

# A Reliability analysis on the applicability range of Caltrans-SDC method to address the P-Delta effects

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**Abstract.** The main objective of this research is to investigate the applicability range on the Caltrans Seismic Design Criteria (SDC) to address the P-Delta effect. Caltrans SDC defines a maximum ratio for bending moment induced by P-Delta effects over the yielding moment capacity of the column. For columns satisfying this criterion predefined target ductility is used to design the columns. Columns with higher P-Delta induced bending moments should be subjected to nonlinear time history analysis to verify their performance. Typically, designers to prevent performing time consuming, nonlinear time history analyses resize the column to remain in the safe margin. This study through rigorous push over analyses identifies the applicability range for the Caltrans-SDC method.

## 1. Introduction

P-Delta effects can have a detrimental impact on the seismic response of bridges because of a reduction in both the shear capacity and initial stiffness of RC bridge columns [1], [2]. The reduction in the initial stiffness imposes an increase in the natural period of the system, and a likely surge in the design displacement demand. In cases that it is not allowed to neglect the P-Delta effects, it is required to consider the dynamic effect of gravity loads acting through lateral displacements in design [3], [4]. Wei et al. [5] provides a detailed description on methodologies to compensate for P-Delta effects on structures. As design codes are progressing towards performance-based metrics, there is an additional need to quantify the destabilizing effect of gravity loads and its effect on the seismic response of bridge columns [6], [7].

## 2. Background

Caltrans SDC [3] provides a procedure that can be used to evaluate whether P-Delta effects can be ignored in design. In design circumstances, not requiring to consider P-Delta effects, structural



components can be designed based on predefined ductility demands. This study intends to find the range which (is valid. In cases which (is not satisfied, increasing the section size or reinforcement ratio can be used to increase the yielding moment capacity of the column, However, Caltrans SDC recommends to perform nonlinear time-history analysis to verify that the column is capable to resist the P-Delta effects.

$$P \times \Delta_r = 0.2 \times M_p^{col} \quad (1)$$

Where,  $\Delta_r$  is the lateral offset between the point of contra-flexure and the base of the plastic hinge, and  $M_p^{col}$  is the idealized plastic moment capacity of a column calculated by M- $\phi$  analysis. If (is satisfied, predefined ductility demands limits the design of structural components. According to Caltrans SDC target displacement ductility for single column and multi-column bents are as follows.

Single Column Bents supported on fixed foundation  $\mu_D \leq 4$

Multi-Column Bents supported on fixed or pinned footings  $\mu_D \leq 5$

### 3. Method

In this research using OpenSees, nonlinear static analysis (Pushover) is performed to obtain the moment-curvature and subsequently the load-deformation for various columns. The main interest is to obtain the value for the applied load and displacement at yielding and ultimate capacity of the column. The value for yielding displacement and load can be obtained by linear interpolation between the yielding and the ultimate point.

Table 1 shows the Column Properties subjected to this study.

**Table 1.** Column Properties

Parameters	
Concrete Strength, $f'_c$ (MPa, ksi)	27.5 (4)
yield Strength, $f_y$ (MPa, ksi)	413 (60.0)
Modulus of elasticity, $E_s$ (MPa, ksi)	$2 \times 10^5$ (29,000)
Longitudinal reinforcing steel: yield strain, $\epsilon_y$	0.0015
Column diameter, $L$ (m, ft)	1.21 (4)
Column aspect ratio, CAR	4 to 12
Cover concrete (cm, in)	5 (2)
Axial load (kips)	389,589,778,973,1168

#### 3.1 Finite element model

Open System for Earthquake Engineering Simulation (OpenSees) [8] is used to investigate the nonlinear load-deformation response of RC bridge columns. The circular cross-section was represented by a fiber-based model and the concrete cover and core sections were modelled with the "Concrete07" uniaxial concrete material class.

### 4. Results

This section looks at nine different column height ratios ranging between four to twelve. longitudinal reinforcement ration is changing between one to four percent. Five different levels of axial load is applied on top of the column. Target displacement ductility level used to design the columns is obtained from the Caltrans SDC, which suggests single column bents supported on fixed foundation can be designed for ductility level 4.

