Object Modeling: Topology and Symmetries

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Abstract: In 3D imaging and modeling, several applications require the definition of conceptual models, which imply the knowledge of the 3D topology. Working in 3D, as well known, is much more complex than in 2D. The combined study of topologic and geometric relations between primary elements and symmetries in the spaces in which the complex objects are located, allows noticing some analogies between the number of the found relations and the number of groups of symmetries.

Keywords: Groups of symmetries, Object based data structure, Object modeling, Topological relations among geometric elements.

1. Introduction

The object modeling allows the widest modalities to represent bodies and figures for their studies and analysis. The relations between characteristic elements (points, lines, surfaces and 3D bodies) determine the validity of this modeling and their sets have correspondences with the groups of symmetry in the mono-dimensional, bi-dimensional and three-dimensional spaces of the formal algebra.

The object (eventually dynamics) modeling is an innovative and technologically advanced instrument for the study of spatial phenomena and their temporal dynamics. Unfortunately, the use of this instrument is still rare; in fact, it remarkably increases the complexity of realization and management of the informative systems.

The production of the cartography has been, for a long time, a refined art. The handicraft experience of the mapmakers completed maps, created from geodetic measurements and densified by surveying operations. The impressive spread of technology is carrying to give over the traditional map production systems, preferring the new and powerful digital systems of map production.

2. Object Modeling

The object paradigm supplies a high level of abstraction of the physical structure of the data; it is

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able to model the system according to the definition of the data (objects), as well as to operate on the concept of attributes and relations, being both stored as an integrating part of the same objects.

The object-based representation permits to create complex objects, like polygons, 3D bodies, etc., analyze and manipulate them like single objects (even if they are combinations of objects). This characteristic eliminates the necessity of clarifying all the geographic and semantic attributes of the objects. Furthermore, the object-based approach allows for easily description of new types of data, defining operations on new objects and structuring the objects in a hierarchical way. In this context, it is of particular interest the use of those tools, as the extensions, able to explore the data, process them in external phases where specific operations are executed, and import the results; because of their high qualification, the dimension and complexity of these external procedures exclude their insertion inside the system.

The development of object-based systems is going in two different directions: adding all the typical functions of the object paradigm to a relational databank, or constructing a new independent system.

The first approach is very robust, because the databanks are a mature product, while the second approach is still an open issue in the scientific and technological research. In both cases, the main objective is the production of a set of procedures able to treat spatially referenced data, as well as common data independent from a reference frame.

The most important procedures for the data management tend to solve these main problems: data storage and search of information according to suitable requirements. In particular, for the management of spatially referenced data it is necessary to create new entities (polygons, 3D bodies, etc.) and to define rules for their processing, like distance, incidence, adjacency, etc. However, some problems of difficult solution, presented by the relational models, are still not completely solved in the object-based systems. At present time, a hybrid system seems to be the best solution for the creation of robust physical (relational) data structures and flexible logical (object-based) data models.

An object can be described as a conceptual entity, easily defined by means of its data and situation context. The situation context includes a set of operations and methods, valid on the object itself. Its state is represented by the values assumed by local variables. Every single object belongs to a class, which defines the type of object. The classes can have own variables, describing the characteristics of the class itself. If classes have common variables and methods, it is possible to define a super – class, which groups the variables and the methods of all these classes. For convention, the universal super – class constitutes the first level of the hierarchical level structure describing the system.

An interesting object-based data structure can be obtained by introducing a linkage between spatially referenced objects and their features, for example, areas with their boundaries, and edges with isolated vertices. Thus, the spatially referenced objects are identified and described by means of their geometric and thematic features. This observation leads to a first formal requirement, in order to construct a formal data structure:

- every object must be associated to an identifier (name or number);
- every identifier must have a link to the attributes of an object.

Objects with common geometric or thematic aspects can be grouped in the same class (set of objects), according to identified by a certain name; the list of the attributes of a class supplies the names of the attributes, whose values are assumed by the objects themselves. This means that every object, belonging to a class, has a list of values, one for each attribute of the class. Common attributes lead to a super – class (i.e. a class of classes), which is associated to the list of attribute values assumed by the classes. It is evident that the construction of a super – class introduces a hierarchy in the level structure of the classes.

