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Investigation of the processes of deformation of super flywheels made by packing its layers on each other

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Abstract. In the work for different cases of nozzles of layers of the super-magnetic pendulum, exact solutions in the stresses are obtained, and estimates of the increase in its specific energy are given. Dependences of tensile stresses in a tape are obtained when winding it into a super-flywheel, an increase in the specific kinetic energy in the super flywheel is observed with a decrease in the relative radius of the inner hole.

1. Introduction

In modern realities, the problem of accumulating energy becomes more important. Simple enough is the technology of energy storage by means of kinetic storage. The main limitation in implementing such a technology is the rotational speed of the rotor part of the kinetic storage, which can lead to an increase in the stress level and its possible further destruction. In the literature, we give some estimates of the specific kinetic energy of rotation of the flywheels of the canonical form [1-9]. For flywheels of arbitrary shape, numerical methods for investigating the specific kinetic energy are proposed in the literature. In a number of works kinetic energy storage devices made of rubber-like materials are proposed, for the calculation of which special methods are proposed [10-14]. Calculation of super-flywheels, made by winding threads or tapes, presupposes methods for their contact interaction [15-21]. In this paper, we obtain some exact solutions in the stresses for super flywheels, made by winding and subsequent gluing of tapes.

2. Kinematics

A tape super flywheel with an acceptable degree of accuracy can be represented in the form of a system of concentric cylinders (Fig. 1), clad against each other with some interference. In this case, even before the start of loading in the super-flywheel, there will be some stress-strain state, which for two cylinders can be determined by calculating the so-called composite cylinders [22].

When calculating a compound cylinder (for definiteness consisting of two cylinders), the main thing is to establish the amount of pressure on the contact surface for a given interference, which is the difference between the outer diameter of the inner cylinder and the inner diameter of the outer cylinder (Fig. 2). It is obvious that the decrease in the outer radius of the inner cylinder and the increase in the inner radius of the outer cylinder are equal to half the interference

$$\left|u_r^{in}\right| + \left|u_r^{out}\right| = \delta/2 \tag{1}$$

Given that the interference is very small compared to the contact surface radius, it can be assumed that the outer radius of the inner cylinder $r^{out}_{in\_cyl}$ and the inner radius of the outer cylinder $r^{in}_{out\_cyl}$ are equal to the radius of the contact surface

$$r^{out}_{in\_cyl} = r^{in}_{out\_cyl} = r_c \,. \tag{2}$$

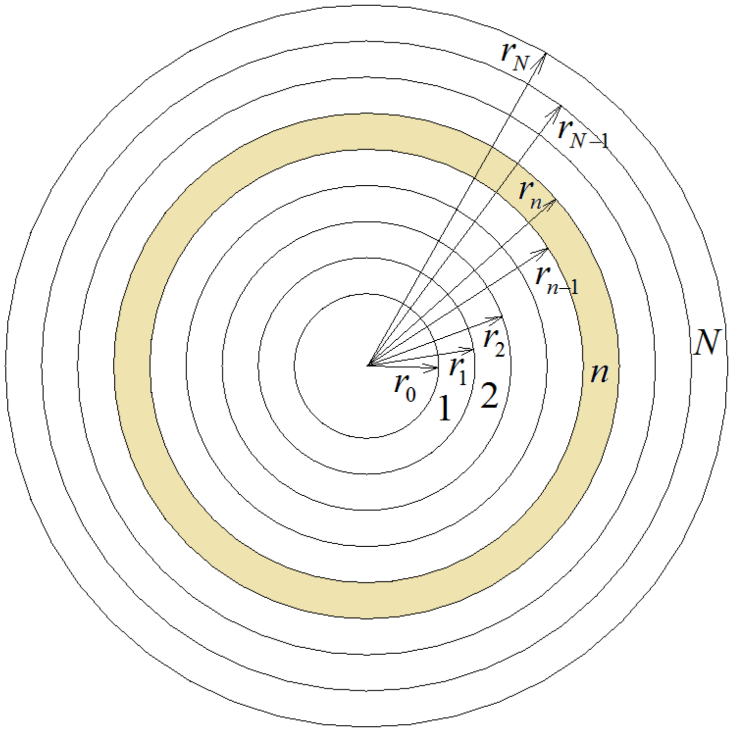

Figure 1. The system of concentric cylinders.