**Table 2.** Yielding displacement

ALR	LRR	$M_y$ (kip.in)	4	5	6	7	8	9	10	11	12
389kips	1	26253	1.92	2.19	2.51	2.96	3.51	4.14	4.80	5.57	6.39
	2	45670	2.96	3.27	3.71	4.31	5.05	5.88	6.79	7.88	9.04
	3	62245	3.76	4.09	4.64	5.32	6.18	7.21	8.37	9.58	10.94
	4	80691	4.63	4.98	5.60	6.39	7.38	8.55	9.90	11.38	13.01
589 kips	1	28445	2.03	2.27	2.61	3.08	3.65	4.23	4.99	5.79	6.65
	2	47400	3.04	3.34	3.80	4.43	5.13	5.98	6.95	8.01	9.20
	3	64628	3.84	4.17	4.73	5.43	6.32	7.31	8.43	9.76	11.14
	4	82067	4.69	5.05	5.66	6.47	7.47	8.65	10.02	11.48	13.20
778 kips	1	30480	2.08	2.35	2.78	3.19	3.73	4.40	5.10	5.93	6.80
	2	49664	3.09	3.42	3.86	4.51	5.22	6.08	7.06	8.19	9.33
	3	66127	3.91	4.25	4.81	5.52	6.43	7.44	8.63	9.89	11.30
	4	83392	4.74	5.10	5.72	6.54	7.57	8.76	10.10	11.62	13.29
973kips	1	32727	2.18	2.43	2.89	3.32	3.89	4.58	5.31	6.17	7.08
	2	51245	3.16	3.46	3.93	4.61	5.37	6.22	7.17	8.31	9.48
	3	67557	3.97	4.31	4.87	5.61	6.48	7.50	8.71	10.03	11.46
	4	84676	4.81	5.17	5.81	6.63	7.67	8.85	10.25	11.75	13.44
1168 kips	1	34636	2.30	2.53	2.92	3.41	4.05	4.71	5.46	6.35	7.29
	2	52799	3.23	3.55	4.05	4.65	5.42	6.32	7.35	8.45	9.72
	3	68956	4.04	4.39	4.95	5.70	6.60	7.65	8.88	10.18	11.63
	4	85942	4.87	5.24	5.86	6.72	7.77	8.96	10.34	11.90	13.62

(All results are in inch)

The P-Delta induced bending moment at target ductility level of four is equal to four times of the yielding displacement (**Table 2**) multiplied by the load at ductility level four (**Table 3**). According to Caltrans SDC if the ratio of bending moment induced by P-Delta effects to the yielding moment capacity of column is less the twenty percent, then structural components shall be designed based on predefined displacement ductility demands.

**Table 3.** Load at ductility level 4(interpolated from idealized bilinear force-displacement diagram)

		Column height ratio								
	$\rho_L$	4	5	6	7	8	9	10	11	12
389kips	1	154.04	122.51	101.75	87.00	76.04	67.53	60.77	55.19	50.60
	2	278.58	218.26	179.73	153.07	133.57	118.34	106.27	96.61	88.45
	3	399.05	307.80	251.79	213.08	185.14	163.90	147.16	133.42	122.05
	4	540.04	411.48	333.88	281.25	243.59	215.27	192.93	174.78	159.88

<b>589 kips</b>	1	164.32	130.58	108.35	92.65	80.99	71.89	64.73	58.79	53.90
	2	288.45	225.76	185.90	158.35	137.89	122.18	109.77	99.73	91.31
	3	408.59	315.34	257.77	218.28	189.72	167.82	150.45	136.56	124.93
	4	NA	418.89	339.46	286.05	247.81	218.76	196.11	177.66	162.64
<b>778 kips</b>	1	173.79	138.00	114.60	97.90	85.54	75.97	68.30	62.10	56.88
	2	298.07	233.15	191.69	163.18	142.06	125.99	113.09	102.70	94.06
	3	417.80	322.38	263.42	223.09	193.76	171.41	153.77	139.41	127.54
	4	NA	425.26	344.71	290.50	251.68	222.19	199.02	180.33	164.96
<b>973 kips</b>	1	183.71	145.76	121.00	103.38	90.32	80.22	72.12	65.52	60.06
	2	307.56	240.01	197.39	168.05	146.28	129.63	116.30	105.62	96.74
	3	426.88	328.95	268.59	227.49	197.58	174.62	156.81	142.25	130.15
	4	NA	431.84	349.99	294.91	255.54	225.51	202.02	183.07	167.44
<b>1168 kips</b>	1	193.53	153.33	127.14	108.69	95.00	84.34	75.83	68.90	63.16
	2	316.22	246.74	202.85	172.35	150.17	133.05	119.56	108.42	99.37
	3	435.61	335.78	274.10	232.10	201.57	178.22	159.91	145.00	132.66
	4	NA	438.12	354.98	299.23	259.16	228.70	204.87	185.66	169.81

