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A Squeeze-film Damping Model for the Circular Torsion Micro-resonators

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Abstract. In recent years, MEMS devices are widely used in many industries. The prediction of squeeze-film damping is very important for the research of high quality factor resonators. In the past, there have been many analytical models predicting the squeeze-film damping of the torsion micro-resonators. However, for the circular torsion micro-plate, the works over it is very rare. The only model presented by Xia et al[7] using the method of eigenfunction expansions. In this paper, The Bessel series solution is used to solve the Reynolds equation under the assumption of the incompressible gas of the gap, the pressure distribution of the gas between two micro-plates is obtained. Then the analytical expression for the damping constant of the device is derived. The result of the present model matches very well with the finite element method (FEM) solutions and the result of Xia's model, so the present models' accuracy is able to be validated.

1. Introduction

Squeeze-film damping, which is a main energy dissipation mechanism of MEMS resonators, plays an important part in the device's dynamic characteristics. There have been many studies over the torsion micro-mirrors in the past few years. The Reynolds equation is a common tool to calculate squeeze-film damping. In early year of 1962, Langlois[1] proposed a general equation for the compressible flow gas film, this equation is the traditional Reynolds equation. Pan et al[2] firstly linearized the nonlinear Reynolds equation under the assumption of small displacement between the fixed substrate and the movable mirror. By means of the solutions of Fourier series, they calculated the expressions of the damping pressure distribution by solving a simplified equation. Darling et al[3] derived the same expression based on the approach of Green's Function. Pandey et al[4-6] derived a modified Reynolds equation for perforated system. However, most of the previous works focused on the performance of the model of rectangular micro-plate, there are few models for the research of the circular micro-plate of a torsion micro-mirror, the one and only model proposed by Xia et al[7]. The modified Reynolds equation proposed by Bao[8] under the assumption of the incompressible gas is considered in this paper, and then extend this equation to the form of cylindrical coordinate. The analytical expression of the damping constant is developed using the Bessel series. Then we verified the accuracy of the model by comparing the present model with the FEM models as well as the result of Xia's model.

2. Problem formulation

The schematic view of the circular micro-plate is shown in Fig.1. R is the radius of the micro-plate, g_0 is the thickness of the air gap. Under the incompressible gas condition, the governing equation proposed by Bao et al[8] is

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{12\mu}{g_0^{\ 3}}\frac{\mathrm{d}h}{\mathrm{d}t} \tag{1}$$

where $p(x,y,t)$ is the air pressure of squeeze-film gap, $p(x,y,t)=p_a+\Delta p(x,y,t)$, p_a is the ambient pressure, μ is the standard viscosity, $h(t)$ is the gap thickness, the expression of $h(t)$ is, $h(t)=g_0+x\theta_0 e^{\omega t}$,where $\theta_0 \mathrm{e}^{\omega t}$ is the angle of the torsion micro-plate.

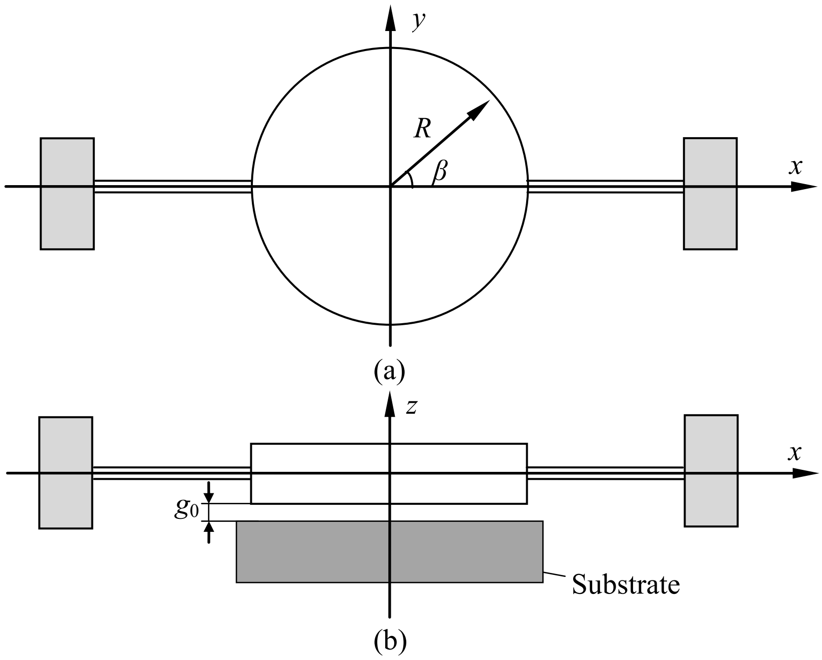

Fig. 1 The schematic drawing of a rigid circular micro-plate.
(a)Top view of the circular micro-plate. (b) Side view of the circular micro-plate.