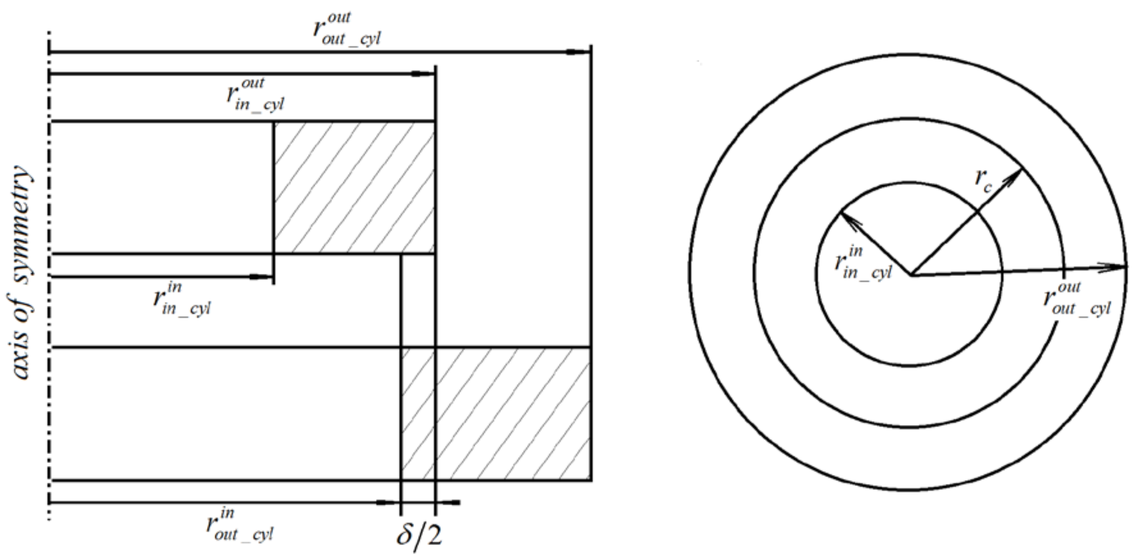

Figure 2. Scheme of attachment of two cylinders with interference.

The contact pressure will be external for the inner cylinder and internal for the outer cylinder. Thus, to determine the stress-strain state, we need a solution of the Lame problem on the deformation of cylinders under the influence of internal and external pressure. Formulas for determining radial displacements and circumferential and radial stresses in a cylinder with internal r_1 and external radius r_2 under internal p_1 and external p_2 pressure in the case of a plane stress state have the form

$$u_r = \frac{p_1 r_1^2\left[(1-\mu)r + (1+\mu) r_2^2 r^{-1}\right]}{E\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2\left[(1-\mu)r + (1+\mu) r_1^2 r^{-1}\right]}{E\left(r_2^2 - r_1^2\right)}, \tag{3}$$

$$\sigma_{rr} = \frac{p_1 r_1^2\left(1 - r_2^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2\left(1 - r_1^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)}, \tag{4}$$

$$\sigma_{\varphi\varphi} = \frac{p_1 r_1^2\left(1 + r_2^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)} - \frac{p_2 r_2^2\left(1 + r_1^2 r^{-2}\right)}{\left(r_2^2 - r_1^2\right)}, \tag{5}$$

where E and μ - is the Young's modulus and Poisson's ratio of the material.