Table 4 presents the results obtained for the Caltrans SDC criterion for ignoring the P-Delta effects. Regions colored in green are where Caltrans allows ignoring the P-Delta effects, while the red region requires performing nonlinear time-history analysis to study the structural behavior with inclusion of the P-Delta effects. Column with high column height ratio and high axial load ratio are the most susceptible columns to P-Delta effects. Results indicate that for these columns Caltrans SDC equation is not satisfied and nonlinear time-history analysis should be implemented to make any decision regarding the adequacy of the design. Typically under these circumstances design firms in order to prevent performing time consuming and highly advanced nonlinear time history analysis with a wide range of earthquake records they prefer to resize the column in order to satisfy the Caltrans equation.

**Table 4.** Ratio of P-Delta induced bending moment to plastic moment capacity

		Column height ratio									
		$\rho_L$	4	5	6	7	8	9	10	11	12
<b>389 kips</b>	1	0.11	0.13	0.15	0.18	0.21	0.25	0.28	0.33	0.38	
	2	0.10	0.11	0.13	0.15	0.17	0.20	0.23	0.27	0.31	
	3	0.09	0.10	0.12	0.13	0.15	0.18	0.21	0.24	0.27	
	4	0.09	0.10	0.11	0.12	0.14	0.17	0.19	0.22	0.25	
<b>589 kips</b>	1	0.17	0.19	0.21	0.25	0.30	0.35	0.41	0.48	0.55	
	2	0.15	0.16	0.19	0.22	0.25	0.29	0.34	0.39	0.45	
	3	0.14	0.15	0.17	0.20	0.23	0.26	0.30	0.35	0.40	
	4	0.13	0.14	0.16	0.18	0.21	0.25	0.29	0.33	0.38	
<b>778 kips</b>	1	0.21	0.24	0.28	0.33	0.38	0.45	0.52	0.61	0.70	
	2	0.19	0.21	0.24	0.28	0.33	0.38	0.44	0.51	0.59	
	3	0.18	0.20	0.23	0.26	0.30	0.35	0.41	0.47	0.53	
	4	0.18	0.19	0.21	0.24	0.28	0.33	0.38	0.43	0.50	
<b>973 kips</b>	1	0.26	0.29	0.34	0.40	0.46	0.55	0.63	0.73	0.84	
	2	0.24	0.26	0.30	0.35	0.41	0.47	0.54	0.63	0.72	
	3	0.23	0.25	0.28	0.32	0.37	0.43	0.50	0.58	0.66	
	4	0.22	0.24	0.27	0.30	0.35	0.41	0.47	0.54	0.62	
<b>1168 kips</b>	1	0.31	0.34	0.39	0.46	0.55	0.64	0.74	0.86	0.98	
	2	0.29	0.31	0.36	0.41	0.48	0.56	0.65	0.75	0.86	

3	0.27	0.30	0.34	0.39	0.45	0.52	0.60	0.69	0.79
4	NA	0.28	0.32	0.37	0.42	0.49	0.56	0.65	0.74

## 5. Result evaluation

In this section a sample column with height of 32 ft with 1 % reinforcement ratio, and 389 kips applied load has been studied under a set of earthquake records. This column fails to satisfy the Caltrans SDC criterion.

Throughout this research ATC Far-Field, ground motion record set is used. The ground motion set is collected from Pacific Earthquake Engineering Research Center (PEER-NGA) database. Table 5 and Table 6 tabulate the characteristics of the ground motion set.

**Table 5.** Ground motion properties

Distance R	R > 10 km
Large Magnitude Events	M > 6.5
Equal Weighting of Events	≤ 2 records per event
Strong Ground Shaking	PGA > 0.2g /PGV > 15 cm/sec
Source Type	Both Strike-Slip and Thrust Fault Sources
Site Conditions	Rock or Stiff Soil Sites Vs > 180 m/s
Record Quality	Lowest Useable Frequency < 0.25 Hz

Magnitude and PGA for the Far-Field earthquake records are tabulated in Table 6.