The following nondimensional variable is introduced for convenience:

$$P(x,y,t)=\frac{\Delta p(x,y,t)}{p_a} \tag{2}$$

Substituting Eq.(2) into Eq.(1), then change the cartesian coordinate to cylindrical coordinate, leads to

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}+\frac{1}{r^2}\frac{\partial^2 P}{\partial \beta^2}=\frac{12\mu\theta_0}{h_0^{\ 3} p_a}\omega\cdot r\sin\beta\cdot e^{\omega t} \tag{3}$$

The boundary conditions is

$$P(R,\beta,t)=0 \tag{4}$$

The expression of $P(R,\beta,t)$ is able be expressed as a Bessel series

$$P(r,\beta,t)=\sum_{k=0}^{\infty}\sum_{q=1}^{\infty}J_k\left(\frac{a_{kq}}{R}r\right)[A_{kq}\sin k\beta+B_{kq}\cos k\beta]\mathrm{e}^{\omega t} \tag{5}$$

where A_{kq} and B_{kq} are complex amplitude to be determined, $J_k(a_{kq}/R\cdot r)$ is the first kind Bessel function of kth order. a_{kq} is the qth zero of $J_k(x)$. Obviously, Eq. (5) meets the boundary condition above. Substituting Eq.(5) into Eq.(3), by comparing each side of the equation, we obtain

$$A_{kq}=\begin{cases}0, & k\neq 1\\ A_{1q}, & k=1\end{cases},\quad B_{kq}=0 \tag{6}$$

So, Eq.(5) is able to be simplified as follows

$$P(r,\beta,t)=\sum_{q=1}^{\infty}A_{1q}\cdot J_1\left(\frac{a_{1q}}{R}r\right)\cdot\sin\beta\cdot\mathrm{e}^{\omega t} \tag{7}$$

Substituting Eq.(7) into Eq.(3), leads to

$$\sum_{q=1}^{\infty}A_{1q}\left[J_1''\left(\frac{a_{1q}}{R}r\right)+\frac{1}{r}J_1'\left(\frac{a_{1q}}{R}r\right)-\frac{1}{r^2}J_1\left(\frac{a_{1q}}{R}r\right)\right]=\frac{12\mu\theta_0}{{g_0}^3 p_a}\cdot\omega\, r \tag{8}$$

Using the orthogonality of the Bessel series, both sides of Eq.(8) multiply $rJ_1\left(\frac{a_{1q}}{R}r\right)$, then integrating the outcome of both sides from r=0 to R, we obtain

$$A_{1q}=-\frac{24\mu\omega\theta_0 R^3}{{g_0}^3 p_a}\cdot\frac{1}{{a_{1q}}^3 J_2(a_{1q})} \tag{9}$$

The squeeze film damping torque is able to be calculated as the integration of the pressure distribution acting on the circular plate, the expression is

$$T_{\mathrm{Squeeze}}=\int_0^R\int_0^{2\pi}(p-p_a)\cdot r\sin\beta\cdot r\mathrm{d}r\mathrm{d}\beta \tag{10}$$

So, the total damping torque is

$$\begin{aligned}T_{\mathrm{damping}}&=\mathrm{e}^{\omega t}\cdot p_a\sum_{q=1}^{\infty}A_{1q}\int_0^R\int_0^{2\pi}r^2 J_1\left(\frac{a_{1q}}{R}r\right)\sin^2\beta\mathrm{d}r\mathrm{d}\beta\\&=-24\pi\mu\omega\theta_0\cdot\frac{R^6\mathrm{e}^{\omega t}}{{g_0}^3}\cdot\sum_{q=1}^{\infty}\frac{1}{a_{1q}^4}\end{aligned} \tag{11}$$