Then the radial displacement of the contact surface of the inner cylinder is determined by the formula

$$u_r^{in}(r_c) = -\frac{p_c r_c^2\left[(1-\mu) r_c + (1+\mu)\left(r_{n\_cyl}^{in}\right)^2 r_c^{-1}\right]}{E\left(r_c^2 - \left(r_{n\_cyl}^{in}\right)^2\right)}, \tag{6}$$

and the radial displacement of the contact surface of the inner cylinder is determined by the formula

$$u_r^{out}(r_c) = \frac{p_c r_c^2\left[(1-\mu) r_c + (1+\mu)\left(r_{n\_cyl}^{out}\right)^2 r_c^{-1}\right]}{E\left(\left(r_{n\_cyl}^{out}\right)^2 - r_c^2\right)}. \tag{7}$$

If the inner and outer cylinders are made of the same material, then the relationship between the contact pressure and interference is obtained in the form

$$p_c = \frac{\delta E}{2r_c} \frac{\left(1 - \left(r_{in\_cyl}^{in} / r_c\right)^2\right)\left(1 - \left(r_c / r_{out\_cyl}^{out}\right)^2\right)}{\left(1 + \left(r_{in\_cyl}^{in} / r_c\right)^2\right)\left(1 - \left(r_c / r_{out\_cyl}^{out}\right)^2\right) + \left(1 - \left(r_{in\_cyl}^{in} / r_c\right)^2\right)\left(1 + \left(r_c / r_{out\_cyl}^{out}\right)^2\right)}. \tag{8}$$

If the thickness of each cylinder t is the same, then the inner and outer radii of the i-th cylinder are determined by the formulas

$$r_i^{in} = r_{i-1} = r_0 + (i-1)t,\; r_i^{out} = r_i = r_0 + it. \tag{9}$$

Then for the n-th cylinder the stresses will be determined by the formulas

$$\sigma_{rr}^n = p_n^{in} \frac{r_{n-1}^2}{r^2} \frac{r^2 - r_n^2}{r_n^2 - r_{n-1}^2} - \sum_{i=n}^{N-1} p_i^{out} \frac{r_i^2}{r^2} \frac{r^2 - r_0^2}{r_i^2 - r_0^2}, \tag{10}$$

$$\sigma_{\varphi\varphi}^n = p_n^{in} \frac{r_{n-1}^2}{r^2} \frac{r^2 + r_n^2}{r_n^2 - r_{n-1}^2} - \sum_{i=n}^{N-1} p_i^{out} \frac{r_i^2}{r^2} \frac{r^2 + r_0^2}{r_i^2 - r_0^2}, \tag{11}$$

where

$$p_i^{in} = \frac{\delta_{i-1} E}{2r_{i-1}} \frac{\left(r_{i-1}^2 - r_0^2\right)\left(r_i^2 - r_{i-1}^2\right)}{\left(r_{i-1}^2 + r_0^2\right)\left(r_i^2 - r_{i-1}^2\right) + \left(r_{i-1}^2 - r_0^2\right)\left(r_i^2 + r_{i-1}^2\right)}, \tag{12}$$

$$p_i^{out} = \frac{\delta_i E}{2r_i} \frac{\left(r_i^2 - r_0^2\right)\left(r_{i+1}^2 - r_i^2\right)}{\left(r_i^2 + r_0^2\right)\left(r_{i+1}^2 - r_i^2\right) + \left(r_i^2 - r_0^2\right)\left(r_{i+1}^2 + r_i^2\right)}, \tag{13}$$

and δ_i is the interference on the outer surface of the n-th cylinder ($\delta_0 = 0$), then

