

Performance Review of Harmony Search, Differential Evolution and Particle Swarm Optimization

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Abstract: Metaheuristic algorithms are effective in the design of an intelligent system. These algorithms are widely applied to solve complex optimization problems, including image processing, big data analytics, language processing, pattern recognition and others. This paper presents a performance comparison of three meta-heuristic algorithms, namely Harmony Search, Differential Evolution, and Particle Swarm Optimization. These algorithms are originated altogether from different fields of meta-heuristics yet share a common objective. The standard benchmark functions are used for the simulation. Statistical tests are conducted to derive a conclusion on the performance. The key motivation to conduct this research is to categorize the computational capabilities, which might be useful to the researchers.

Keywords: Differential Evolution, Harmony Search, Optimization, Particle Swarm Optimization.

1. INTRODUCTION

Optimization is considered as a mathematical problem. It shows a great role in the development of an intelligent system. There exist several heuristic algorithms are proposed and they are utilized to solve complex optimization problems [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [26] [27] [28]. Heuristic algorithms are mainly classified into 2 main classes: Evolutionary Algorithms (EAs) and Swarm Intelligence (SI) algorithms. Besides these algorithms, there exist few more algorithms. They work on principle of different natural phenomenon such as Gravitational Search (GS), Biogeography Based Optimization (BBO) and Harmony Search (HS). There exist several literatures presents the working and applications of these algorithms, but there is rarely of literatures that demonstrate the performance comparison of these algorithms, which must for an effective utilization of these algorithms. Considering this view, we present the performance comparison of 3 meta-heuristic algorithms. To be specific, we concentrated on 3 algorithms: HS, Differential Evolution (DE) and Particle Swarm Optimization (PSO). As discussed, these algorithms are originated altogether from different class of heuristic algorithms, but sever for a common goal that is to find an optimal solution. Extensive computer simulations are performed considering the standard benchmark functions. The results are collected for the analysis. In order to derive an exact conclusion, we perform various statistical tests. The underlying motivation to conduct this research is categorize the computational capability of HS, DE and PSO with a believe that the outcome might be useful to the researchers who are intended to apply these algorithms.



We organize the paper as outlined here: Section II shows a discussion of the heuristic algorithms adapted to conduct this study. The description of the standard benchmark functions is given in Section III. Section IV gives the description of the simulation model, results and discussion of the results. At last, we give a conclusion in Section V.

2. HEURISTIC ALGORITHMS ADAPTED

In this section, we briefly discuss three heuristic optimization algorithms: HS, DE and PSO. HS is an optimization algorithm developed by Geem et al. [4]. It is a music based optimization approach. HS is utilized for function optimization, pipe network optimization, data optimization, classification system and many more [5]. A comprehensive description of HS with its applications is presented in [6].

DE was proposed by Storn and Price [7]. It gained popularity due to its computational ability. It is successfully applied to an optimal design of heat exchanges, batch fermentation process, optimization of nonlinear chemical process and function optimization [8] [9].

Elberhart and Kennedy [10] have presented the PSO. It has ability to handle an optimization problem that has a large search space. It works on a flock of birds of a group of people [11]. In PSO algorithm, a population also referred as a swarm, which consists of a number of individuals. It is different from any other optimization algorithm as it uses a population of potential solution during the search process. PSO algorithm is very effective to wide range of optimization problems [12] [13] [14] [15].

3. BENCHMARK FUNCTIONS USED

Standard benchmark functions are considered to perform to computational experiments are depicted in Table I [1] [2] [3]. Table I shows the benchmark functions with their formulations such as dimensions (D), search space (SS), and characteristic (U: Unimodal, M: Multimodal, S: Separable, N: Non-separable).

