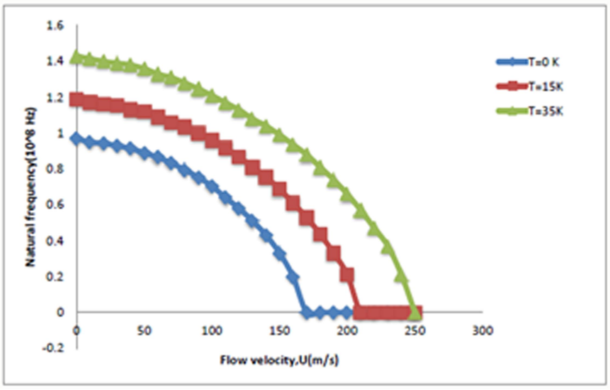


**Figure 4.** Natural frequency v/s flow velocity (At $e_0a/L$=0.05, K=0.1 MPa)

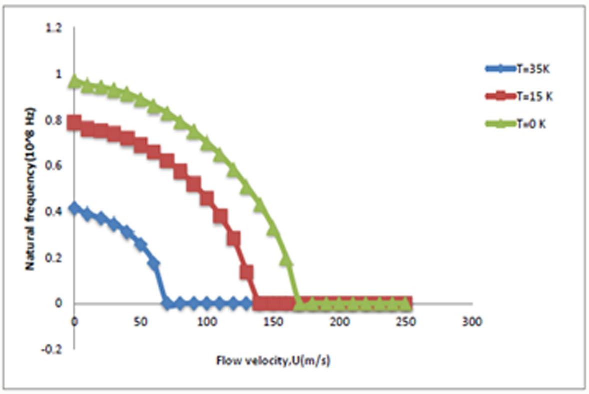


**Figure 5.** Natural frequency v/s flow velocity (At $e_0a/L$=0.05, K=0.1 MPa)

Figure 5 is drawn with the value of the nonlocal parameter and elastic force constant as 0.05 and 0 MPa respectively in the high-temperature region, which means the coefficient of thermal expansion has positive values. When the flow velocity is lower than the critical flow velocity, the increase of the temperature change (from 0K to 35K) tend to decrease the natural frequencies of the singe-walled carbon nanotube (from $1\text{x}10^8$ Hz to $0.4x10^8$ Hz) and critical flow velocity (from 180 m/s to 70 m/s). Moreover, the natural frequency and critical flow velocity are much lesser when compared with Figure 2.

*3.2.2 Effect of the Nonlocal parameter.* Figure 6 is drawn in a high-temperature region which represents the variation of the fundamental frequency of fluid conveying single-walled carbon nanotube with flow velocity for different values of $e_0a/L$. We observe that the nonlocal parameter ($e_0/L$) increases and natural frequency decrease at T=35K and K=0 MPa. Also, the nonlocal parameter ($e_0/L$) increases from 0 to 0.1 at zero flow velocity, natural frequency decreases from 0.46 x $10^8$ Hz to $0.35x10^8$ Hz and critical flow velocity reduces from 80 m/s to 60 m/s.

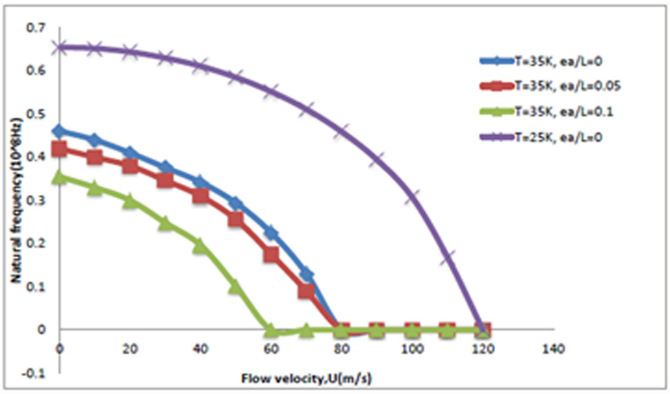


**Figure 6.** Natural frequency v/s flow velocity (At K=0 MPa, Simply supported B.C).

The variation of $e_0a/L$ from 0 to 0.05, have a significant effect on the natural frequency at zero flow velocity but does not have very significant effect on critical flow velocity. When dealing with $e_0a/L=0$ the nonlocal beam theory reduces to local beam theory. A curve is drawn at T=25K and with $e_0a/L=0$, which corresponds to a local beam theory. We clearly observe that in local beam theory the reduction of the natural frequencies and critical flow velocities happens when the nonlocal parameter is introduced.