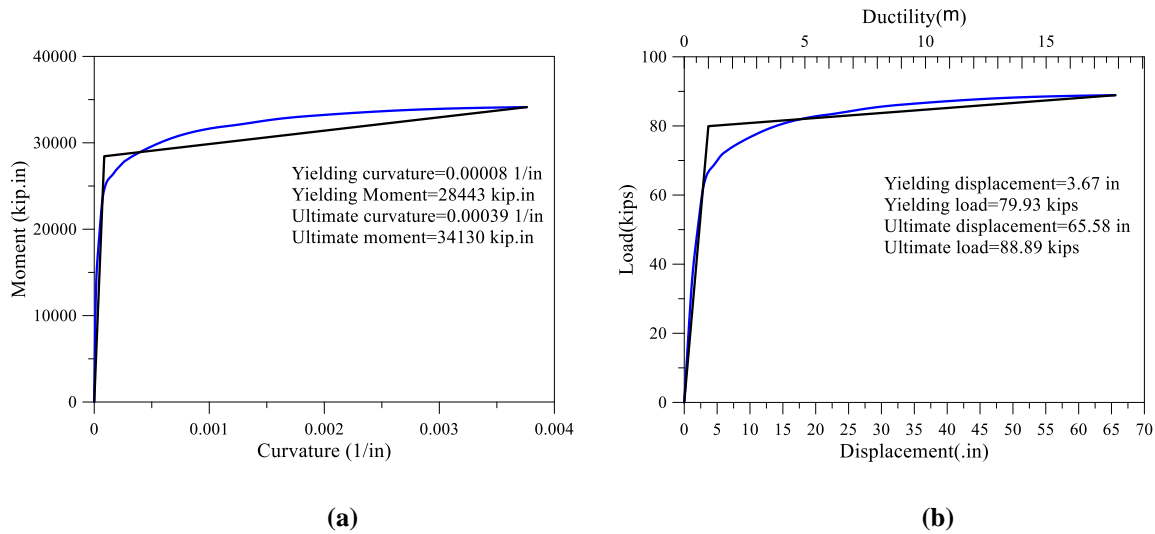
**Table 6.** Ground motion records

EQ ID	Earthquake			PGA (g)	EQ ID	Earthquake			PGA (g)
	M	Year	Name			M	Year	Name	
12011	6.7	1994	Northridge	0.52	12092	7.3	1992	Landers	0.42
12012	6.7	1994	Northridge	0.48	12101	6.9	1989	Loma Prieta	0.53
12041	7.1	1999	Duzce, Turkey	0.82	12102	6.9	1989	Loma Prieta	0.56
12052	7.1	1999	Hector Mine	0.34	12111	7.4	1990	Manjil, Iran	0.51
12061	6.5	1979	Imperial Valley	0.35	12121	6.5	1987	Superstition Hills	0.36
12062	6.5	1979	Imperial Valley	0.38	12122	6.5	1987	Superstition Hills	0.45
12071	6.9	1995	Kobe, Japan	0.51	12132	7.0	1992	Cape Mendocino	0.55
12072	6.9	1995	Kobe, Japan	0.24	12141	7.6	1999	Chi-Chi, Taiwan	0.44
12081	7.5	1999	Kocaeli, Turkey	0.36	12142	7.6	1999	Chi-Chi, Taiwan	0.51
12082	7.5	1999	Kocaeli, Turkey	0.22	12151	6.6	1971	San Fernando	0.21
12091	7.3	1992	Landers	0.24	12171	6.5	1976	Friuli, Italy	0.35

The procedure is as the following steps:

### Step1: Perform pushover analysis and obtain load-deformation diagram

Figure 1 shows the result obtained from pushover analysis.



**Figure 1.** Moment-curvature and pushover analysis (Col spec: Col Height=32 ft., Axial load=389 kips,  $\rho_L=1\%$ ).

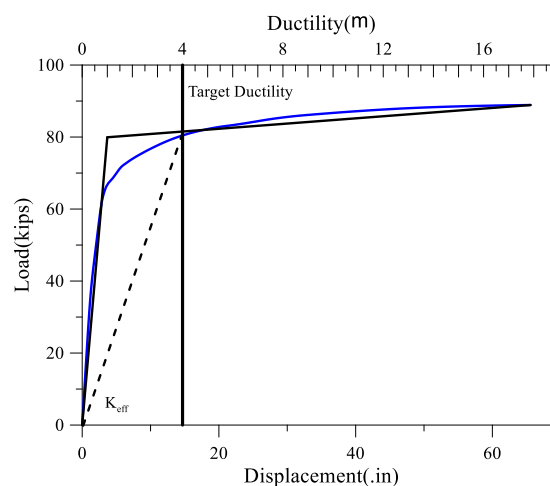
### Step2: Compute the target spectral displacement