$$\sigma_{rr}^{n}=\frac{\delta_{n-1}E}{2}\frac{\left(r_{n-1}^{2}-r_{0}^{2}\right)r_{n-1}}{\left(r_{n-1}^{2}+r_{0}^{2}\right)\left(r_{n}^{2}-r_{n-1}^{2}\right)+\left(r_{n-1}^{2}-r_{0}^{2}\right)\left(r_{n}^{2}+r_{n-1}^{2}\right)}\frac{r^{2}-r_{n}^{2}}{r^{2}}-$$
$$-\sum_{i=n}^{N-1}\frac{\delta_{i}E}{2}\frac{\left(r_{i+1}^{2}-r_{i}^{2}\right)r_{i}}{\left(r_{i}^{2}+r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)+\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}+r_{i}^{2}\right)}\frac{r^{2}-r_{0}^{2}}{r^{2}}$$
$$\sigma_{\varphi\varphi}^{n}=\frac{\delta_{n-1}E}{2}\frac{\left(r_{n-1}^{2}-r_{0}^{2}\right)r_{n-1}}{\left(r_{n-1}^{2}+r_{0}^{2}\right)\left(r_{n}^{2}-r_{n-1}^{2}\right)+\left(r_{n-1}^{2}-r_{0}^{2}\right)\left(r_{n}^{2}+r_{n-1}^{2}\right)}\frac{r^{2}+r_{n}^{2}}{r^{2}}-$$
$$-\sum_{i=n}^{N-1}\frac{\delta_{i}E}{2}\frac{\left(r_{i+1}^{2}-r_{i}^{2}\right)r_{i}}{\left(r_{i}^{2}+r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)+\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}+r_{i}^{2}\right)}\frac{r^{2}+r_{0}^{2}}{r^{2}}. \quad (14)$$

The circumferential stresses in winding in the i-th turn are determined in the form can be defined as the circumferential stresses in the i-th cylinder when it is pressed with interference onto the already assembled part of the super flywheel

$$\sigma_{\varphi}^{i}=\frac{p_{i}^{in}r_{i-1}^{2}\left(1+r_{i}^{2}r^{-2}\right)}{\left(r_{i}^{2}-r_{i-1}^{2}\right)}=\frac{\delta_{i-1}E}{2r_{i-1}\left(r_{i}^{2}-r_{i-1}^{2}\right)}\frac{\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(r_{i}^{2}-r_{i-1}^{2}\right)r_{i-1}^{2}\left(1+r_{i}^{2}r^{-2}\right)}{\left(r_{i-1}^{2}+r_{0}^{2}\right)\left(r_{i}^{2}-r_{i-1}^{2}\right)+\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(r_{i}^{2}+r_{i-1}^{2}\right)}=$$
$$=\frac{\delta_{i-1}E}{2}\frac{r_{i-1}\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(1+r_{i}^{2}r^{-2}\right)}{\left(r_{i-1}^{2}+r_{0}^{2}\right)\left(r_{i}^{2}-r_{i-1}^{2}\right)+\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(r_{i}^{2}+r_{i-1}^{2}\right)}. \quad (15)$$

If the tightness of all the same, the formula for the radial and circumferential stress to the n-th cylinder will take the form

$$\sigma_{rr}^{n}=\frac{\delta E}{2}\left(\frac{\left(r_{n-1}^{2}-r_{0}^{2}\right)r_{n-1}}{\left(r_{n-1}^{2}+r_{0}^{2}\right)\left(r_{n}^{2}-r_{n-1}^{2}\right)+\left(r_{n-1}^{2}-r_{0}^{2}\right)\left(r_{n}^{2}+r_{n-1}^{2}\right)}\frac{r^{2}-r_{n}^{2}}{r^{2}}-\right.$$
$$\left.-\sum_{i=n}^{N-1}\frac{\left(r_{i+1}^{2}-r_{i}^{2}\right)r_{i}}{\left(r_{i}^{2}+r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)+\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}+r_{i}^{2}\right)}\frac{r^{2}-r_{0}^{2}}{r^{2}}\right), \quad (16)$$
$$\sigma_{\varphi\varphi}^{n}=\frac{\delta E}{2}\left(\frac{\left(r_{n-1}^{2}-r_{0}^{2}\right)r_{n-1}}{\left(r_{n-1}^{2}+r_{0}^{2}\right)\left(r_{n}^{2}-r_{n-1}^{2}\right)+\left(r_{n-1}^{2}-r_{0}^{2}\right)\left(r_{n}^{2}+r_{n-1}^{2}\right)}\frac{r^{2}+r_{n}^{2}}{r^{2}}-\right.$$
$$\left.-\sum_{i=n}^{N-1}\frac{\left(r_{i+1}^{2}-r_{i}^{2}\right)r_{i}}{\left(r_{i}^{2}+r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)+\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}+r_{i}^{2}\right)}\frac{r^{2}+r_{0}^{2}}{r^{2}}\right). \quad (17)$$

