Comparative Performance Analysis of Sliding Mode and Q-Controller Algorithms for Buck Converter

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Abstract: The switched mode dc-dc converters are some of the simplest power electronic circuits which have received an increasing deal of interest in many areas due to their high efficiency and small size. These converters are non-linear and time-variant in nature; hence the analysis, control and stabilization are the main factors that need to be considered. Many control methodology are used for control of switch mode dc-dc converters but the optimum one is always in demand. This paper presents the linearization of the Buck converter model and a comparison between a linear and non-linear control algorithms for better output voltage regulation along with robustness to change in input voltage and load parameters. The computer –aided design software tool Matlab/Simulink is used for the simulations and the results are presented.

Keywords: Buck Converter, Modelling, Q-Parameterization, State-space averaging, Sliding Mode Controller.

I. INTRODUCTION

DC-DC converters represent a challenging field for sophisticated control techniques' application due to their intrinsic nature of nonlinear, time-variant characteristics. As these converters work due to switching, hence the control action is implemented by varying the duty cycle. As the power converters exhibit variable structure system characteristics, the application of proper control law becomes complicated. Thus the conventional PWM controlled power electronic circuits are simplified based on state-space averaging technique and the system being controlled operates optimally only for a specific condition.

This paper is divided into three main sections: a modelling section, where the necessary buck converter dynamics are established based on it's on and off states and the linearization process is applied. The second section deals with the analysis of the system without controller and the third section includes the application of control law and analysis of the closed loop system for both types of controllers, the Q-controller and the Sliding Mode controller(SMC) . The main advantage of the Q- parameterization is that all closed loop sensitivity functions are affine w.r.t to a linearly parameterized transfer function Q. A control law suitable for the dc-dc power converters must cope with their intrinsic nonlinearity and wide input voltage and load variations ensuring stability in any operating condition. Thus a analysis based on the performance of the system using a linear and a non-linear controller is made and in fact a standard solution for this problem is to guarantee the closed loop stability and proper regulation in presence of considered parameter variations.

II MODELING:

The inherent switching operation of the converters results in the circuit components being connected together in periodically changing configurations, each configurations being described by a separate set of equations. The transient analysis and control design is therefore difficult since a number of equations must be solved in sequence.

The non-linear system is characterized as linear model within certain range and time-period using state-space averaging (SSA) technique which is the most common solution [1].

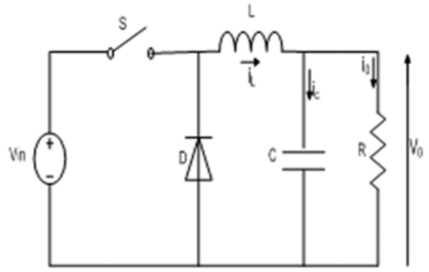

Fig 1: DC-DC Buck Converter Topology

Fig.1 shows the general Buck converter structure for which the dynamics of the system is formulated.
For the on and off conditions of switch S, the respective dynamic equations are given by:

$$\frac{di_L}{dt} = -\frac{v_C}{L} \quad , \quad \frac{dv_C}{dt} = \frac{i_L}{C} - \frac{v_0}{RC} \tag{1}$$

$$\frac{di_L}{dt} = \frac{v_{in}}{L} - \frac{v_0}{L} \quad , \quad \frac{dv_C}{dt} = \frac{i_L}{C} - \frac{v_0}{RC} \tag{2}$$

The state-space average model of the converter is formed by taking a weighted average of the equations (1)-(2) and is expressed in the state-space format as.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3}$$

A, B, C are the overall state, input and output matrices respectively which are obtained by summation of on and off states, mathematically represented as equation (4)

$$\begin{aligned} A &= A_1 d + A_2(1-d) \\ B &= B_1 d + B_2(1-d) \\ C &= C_1 d + C_2(1-d) \end{aligned} \tag{4}$$

and from the converter dynamics the state matrix 'x' and the system matrices A,B and C are given as:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_L \\ v_C \end{bmatrix} \qquad A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B = \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{5}$$