Table I. Standard benchmark function adapted

S. N	Function	Formulation	D	SR	C
F01	Booth Function	$f(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	2	[-10, 10]	MS
F02	Rastrigin Function	$f(x) = A_n + \sum_{i=1}^n [x_i^2 - A \cos(2\pi x_i)]$	30	[-5.12, 5.12]	MS
F03	Schwefel Function	$f(x) = 418.9829d - \sum_{i=1}^d x_i \sin(\sqrt{ x_i })$	30	[-100, 100]	UN

4. SIMULATION MODEL, RESULTS AND ANALYSIS

To evaluate the performance, extensive experiments are performed. The standard benchmark functions enlisted in Table I are implemented using MATLAB 2015a, Intel ® Core™ i7-3632 QM, 2.20 GHz, x64 based processor, RAM -8 GB. Each algorithm is implemented 20 times. Table II presents the best, average and worse results received for each method. The results show that PSO is showing better performance. Table III depicts the mean value and standard deviation. We can see that the performance of PSO is superior over HS and DE.

Table II. The best, average and worst results obtained by HS, DE and PSO

Function	Factor	HS	DE	PSO
F01	B	6.07238E-07	0.315841106	0.2215424
	M	0.000110246	523.2587633	621.8396674
	W	6.03E-04		6232.3199
F02	B	0	2.376470556	0.0008155
	M	0.577971116	134.4679	0.0624826
	W	1.989920381		0.5022965
F03	B	513.2464588	726.5836864	789.8026336
	M	590.247288	737.8034	789.9504101
	W	651.4546808		790.3974054

B: Best result, A: Average result, W: Worst result

Table III. Performance matrix for each algorithm (M: Mean, S.D.: Standard deviation)

Function	Factor	Algorithms		
		HS	DE	PSO
F01	M	0.00011025	523.2587633	621.8396674
	S.D	0.00020797	487.6083258	1250.108898
F02	M	0.57797112	134.4679	0.0624826
	S.D	0.83112218	104.445407	0.1056152
F03	M	590.247288	737.8034	789.9504101
	S.D	60.6408758	60.29352953	0.1492436

Table IV. ANOVA Table for CostFuncValue

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		.009	2	.004	5.858	.008
	Linear Term	Contrast	.008	1	.008	10.300	.003
		Deviation	.001	1	.001	1.416	.244
Within Groups			.020	27	.001		
Total			.028	29			

Table V. Multiple Comparisons Tests at the 0.05 confidence level for CostFuncValue

Test Type	(I) Algorithm	(J) Algorithm	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	HS	DE	.03190451830*	.0121065666 0	.036	.0018872829	.0619217537
		PSO	.03885522420*	.0121065666 0	.009	.0088379888	.0688724596
	DE	HS	-.03190451830*	.0121065666 0	.036	-.0619217537	-.0018872829
		PSO	.00695070590	.0121065666 0	.835	-.0230665295	.0369679413
	PSO	HS	-.03885522420*	.0121065666 0	.009	-.0688724596	-.0088379888
		DE	-.00695070590	.0121065666 0	.835	-.0369679413	.0230665295
LSD	HS	DE	.03190451830*	.0121065666 0	.014	.0070638955	.0567451411
		PSO	.03885522420*	.0121065666 0	.003	.0140146014	.0636958470
	DE	HS	-.03190451830*	.0121065666 0	.014	-.0567451411	-.0070638955
		PSO	.00695070590	.0121065666 0	.571	-.0178899169	.0317913287
	PSO	HS	-.03885522420*	.0121065666 0	.003	-.0636958470	-.0140146014
		DE	-.00695070590	.0121065666 0	.571	-.0317913287	.0178899169

A statistical test is performed to derive a conclusion considering the hypothesis as outlined below.

$$H_0 : \mu_{HS} = \mu_{DE} = \mu_{PSO} \text{ and}$$

H_A : No significant difference at 95% confidence interval.

F-test is conducted. It uses ANOVA and gives the F-ratio. The F-ratio is also referred as p-value. If it is less than or equal α level (0.05), then we reject H_0 . Table IV presents the result received from the F-test for F01.