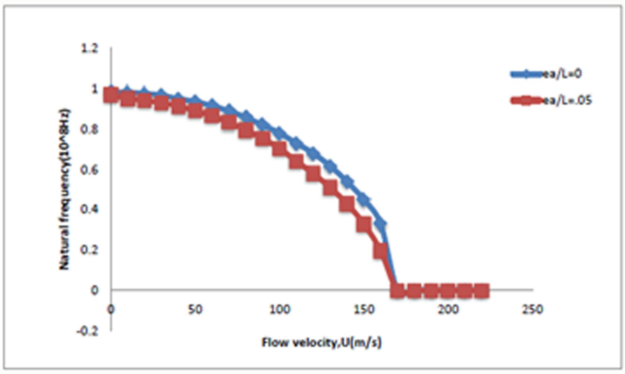


**Figure 7.** Natural frequency v/s flow velocity (At T=0 and K=0 MPa, Simply supported B.C).

We can observe that the nonlocal parameter ($e_0/L$) increases and natural frequency decrease again. At zero flow velocity, the nonlocal parameter ($e_0/L$) is increased from 0 to 0.05, but it does not increase the natural frequency so much. If we compare above graph at $e_0a/L=0.05$ from fixed-fixed condition (Figure 3) we notice that the natural frequency of later case has a higher value ($1.84x10^8$ Hz) than former ($1x10^8$ Hz) and critical flow velocity has a higher value (210 m/s) than former (170 m/s). The reason is only that the fixed end gives an extra stiffness to the beam.

## 4. Conclusion

This paper presents an analytical model for studying the effects of temperature change, the nonlocal parameter on the natural frequency of single-walled carbon nanotube conveying fluid for various boundary conditions.
Several results are presented on the variation of the fundamental natural frequency of single-walled carbon nanotube with flow velocity for various parameter values. It is found that at low or room temperature, the fundamental natural frequency and critical flow velocity for the single-walled carbon nanotube increase as the temperature change increases, on the contrary, while at high temperature the fundamental natural frequency and critical flow velocity for the single-walled carbon nanotube decrease as the temperature change increases.

**References**

[1] Iijima, Sumio 1991 *Nature* **58** 6348.
[2] Ball P. 2001 *Nature,* **414** 6860.
[3] Thostenson E. T., Ren Z., & Chou T. W. 2001*Composites science and technology*, ***61*(13)** 1899.
[4] Zhou, S. J., and Z. Q. Li. 2001 *Journal of Shandong university of technology* **31** 401.
[5] Pop E., Mann D. A., Goodson K. E., & Dai H. 2007 *Journal of Applied Physics*, **101(9)** 093710.
[6] Treacy M. J., Ebbesen T. W., & Gibson J. M. 1996 *Nature,* **381** 678.
[7] Pop, E., Mann, D., Wang, Q., Goodson, K., & Dai, H. (2006) *Nano letters*, ***6*(1)** 96.
[8] Eringen, A. C. 1983 *Journal of Applied Physics* **54 (9)** 4703.
[9] Sudak, L. J. 2003 *Journal of Applied Physics* **94(11)** 7281.
[10] Wang, Q., and V. K. Varadan 2005 *Smart Materials and Structures* **14(1)** 281.
[11] Yakobson, Boris I., C. J. Brabec, and J. Bernholc 1996 *Physical review letters* **76(14)** 2511.
[12] Sears, A., and Romesh C. Batra 2006 *Physical Review B* **73(8)** 085410.
[13] Reddy, J. N. 2007 *International Journal of Engineering Science* **45(2)** 288.
[14] Kumar, Ravi, and Sumit Deol 2016 *Int. j. sci. & Res.,* **5(3)** 1768.
[15] Ravi Kumar B., and K. Palaksha Reddy 2017 *Rasayan J. of Chem.* **10(2)** 481.
[16] Pradhan, S. C., and G. K. Reddy 2011*Computational Materials Science* **50(3)**1052.
[17] Zhou, J. K. 1986 *Huazhong Science & Technology University Press, China*.
[18] Chen, Cha'O. Kuang, and Shing Huei Ho. 1999 *Applied Mathematics and computation* **106(2)** 171.
[19] Reddy, J. N., and Pang, S. D. 2008 *Journal of Applied Physics* **103 (2)** 023511.
[20] Chang, T-P. 2012 *Applied Mathematical Modelling* **36(5)** 1964.