In order to simplify the model, it is linearized by considering small variations in the variables. The steps include representation of each variable by the sum of a DC component and small-signal or AC component thereby implementing the Laplace Transform[2].Neglecting the higher order terms and rearranging we get the linearized model for duty cycle and input voltage as the input. As in this paper the control is implemented by varying the duty cycle equation (6) is our requirement.

$$\frac{v_O(s)}{d(s)} = C[sI - A]^{-1} \cdot [(A_1 - A_2)X + (B_1 - B_2)V_{in}] + (C_1 - C_2)X \tag{6}$$

Using the data sheet values [3] and equation (6), the transfer function of the buck converter for duty cycle as input is :

$$\frac{v_O(s)}{d(s)} = \frac{4 \times 10^8}{s^2 + 250s + 3.33 \times 10^7} \tag{7}$$

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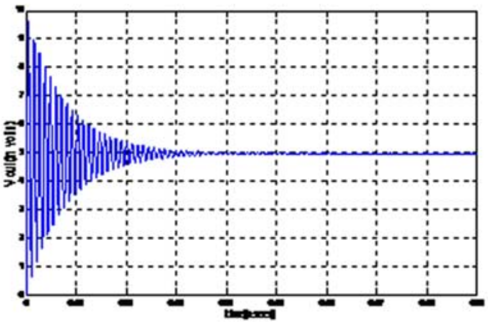

Fig.2:Output voltage of Buck Converter without controller

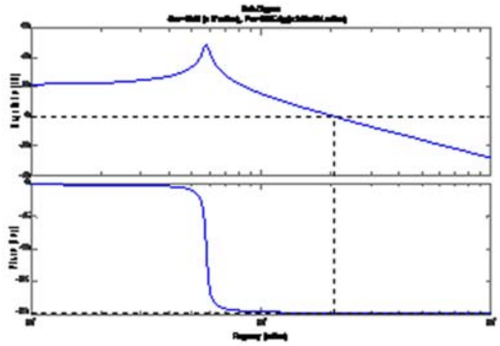

Fig.3: Bode diagram of plant without controller

The dc-dc switching converter with an ideal switch and gain blocks is constructed as a SIMULINK model. In order to facilitate the simulation analysis and controller verification, the pulse-width-modulation signal is built in the system to control the ideal switch. Fig.2 shows the output voltage waveform of the PWM converter without controller. Switching frequency of 55 kHz and duty cycle of 0.4 is applied to the model of the switching converter. The Bode plot in Fig.4 shows that the open loop system is absolutely stable with Gain Margin (GM) = Infinite(Inf) dB at (Inf rad/sec) and Phase Margin(PM)=0.7454 deg at(2.08e4rad/sec).Phase Margin, of the open loop system is very low hence less robust to parameter variations.

III. CLOSED-LOOP DESIGN

A successful closed-loop design needs to consider major control limitations. Two major issues will influence the control performance, namely: 1) the time-variant characteristics and 2) the input and load perturbations. The simulated results will illustrate the comparison between the controllers in terms of fast dynamic response of the output voltage and robustness to load and input voltage variations. Fig.4 illustrates the closed loop design where the output of the converter subsystem is compared with the set point and the error is fed to the controller to generate the control signal. The two control law presented in the paper show stabilized regulation of the output voltage thereby reducing the transients and also satisfies the robustness nature of the controller.

A. Q-Parameterization

Q Parameterization basically utilizes the model inversion phenomena for set point tracking and allows designers to choose the closed loop parameters of undamped natural frequency $_{cl}$ and damping factor $_{cl}$ [4].As the nominal model is a lightly damped system the damping ratio is selected to be 0.7 and the undamped natural frequency is 28571.43 rad/s.The Q function parameter must be a stable and proper transfer function in order to ensure the design of a stable controller. Stability of Q(s) implies the internal stability of the system which is an important aspect for stable systems hence it is essential to use a filter along with Q(s).The controller transfer function is thus derived as (8)-(9)