The target displacement at the target ductility can be obtained using (2). Design target ductility of four is suggested by Caltrans SDC for single column bent supported on fixed foundation.

$$SD_{cap} = \mu \cdot \Delta_y = 14.68 \text{ in} \quad (2)$$

$$\mu = 4$$

Figure 2 shows the procedure to compute the effective stiffness at target ductility. Force-displacement analysis provides the load and displacement values at yielding and ultimate points.



**Figure 2.** Effective stiffness at target ductility of 4.

### Step3: Compute the effective period of the column at the target ductility

The lateral load at the target ductility can be obtained using (.

$$F_{target} = F_y + \frac{F_u - F_y}{\Delta_u - \Delta_y} (\Delta_y(\mu - 1)) = 81.52 \text{ kips} \quad (3)$$

Effective stiffness at target ductility can be obtained using (.

$$K_{eff} = \frac{F_{target}}{\mu \Delta_y} = 5.55 \text{ kips/inch} \quad (4)$$

The effective period of structure can be obtained using

$$T_{eff} = 2\pi \sqrt{\frac{W}{g K_{eff}}} = 2.67 \text{ sec} \quad (5)$$

#### Step4: Compute the earthquake record scale factor

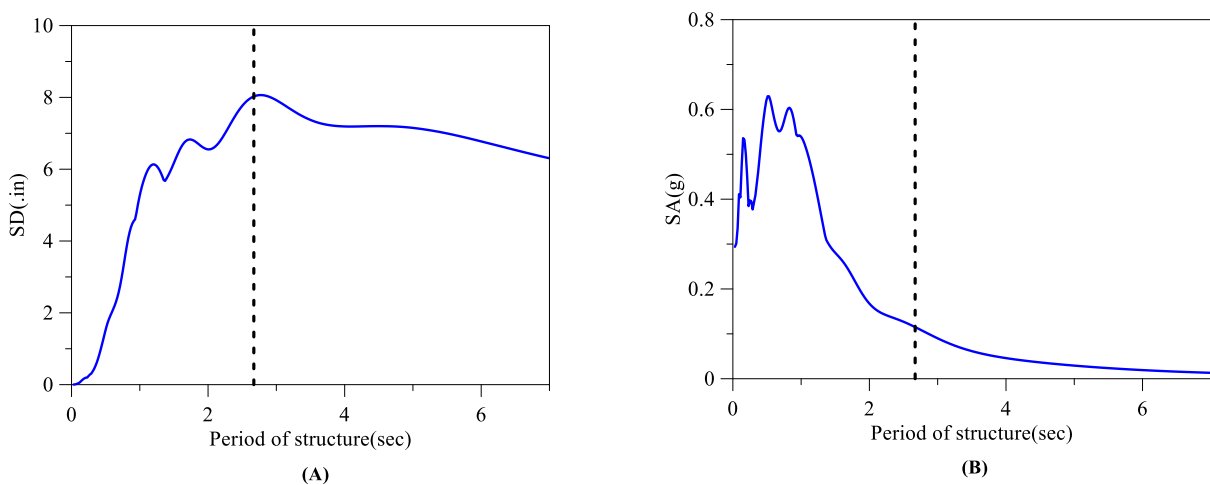
Expression proposed by Chopra and Goel (1999),  $\zeta_{eq}^{\wedge} = \zeta + \zeta_{eq}$  was used for total viscous damping ratio.

$$\zeta_{eq} = \frac{1}{\pi} \left( 1 - \frac{0.95}{\sqrt{\mu}} - 0.05\sqrt{\mu} \right) \quad (6)$$

$$\mu = 4, \zeta_{eq} = 0.185$$

Where,  $\mu$  is the target ductility level and  $\zeta$  is the elastic viscous damping and  $\zeta_{eq}$  is equivalent viscous damping.

Newmark integration method is used to develop spectral displacement and spectral acceleration versus period of structure. Figure 3(A) shows the spectral displacement versus period of the structure for the earthquake record with EQID=120111 with 18.5% damping ratio. Figure 3 (B) shows the spectral acceleration versus time. For a structure with the period of 6.14 sec the spectral displacement is 8.02 inch.