The circumferential stresses in winding in the i-th turn are determined in the form

$$\sigma_{\varphi}^{i}=\frac{\delta E}{2}\frac{r_{i-1}\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(1+r_{i}^{2}r^{-2}\right)}{\left(r_{i-1}^{2}+r_{0}^{2}\right)\left(r_{i}^{2}-r_{i-1}^{2}\right)+\left(r_{i-1}^{2}-r_{0}^{2}\right)\left(r_{i}^{2}+r_{i-1}^{2}\right)}\approx$$
$$\approx\frac{\delta E}{2}\frac{(i-1)\left(2r_{0}+(i-1)t\right)}{2r_{0}^{2}+2r_{0}(i-1)t+(i-1)^{2}t^{2}+(i-1)\left(2r_{0}+(i-1)t\right)\left(r_{0}+(i-1)t\right)}. \quad (18)$$

If we assume that the contact pressure between all the cylinders are the same and are equal to p_c, then in this case the interference between the n-th cylinder and the $(n+1)$-th will be determined by the formula

$$\delta_{i}=\frac{2p_{c}r_{i}}{E}\frac{\left(r_{i}^{2}+r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)+\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}+r_{i}^{2}\right)}{\left(r_{i}^{2}-r_{0}^{2}\right)\left(r_{i+1}^{2}-r_{i}^{2}\right)}, \quad (19)$$

the ratio of stress take the form of

$$\sigma_{rr}^{n} = p_c\left[\frac{r^2-r_n^2}{r^2}\frac{r_{n-1}^2}{\left(r_n^2-r_{n-1}^2\right)}-\frac{r^2-r_0^2}{r^2}\sum_{i=n}^{N-1}\frac{r_i^2}{\left(r_i^2-r_0^2\right)}\right], \tag{20}$$

$$\sigma_{\varphi\varphi}^{n} = p_c\left[\frac{r^2+r_n^2}{r^2}\frac{r_{n-1}^2}{\left(r_n^2-r_{n-1}^2\right)}-\frac{r^2+r_0^2}{r^2}\sum_{i=n}^{N-1}\frac{r_i^2}{\left(r_i^2-r_0^2\right)}\right]. \tag{21}$$

The circumferential stresses in winding in the i-th turn are determined in the form

$$\sigma_{\varphi}^{i} = \frac{p_c r_{i-1}^2\left(1+r_i^2 r^{-2}\right)}{\left(r_i^2-r_{i-1}^2\right)} \approx \frac{p_c r_{i-1}}{t} = \frac{p_c\left(r_0+(i-1)t\right)}{t}. \tag{22}$$

If we take the ratio

$$\delta_i = \frac{2p}{Er_i}\frac{\left(r_i^2+r_0^2\right)\left(r_{i+1}^2-r_i^2\right)+\left(r_i^2-r_0^2\right)\left(r_{i+1}^2+r_i^2\right)}{\left(r_{i+1}^2-r_i^2\right)}, \tag{23}$$

the ratio of stress take the form

$$\sigma_{rr}^{n} = p\left[\frac{\left(r_{n-1}^2-r_0^2\right)}{\left(r_n^2-r_{n-1}^2\right)}\frac{r^2-r_n^2}{r^2}-\frac{\left(r^2-r_0^2\right)(N-n)}{r^2}\right], \tag{24}$$

$$\sigma_{\varphi\varphi}^{n} = p\left[\frac{\left(r_{n-1}^2-r_0^2\right)}{\left(r_n^2-r_{n-1}^2\right)}\frac{r^2+r_n^2}{r^2}-\frac{\left(r^2+r_0^2\right)(N-n)}{r^2}\right]. \tag{25}$$