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)} \tag{8}$$

$$C(s) = \frac{s^2 + 250s + 0.33 \times 10^8}{0.488s^2 + 19600s} \tag{9}$$

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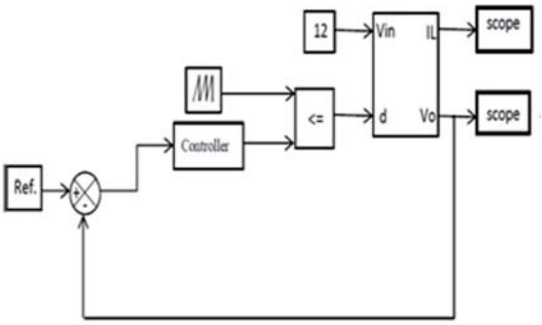

Fig.4: Closed loop design with Q-controller

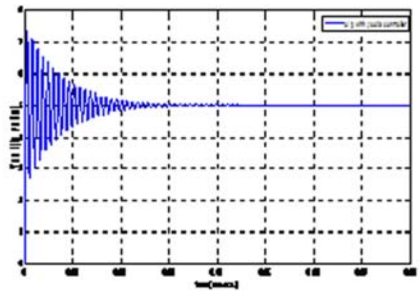

Fig. 5: Output Voltage of plant with Q- Controller

B. Sliding Mode Control

Conventional Siding Mode Control (CSMC) is a non-linear control law derived from a variable structure control (VSC) system theory.VSC system is a class of systems where the control law itself changes during the control process according to some pre-defined rules that depend on the system states.

The basic principle of SMC is to employ certain manifold as a reference path such that the trajectory of the controlled system is directed to a desired equilibrium point [5] .The Sliding mode controller works in three fundamental steps: a) selecting a sliding surface in a fictious plane in three dimensional space (Hitting condition), ii) sliding on the surface and staying on it or near the surface (Existence condition) and iii) converging to a stable equilibrium point (stability condition).

The first step towards designing the SMC is selection of a suitable sliding surface. In this paper a first-order sliding surface (S) involving the voltage error dynamics is considered which is the cost function and is to be minimized. Considering the output voltage as the control variable, the state-variables to be controlled is expressed as:

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_{ref} - v_0 \\ \frac{d}{dt}(V_{ref} - v_0) \end{bmatrix}$ where V_{ref} and v_{o} denote the reference and sensed instantaneous output voltages, respectively. x_1 is the voltage error and x_2 the derivative of voltage error. The sliding surface (S) selected for the converter is of first order and is given by: $S = \left(\frac{d}{dt} + \lambda\right)\tilde{x}$ where $\tilde{x} = V_{ref} - v_0$ is the output voltage error and λ is the sliding co-efficient. Since our aim is to regulate the output voltage, the sliding surface is expressed as parameter of voltage and using the converter dynamics is thus designed as

$$S = \left(\frac{d}{dt} + \lambda\right) \cdot \left(V_{ref} - v_0\right) \quad (10)$$

$$S = V_{ref} - \dot{v}_0 + \lambda V_{ref} - \lambda v_0 \quad (11)$$

$$S = \lambda V_{ref} - \frac{1}{C} i_L + \left(\frac{1}{RC} - \lambda\right) v_0 \quad (12)$$

Next design step includes the calculation for hitting condition. As our reference is the output voltage and S has been formulated using the output and reference voltage the condition should be such that if the sensed output voltage is much lower than the reference voltage, i.e., *S* is positive, the switching action required for the compensation is to turn on the switch so that energy is transferred from the input source to the inductor and if the sensed output voltage is much higher than the reference voltage, i.e., *S* is negative, the switching action is to turn off the switch so that energy transfer between the source and the inductor is discontinued. In order to satisfy this condition, the non-linear control function should behave like a signum function as given in equation (13)

$$u_n = sign(S)$$

Thus the SM control signal u consists of two components a nonlinear component u_n and an equivalent component u_{eq}. The existence of the controlled system on the sliding surface implies that $\dot{S} = 0$