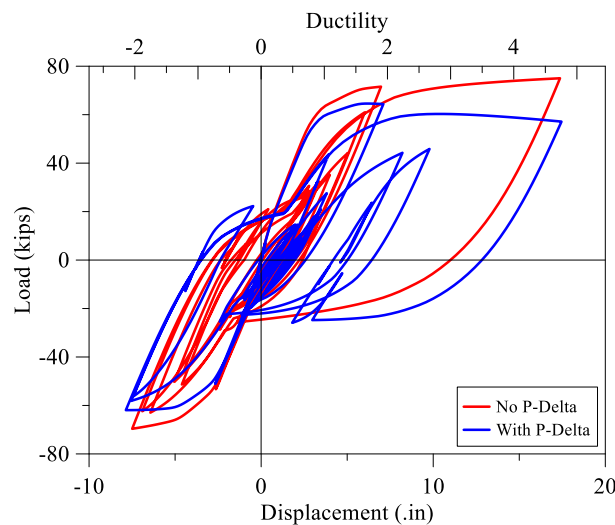
**Figure 3.** Response spectrum (EQID=120111,  $\zeta_{eq} = 0.185$ ).

The earthquake scale factor is the ratio of spectral displacement computed in previous step to the spectral displacement for the non-scaled earthquake record (at the effective period).

$$scalefactor = \frac{SD_{capacity}}{SD_{demand}} = \frac{14.68}{8.02} = 1.83 \quad (7)$$

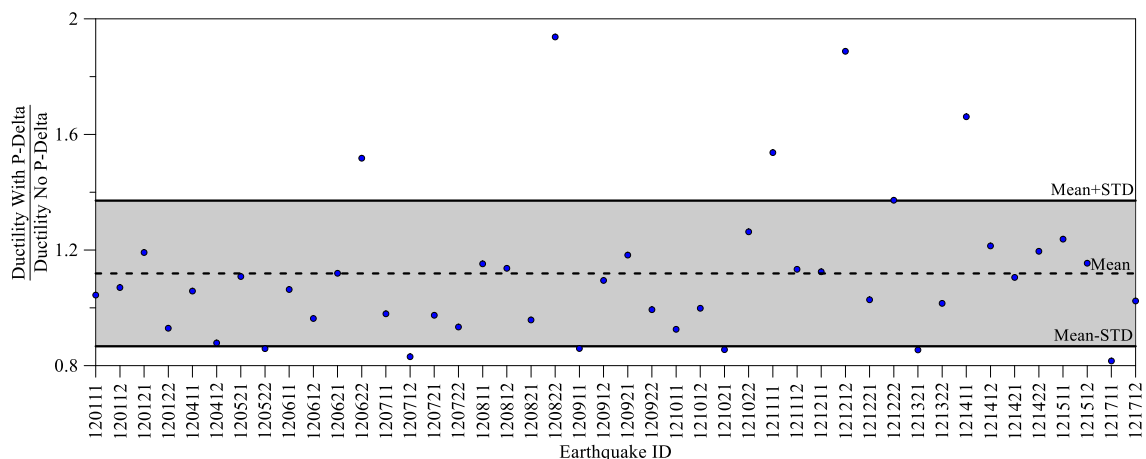
#### Step5: Perform nonlinear time-history analysis

In this stage the earthquake record is amplified by the by the earthquake scale factor. Nonlinear time history response for the structure is presented.



**Figure 4.** Dynamic inelastic time history analysis (EQID=120111)

Same analysis is performed on all the earthquake records introduced in Table 6. Figure 5 shows the ratio between the displacement ductility with P-Delta effects over displacement ductility ignoring the P-Delta effects for all earthquake records. The result from nonlinear time-history analysis indicates that although this column does not comply with the Caltrans SDC criterion for ignoring the P-Delta effects, the column is stable and the damage ductility is close to target ductility of four.



**Figure 5.** Obtained results for all earthquake records.



The mean value for ductility with P-Delta effects over the ductility without P-Delta effects for the studied column is 1.119 with standard deviation of 0.25.

## 6. Conclusion

Following observations have been made as the result of this research.

- Caltrans SDC method for columns with high aspect ratios(10-12) was not applicable, although these columns are most susceptible to P-Delta effects.
- Caltrans SDC encourages designers to use higher longitudinal reinforcement ratio or bigger section sizes instead of performing nonlinear time-history analysis.
- This method does not explicitly address the instability of the column.

## References

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