The circumferential stresses in winding in the i-th turn are determined in the form

$$\sigma_{\varphi}^{i} = \frac{p\left(r_{i-1}^2-r_0^2\right)\left(1+r_i^2 r^{-2}\right)}{\left(r_i^2-r_{i-1}^2\right)} \approx \frac{p(i-1)\left(2r_0+(i-1)t\right)}{r_0+(i-1)t}. \tag{26}$$

3. An estimate of the increase in the energy capacity of a super flywheel under a monotonically varying interference

The circumferential stresses in the n-th cylinder of the untwisted flywheel are determined by the formula

$$\sigma_{\varphi\varphi}^{n} = p\left[\frac{(n-1)\left(2r_0+(n-1)t\right)}{\left(2r_0+2nt-t\right)}\frac{r^2+\left(r_0+nt\right)^2}{r^2}-\frac{r^2+r_0^2}{r^2}(N-n)\right]+ \\ +\frac{3+\mu}{8}\rho\omega^2 r_0^2\left(1+k^2\left(1+r_0^2 r^{-2}\right)-\frac{1+3\mu}{3+\mu}r^2 r_0^{-2}\right). \tag{27}$$

To estimate the specific energy intensity of the super flywheel, whose layers are stacked with interference, we equate the kinematic energy with the circumferential stresses along the inner and outer edges of the super flywheel

$$\sigma_{\varphi\varphi}\left(r_0\right) = \frac{\rho\omega^2 r_0^2}{4}\left((3+\mu)k^2+1-\mu\right)-2p(N-1)=. \\ =\frac{\rho\omega^2 r_0^2}{4}\left(3+\mu+k^2(1-\mu)\right)+\frac{2p(N-1)\left(2r_0+(N-1)t\right)}{\left(2r_0+2Nt-t\right)}=\sigma_{\varphi\varphi}\left(r_N\right). \tag{28}$$

From here

$$2p(N-1) = \frac{\rho\omega^2 r_0^2}{2}\frac{2r_0+2Nt-t}{4r_0+3Nt-2t}(1+\mu)\left(k^2-1\right) \tag{29}$$

and

$$\sigma_{\varphi\varphi}^{0}=\sigma_{\varphi\varphi}^{0}\left(r_0\right)=\frac{\rho\omega^2 r_0^2}{4}\left[\left(\left(3+\mu\right)k^2+1-\mu\right)-\frac{2k-t/r_0}{1+3k-2t/r_0}2\left(1+\mu\right)\left(k^2-1\right)\right]. \qquad (30)$$

If the number of winding layers is large, then

$$\sigma_{\varphi\varphi}^{0}\approx\frac{\rho\omega^2 r_0^2}{4}\left[\left(\left(3+\mu\right)k^2+1-\mu\right)-\frac{2k}{1+3k}2\left(1+\mu\right)\left(k^2-1\right)\right]=$$
$$=\frac{\rho\omega^2 r_0^2}{4}\left[\frac{1-\mu+k\left(7+\mu+k\left(3-k\left(-5+\mu\right)+\mu\right)\right)}{1+3k}\right]. \qquad (31)$$

If the flywheel is solid, then the stresses on the inner surface of the flywheel will coincide with those for the super flywheel at angular velocity ω_s

$$\frac{\rho\omega_s^2 r_0^2}{4}\left(\left(3+\mu\right)k^2+1-\mu\right)=\frac{\rho\omega^2 r_0^2}{4}\left[\frac{1-\mu+k\left(7+\mu+k\left(3-k\left(-5+\mu\right)+\mu\right)\right)}{1+3k}\right]. \qquad (32)$$