From equation (10)-(12)

$$\dot{S} = \left(\frac{1-\lambda RC}{RC^2}\right)i_L - \left(\frac{L-\lambda RLC - RC^2}{RC^2 L}\right)v_0 - \left(\frac{v_{in}}{LC}\right)u \quad (13)$$

and equating it to zero the equivalent control signal is given by equations in (14) and (15)

$$u_{eq} = \left(\frac{L-\lambda RLC}{RCV_{in}}\right)i_L - \left(\frac{L-\lambda RLC - R^2C}{R^2CV_{in}}\right)v_0 \quad (14)$$

$$u_{eq} = \alpha_1 i_L - \alpha_2 v_0 \quad (15)$$

$$\alpha_1 = \frac{L-\lambda RLC}{RCV_{in}} \quad and \quad \alpha_2 = \frac{L-\lambda RLC - R^2C}{R^2CV_{in}}$$

The third and final stage of the design includes the existence condition in which the controlled plant dynamics should stay on S and converge to the equilibrium point of zero. The time derivative of S must be negative definite to ensure the stability of the system and to make the surface *S* attractive. Therefore the condition required to be fulfilled is as given by $\lim_{S\to 0} S \cdot \dot{S} < 0$ .The most optimized value of λ to satisfy this condition is 200.The closed loop system along with the controller is structured in the simulink environment and the time domain performance of the system is analysed from the result.

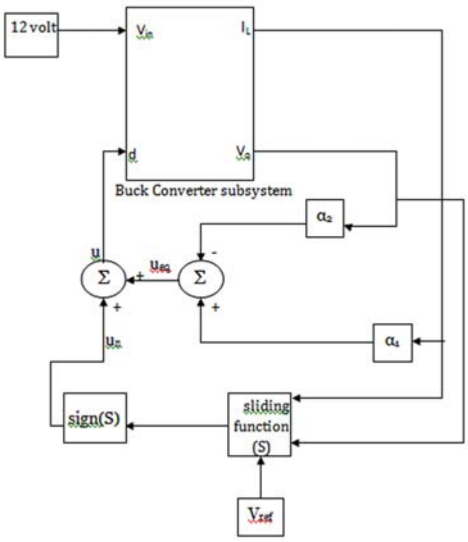

Fig.6: Closed loop Simulink model with SMC

IV.PERFORMANCE EVALUATION BASED ON NUMERICAL SIMULATIONS

The effectiveness of both the controller designs to follow the reference is demonstrated by simulation results shown in Figures 5 and 7.Table I gives the comparison between the open loop plant and the controllers in time domain. The peak overshoot (Mp) expressed as percentage value is given as equation (16)

$$\%M_p = \frac{Peak\ value - Steady\ State\ value}{Steady\ State\ value} \times 100 \quad (16)$$

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The steady state value of the output voltage of the converter for this paper is 5 volt and from Fig.5 above, the peak value of the output voltage obtained from the simulation result using Q-controller is 7.4 volt. Thus the Q-controller reduces the peak overshoot to 48% and SMC reduces it to 0.3% calculated using equation (16) and Fig.7. The settling time is calculated from the waveform within 2% tolerance band. Steady-state error for both the controller is zero which indicates absolute tracking of the set point value. Sliding Mode controller settles the system faster but with a negligible margin as compared to Q- controller. The robust stability is studied by varying the input voltage by step change from 12 to 24 volt. The results are shown in Fig 8 - 9.The figures illustrate that when the input voltage is varied from 12 to 24 volt. the output voltage of the converter with Q- controller varies with large transients from 0.05secs upto 0.08secs and then settles down whereas with SMC the variation is negligible. The output voltage varies considerably for a load variation from 20 to 40 using SMC and the Q- controller show negligible variation in the output voltage for small change in load resistance as shown in
Fig.10 -11.