As obtained p-value < 0.05, hence we rejected H_0 . In this situation, we applied multiple comparison tests (Posthoc test). Two different Posthoc tests: TukeyHSD and LSD tests are conducted. Table VII shows the response of multiple comparison tests. The 1st column “(I) Algorithms” and 2nd column “(J) Algorithms” shows the different combinations through which the algorithms can be compared.

The astrix (*) mark presented in the 4th column indicates the difference is significant, can also be verified via a p-value. If the obtained p-value < 0.05, then that combination of algorithms is significantly different. The data presented in Table V indicates that the performance of HS, DE and PSO is significantly different.

Table VI. Homogeneous Subset Table for CostFuncValue

Test Type	Algorithm	Sample Size	Subset for p-value = 0.05	
			1	2
Tukey HSD ^a	PSO	10	.0017698885	
	DE	10	.0087205944	
	HS	10		.0406251127
	Sig.		.835	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 10.000.

Homogeneity test is performed. It tests the similarity through the Tukey HSD test. The estimated marginal means of the approaches in homogeneous groups are shown in Table VI for F01. The mean values of PSO and DE fall in the same group indicate that their performance is almost similar, whereas HS is in the different group at 0.05 confidence interval.

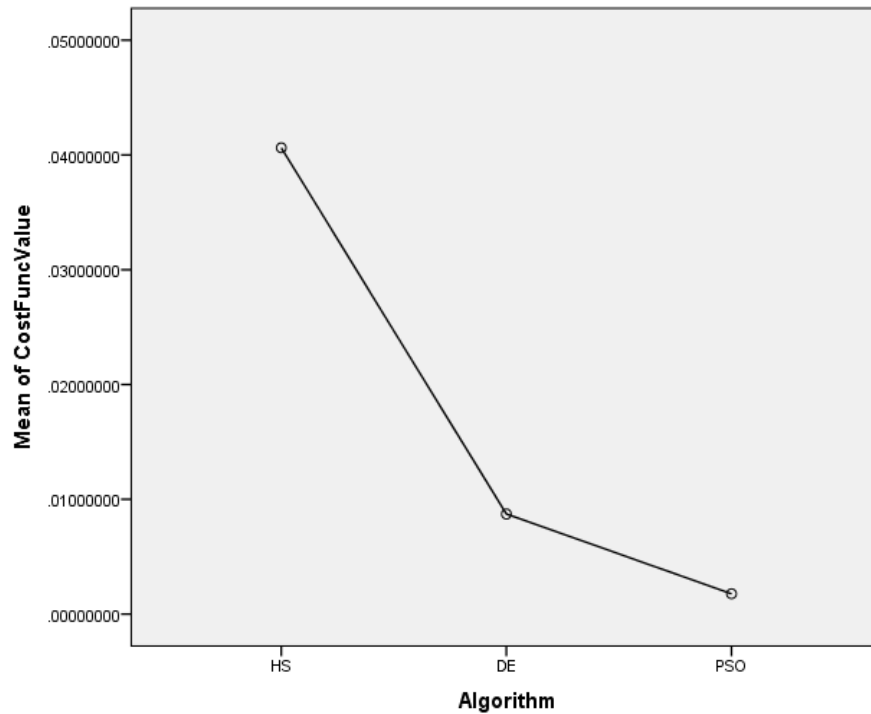
**Figure 1.** Mean of cost function Vs. Algorithms plot of Booth Function

Fig. 1 presents the mean of the cost function vs. algorithms plot for F01. It revealed the superiority of the PSO over HS and DE.

5. CONCLUSIONS

This paper showed the performance comparison of 3 heuristic algorithms. Standard benchmark functions have been considered for implementation. Extensive experiments have been performed and then we analyzed the results in a comprehensive manner. Statistical tests have been conducted that demonstrate the superiority of PSO over other two algorithms.

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