So the corresponding damping constant C_θ is

$$C_\theta=-\frac{T_{\mathrm{damping}}}{\omega\theta_0\mathrm{e}^{\omega t}}=\frac{24\pi\mu R^6}{{g_0}^3}\cdot\sum_{q=1}^{\infty}\frac{1}{a_{1q}^4} \tag{12}$$

3. Validation

Now, we verify the accuracy of the present model by comparing the present model with the FEM models and also the results of Xia's model. In ANSYS, FLUID136 elements are used for modeling the squeeze-film. The parameters and dimensions of the circular micro-plate are listed in Table1.

Table1. The parameters and dimensions of the circular micro-plate

Parameters	Description	Values
R	Radius of the circular micro-plate	500μm
T_p	Thickness of the circular micro-plate	10μm
g_0	Gap spacing	5μm
p_a	Ambient pressure	101325 Pa
ps	Density	$2330\mathrm{kg\cdot m^{-3}}$
u	Viscosity coefficient	1.85×10^{-5} $\mathrm{N\cdot s\cdot m^{-2}}$
f	frequency	3000Hz

Fig.2 shows the comparison of damping constants over the results of present model and FEM models and the results of Xia's model as a function of R. Other parameters of the model set as Table1 shows. We varied the radius of the circular micro-plate from 50μm to 1000μm. We find the theoretical model in this paper agrees very well with FEM results as well as Xia's results in a wide range of radius, the discrepancy of the results is very small. We also find the damping constant increasing as the increase of the radius of the circular plate. Because the contact area between the plate and the gas is increasing exponentially as the increase of the size of circular micro-plate, so the corresponding damping constant is increasing.

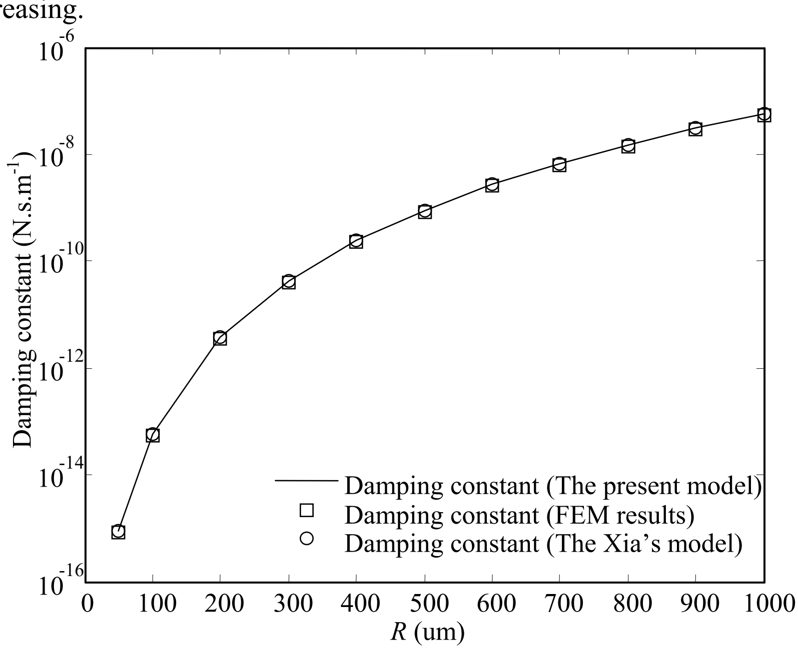

Fig. 2 The comparison of damping constants over the results of present model and the FEM models and the Xia's model under different *R*.

Fig.3 shows the comparison of damping constants over the results of present model and the FEM models and the results of Xia's model as a function of gap spacing from 2μm to 20μm. Similarly, the theoretical model in this paper agrees very well with FEM results as well as Xia's results in a wide range of gap spacing. We also find the damping constant decreasing as the increase of the gap spacing. Because the gas is easier to escape from the gap between two plates as the gap spacing is increasing, so the corresponding damping constant is decreasing.