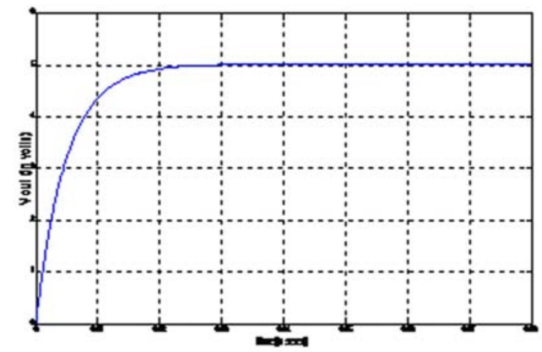

Fig.7: Output Voltage of plant with SMC Controller

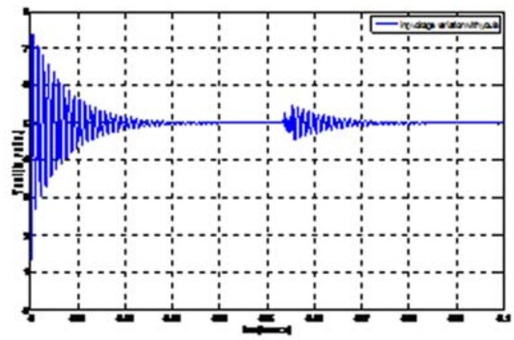

Fig. 8: Results for input voltage variations using Q-controller

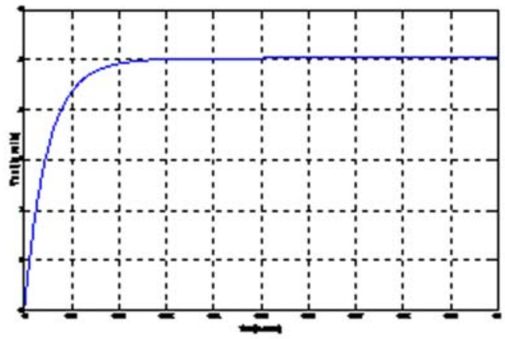

Fig. 9: Results for input voltage variation (SMC controller)

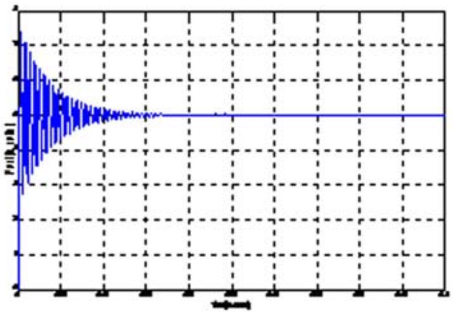

Fig. 10: Results for load variation (Q-controller)

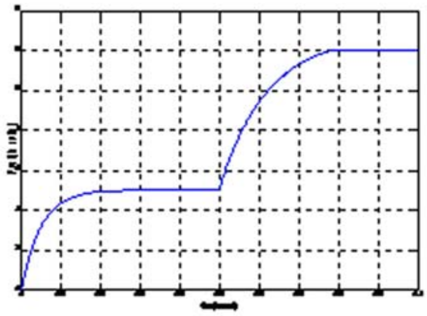

Fig. 11: Results for load variation (SMC controller)

Table I: Time Domain parameters

Performance in time domain	**Plant without controller**	**Q- controller**	**SMC Controller**
Rise Time	0.0005 sec	**0.00025 sec**	0.028secs
Peak Overshoot	92%	**48%**	**0.3%**
Peak Time	0.0005sec	**0.0005 sec**	0.036sec
Settling Time	0.035 sec	0.025 sec	**0.018sec**
Steady-state error	0.04	0	0

V.CONCLUSION

This paper has presented the modelling and analysis of dc-dc Buck converter. It also presented the use of Q-controller Parameterization to design a control system for regulating the output voltage as well proving robustness to the system. The switching mode converters present non-linear and time-variant characteristics hence is important to approximate the non-linear model for ease in application of linear controllers. With linear control techniques there is always a trade-off between system performance and robustness. According to the analysis the closed loop design is optimized for Q- controller as well as SMC, both provides better regulation to parameter variations. Table I shows that overshoot and settling time both are reduced using the Sliding Mode Controller whereas Peak time and Rise time are reduced using the Q- controller.

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