Then the ratio of the squares of the angular velocities (as well as the kinetic energies) will be equal to

$$\frac{\omega^2}{\omega_s^2}=\frac{\left(1+3k\right)\left(\left(3+\mu\right)k^2+1-\mu\right)}{1-\mu+k\left(7+\mu+k\left(3-k\left(-5+\mu\right)+\mu\right)\right)}. \qquad (33)$$

For a small hole $\left(k\to\infty\right)$, the ratio of the squares of the angular velocities will be

$$\frac{\omega^2}{\omega_s^2}=\frac{3\left(3+\mu\right)}{5-\mu}. \qquad (34)$$

4. Numerical example

The stresses in the layers were calculated for a super flywheel with the following geometric and mechanical characteristics [23, 24]: the Young's modulus $\AA=200000\,MPa$, Poisson's ratio $\mu=0.3$, internal radius $r_0=1\,m$, external radius $r_N=2\,m\left(k=2\right)$, tensile strength $\sigma_{ts}=1100\,MPa$, material density $\rho=7860\,kg/m^3$. In the continuous flywheel for these geometric and mechanical parameters, the angular velocity $\omega=200\ rad/s$, while the maximum stress in the flywheel (hoop stresses on the inner surface of the flywheel) are equal to $\sigma_{\varphi\varphi}^{\max}=1092.54\,MPa$.

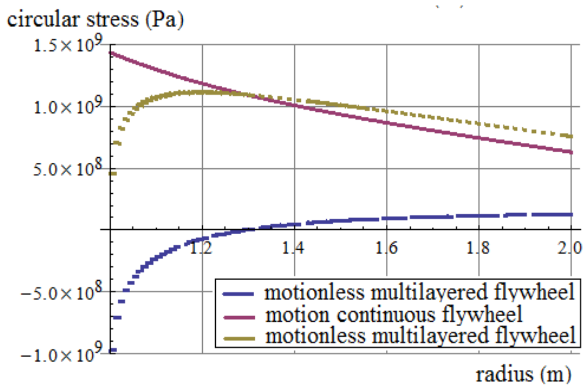

Figure 3. **C**ircular stresses for the case of constant interference.

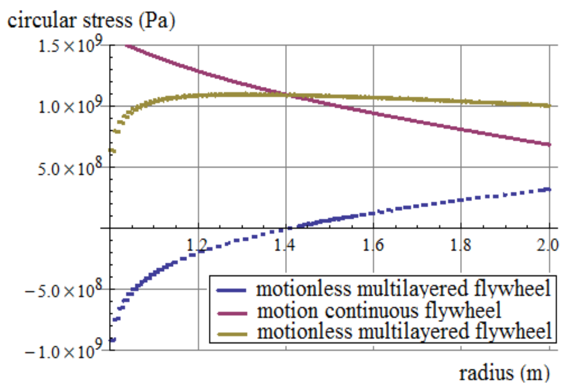

Figure 4. **C**ircular stresses for the case of equal contact.

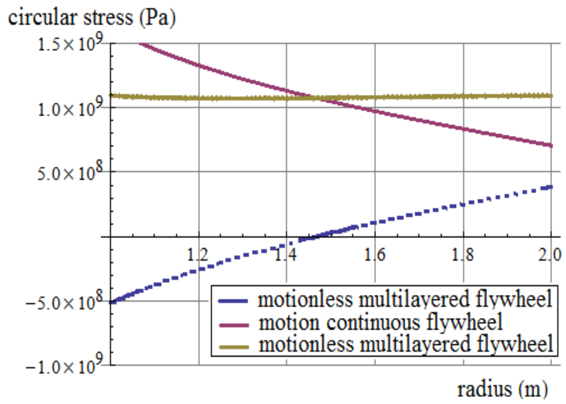

Figure 5. Circular stresses for case of a monotonically changing interference.