At the top level, the super – class with its list of attributes is placed; at the lower levels, the lists of the attributes with their values are subdivided, because some attributes are estimated at the intermediate level of class, while the other ones are estimated only at the successive level. At the intermediate level, we place the class whose list of the attributes is subdivided in two parts: the former contains the values of the attributes associated to the super – class; the latter contains the list of the attributes of the class itself. At the lowest level, we place the object whose list of the attributes is estimated through the lists of the attributes of the super – class or the belonging class. The estimation of an attribute, at the class level, implies that all the objects belonging to the same class have the same value for the selected attribute. Furthermore, it is possible to introduce more hierarchical levels: every level inherits the list of the attributes of the higher level and transmits the list of the attributes to the lower level. At the lowest level of the hierarchy, all the attributes must be estimated.

Classes of objects and objects are defined so that every object belongs to one (and only one) class; this requirement is equivalent to the convention according to which the classes must be mutually exclusive. The objects can be classified according to their geometric or thematic features; therefore, a second convention states that a class contains only objects of a certain type. Sometimes this convention seems to be too rigid; however, it allows for the construction of a very simple and transparent structure. This can be applied in various applications, as shown in Fig. 1 and Fig. 2.

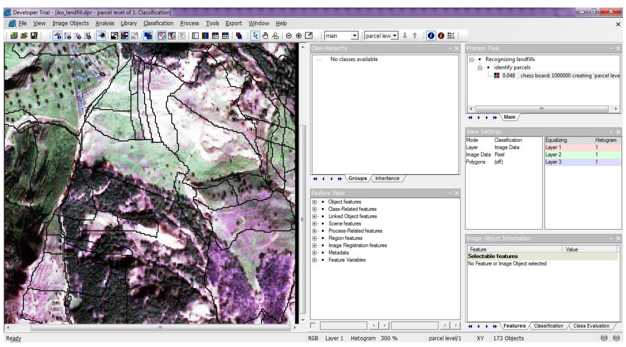

Fig. 1 - Object-based approach: Segmentation with thematic features (parcels).

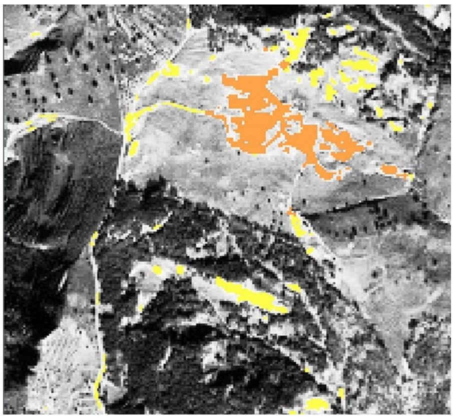

Fig. 2 - Object-based approach: Classification of landfills.

For some rare cases, a possible solution is the construction of complex objects. The last convention implies a relation of the type: many to one between objects and classes.

Using this object-based approach, geometric structures with different complexity can be described by objects belonging to suitable classes of features [4] – [6]. For instance:

- a punctual structure, by means of an object belonging to the class of the points;
- a liner structure, by means of an object belonging to the class of the lines;
- a bi-dimensional structure, by means of an object belonging to the class of the surfaces;
- a three-dimensional structure, by means of an object belonging to the class of the 3D bodies.

In order to construct a formal data structure, it is necessary to identify the geometric features of the object and their relations. This can be done with different methods. A possible approach uses the graph theory, putting into a one-to-one relation the thematic and the corresponding geometric elements, and describing the topological relations between the various elements. The elements can be described in two different ways:

- in a parametric form, i.e. through parameters, which define the equation of a mathematical curve;
- through a sequence of points connected by a polyline, being the link between two consecutive points a segment of one straight line.

The second solution implies the introduction of new types of points beyond the nodes: these new points only contain an information concerning the point positions. In both cases, the following conventions are adopted:

- when a complex object is analyzed through a graph, all the points describing its geometry are treated as nodes, having each node the position given by the coordinates of the corresponding point;
- by using the duality principle, all the planar figures derived by the decomposition of a complex figure are treated like nodes of a dual graph, having each node the position given by the coordinates of the centroids of the same figure;
- by using the duality principle, the 3D bodies, derived by the decomposition of a complex solid object, are treated like nodes of a dual graph, having each node the position given by the coordinates of the centroids of the same body;
- all the segments of straight lines are represented by edges of the graph and each edge

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has an initial and a final node ;

- by using the duality principle, all the surfaces in 3D space are represented by edges of a dual graph and each edge has an initial and a final node .

In order to avoid geometric ambiguities, two new conventions are introduced:

- for each couple of nodes, there is no more than one edge, connecting them;
- the edges cannot be intersected in the simple case of a planar graph; the same requirement is imposed for all the planar sub – graphs, derived by a possible decomposition of a spatial graph.