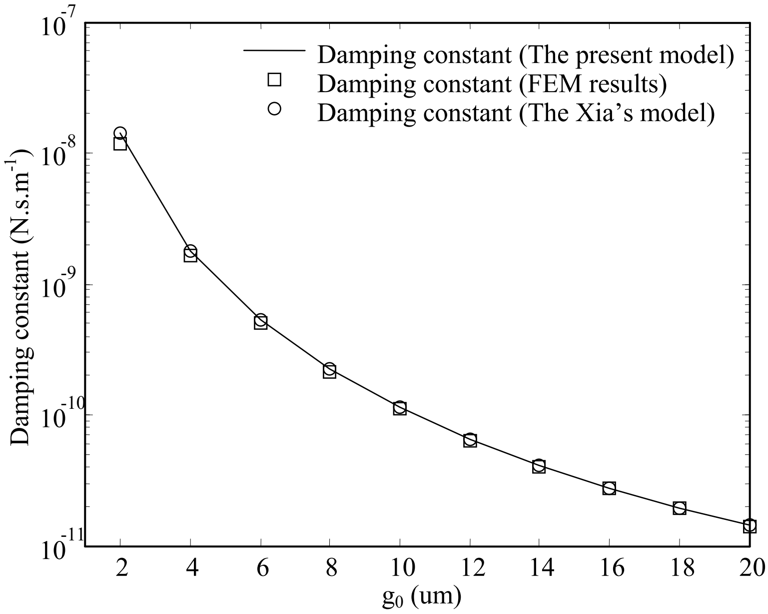

Fig. 3 The comparison of damping constants over the results of present model and the FEM models and the Xia's model as a function of g_0.

Fig.4 gives the rate of convergence for the circular micro-plate. As it shows, the difference between N=5 and N=20 is very small. For those equations we proposed, the sum of 5 terms is enough to achieve convergence. Therefore, we use the sum of 20 terms to calculate the squeeze-film damping in

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this paper. This simplification of the series calculation not only meets the requirement of accuracy but also save time on the calculation.

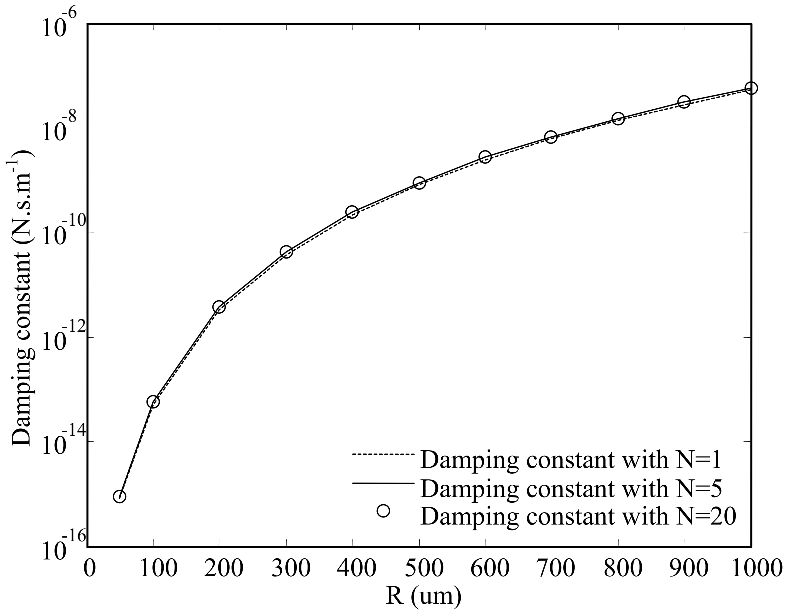

Fig.4 The comparison of damping constant in the cases of N = 1, 5 and 10

4. Conclusions

In this paper, an analytical model for calculating the squeeze-film damping of the circular torsion micro-plate is given. The gas of the squeeze-film gap between two micro-plates is considered as incompressible. The pressure distribution of the gas is approximated through the Bessel series solutions. Then we obtained the analytical expression of the damping constant of the device. Through comparison of damping constants over the results of present model and the FEM models and the Xia's model as a function of the radius of the circular micro-plate and gap spacing, we find the analytical model in this paper agrees very well with FEM results as well as Xia's results, the discrepancy of the results is very small. In this paper, to simplify the calculation of the squeeze-film damping, the sum of 20 terms is enough to realize convergence and accuracy.

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