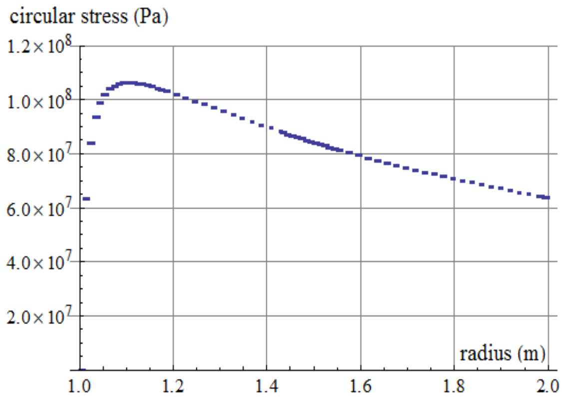

Figure 6. Circumferential stresses in the layers of the super flywheel when winding for constant interference.

In Figure 3, for the case of the same interference, the diagrams of the circumferential stresses in a 100-layer super flywheel are given for the maximum possible angular velocity and the diagram of the circumferential stresses in a continuous flywheel for the same angular velocity. Figures 4 and 5 show the corresponding diagrams of the circumferential stresses for the case of equal contact pressures and the case of a monotonically varying interference.

The circumferential stresses in the layers of the super flywheel when winding for constant interference δ, constant contact pressure p_c and a constant monotonic interference parameter p for the case of 100 turns are shown in Figures 6-8, respectively.

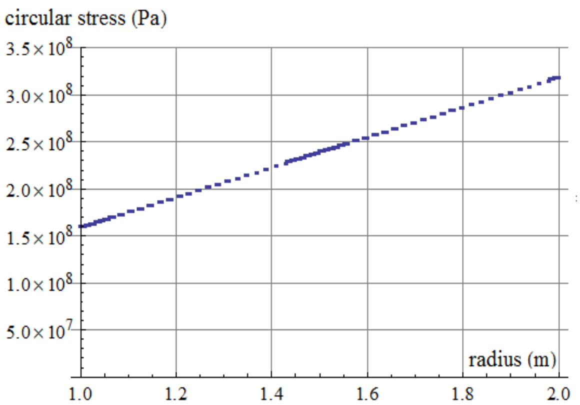

Figure 7. Circumferential stresses in the layers of the super flywheel when winding for constant contact pressure.

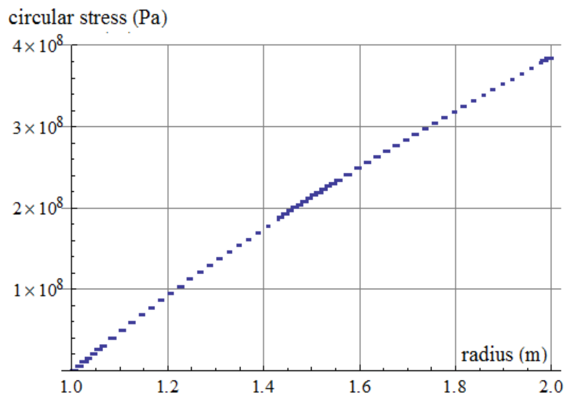

Figure 8. Circumferential stresses in the layers of the super flywheel when winding for case of a monotonically changing interference.

5. Conclusion

Analysis of the results shows that the manufacture of super-flywheels with concentric packing on each other with the tightness of thin-walled cylinders (with their further gluing or welding) with an appropriate choice of interference can lead to an increase in the specific energy capacity of the super-flywheel design. The relative decrease in the radius of the inner opening of the super flywheel, the increase in the number of layers (thin-walled cylinders planted on each other) and the increase in the

Poisson's ratio of the material from which the super flywheel is made also leads to an increase in the specific energy intensity. Precise solutions for the distribution of stresses in the layers of the super flywheel are obtained.

Acknowledgments
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