The connection between geometric and thematic elements is performed through identifiers. The successive step, in the definition of a formal data structure for a complex object, consists in the analysis of the linkage between the geometric elements (nodes, edges) and the thematic elements (points, lines, surfaces and 3D bodies):

- the linkage between features, like points (thematic elements) and nodes (geometric elements) consists in the following condition: every point is represented by one node (and one only);
- if one node does not represent any feature, a null identifier is used.

The connection between the other geometric and thematic elements with more complexity is set up as follows:

- an edge can be part of a characteristic line;
- if an edge does not belong to a characteristic line, a null identifier is imposed;
- in the simple case(planar graph or planar subgraphs derived by a decomposition of a spatial graph) an edge has always one (and only one)characteristic area at its right and one (and only one) characteristic area at its left;
- a face (i.e. an edge, by using the duality principle) can be part of a characteristic surface (i.e. a characteristic line, by using the same duality principle);
- if a face does not belong to a characteristic surface, a null identifier is imposed;
- by using the duality principle, a surface in 3D space has always one (and only one) characteristic 3D body at its right and a characteristic 3D body 3D (and only one) at its left;
- if an edge (or a face) is a boundary element, one of the two characteristic surfaces (or one of the two characteristic 3D bodies) is called external element;
- if a characteristic line (or a characteristic surface) is a boundary element between two characteristic surfaces (or two characteristic 3D bodies), the edges (or the faces) belonging to this boundary are part of the same characteristic line (of the same characteristic surface);
- on the contrary, if an edge (or a face) is not part of a characteristic line (or a characteristic surface), or it does not belong to a certain boundary, it happens in a different way. The edge and its characteristic line (or the face and its characteristic surface) intersect a characteristic surface (or a characteristic 3D body), and the right – left linkage is referred to the same surface (or to the same 3D body).

Further developments in a formal structure for a complex object introduce two types of topological relations:

- the former is expressed by from the graph structure;

- the latter presents the linkage between the geometric elements and thematic one.

Finally in the case of 3D bodies modeling [7], [8], it is necessary to establish a cross - connection table between edges and faces (or characteristic lines and characteristic surfaces), so that a topological linkage between primary graphs and dual graphs is present. In fact, while this linkage is directly defined by the edges in the planar graphs, the primary and dual spatial graphs are completely separated (if they are not previously decomposed in planar subgraphs); as a consequence, the above mentioned cross - connection table is strictly required [9], [10].

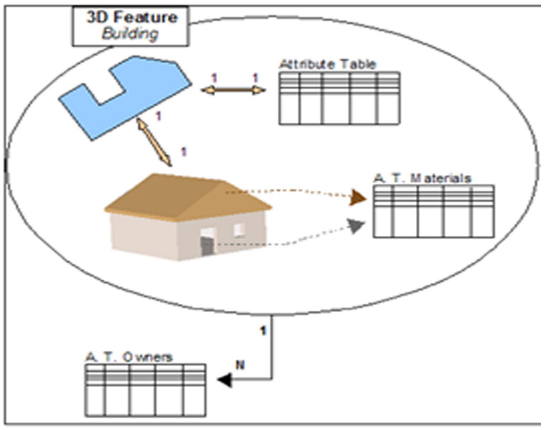

Fig. 3 - 3D-extension of feature.

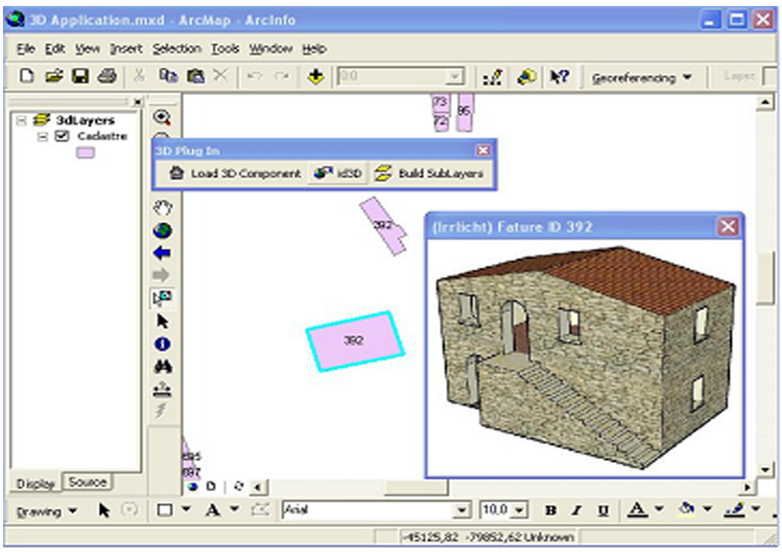

Fig. 4 - 3D in ArcGIS.

3. Relations among the object elements

The very well defined complex objects are in mutual relations in the space, where they are located. In order to better define this concept, terms like at the right or at the left of, over or under to, inside or outside to, intersects, etc. are usually adopted. In the previously analyzed formal data structure, two types of topological relations are introduced:

- relations within geometric data, described by graphs (nodes and edges) in the vectorial structures and by cells (pixels and voxels) for the rasters;
- relations within objects, described by the linkage between primary elements (points, lines, surfaces and 3D bodies) and the same primary elements and the complex objects, formed by the primary elements themselves.

In order to determine the proper application context of each element, some properties are suitably defined. While no particular relations are established for the points, on the other hand the lines and the surface are supposed continuous, open (i.e. simply connected) or closed (i.e. not simply connected, but with a single hole) and the 3D bodies are assumed continuous and simply connected. Therefore, unless different indications, all the elements are finite in their geometric dimensions; moreover the lines have a fractal dimension (i.e. the effective dimension occupied in the space) equal to one. While the surfaces have a fractal dimension equal to two and their eventual closure does not take into account any pathological cases, like the ring of Moebius and the bottle of Klein.

In order to obtain a broad starting point, very general relations among complex objects are defined, as follows:

- 3 classes of relations in the mono-dimensional space: between points, points and lines, lines;
- 6 classes of relations in the bi-dimensional space: beyond to those of the previous case, between points and surfaces, lines and surfaces, surfaces;
- 10 classes of relations in the three-dimensional space: beyond to those of the previous cases, between points and 3D bodies, lines and 3D bodies, surfaces and 3D bodies, 3D bodies.

In one dimension, the topological relations are seven, while passing to two dimensions; ten new relations join to the ones previously defined for the mono-dimensional case, reaching the number of 17 topological relations. Finally, passing to three dimensions, 15 new relations join the other ones previously defined for the bi-dimensional case, reaching the number of 32 topological relations. The following lists present the main geometric relations among the primary elements:

- 10 geometric relations, for the mono-dimensional case;
- 32 geometric relations, for the bi-dimensional case;
- 230 geometric relations, for the three-dimensional case.

We can notice that the long list could further be lengthened, if we also take into account particular conditions of external or inner tangency, intersection and superimposition. Furthermore these conditions can be simple, double or multiple and can be punctual, liner and areal. Therefore taking into account the length of the present list and the richness of the proposed under – classifications, 4783 cases could be found according to the cardinality of the recently discovered four-dimensional symmetry group.

4. Conclusions

The study of the topological and geometric relations among primary elements and the groups of symmetries, in the spaces where the complex objects are located, shows particularly curious identities between the number of the found relations and the cardinality of the groups of symmetries. In fact, as 7 are the topological relations in one dimension, as many are the elements of the group of liner symmetries, which have one single direction of translation. The same analogy is evident in two dimensions where, in correspondence to 17 topological relations, an identical number of elements forms the group of symmetries in the plan, considering two directions of translation. Still to the 32 topological relations, characterized in three dimensions, correspond the elements of the group of symmetries in the 3D space, considering three directions of translation and the crystallographic restriction. Furthermore considering the main geometric relations (being 4 the number of the elements of the group of liner symmetries, considering the crystallographic restriction), we can notice like:

- 10 (number of elements in mono-dimensional case) corresponds to the number of the elements of the group of symmetries in the plan, considering the crystallographic restriction;
- 32 (number of elements in the bi-dimensional case) corresponds to the number of the elements of the group of symmetries in the 3D space, again considering the crystallographic restriction;
- 230 (number of elements in e three-dimensional case) corresponds to the number of the elements of the group of symmetries in the 3D space, without any restriction.

However, we notice that the fundamental differences between the symmetries and the topological and geometric relations among objects. Firstly the symmetry form groups (i.e. they have a composition law and the association property, the identity and the inverse element of each element of the group according to the composition law), while the topological and geometric relations only constitute sets. Successively, the sets of the topological and geometric relations are included each one in the other, in agreement with the increasing of the dimension of the space, where the complex objects are located, while the groups of symmetries are all separate among them. Finally, the present considerations want only to express a qualitative point of view, while mathematical implications would often require to proof complex theorems.

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