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Numerical estimation of various influence factors on a multipoint hydrostatic leveling system

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Abstract. Among various methods that allow controlling quasi-static vertical displacements of structures, the hydrostatic leveling method remains relevant. A multi-point hydrostatic leveling systems allows controlling the vertical displacement field with the required spatial resolution. It is assumed that the liquid level in the each measuring vessel has the same absolute elevation. However, it is influenced by various external factors, which are difficult to eliminate when implementing the method in real constructions. Consequently, it is necessary to assess the influence of these factors and develop methods aimed at their reduction or compensation. A mathematical model, describing the viscous incompressible fluid motion located in a multi-point level with absolutely rigid walls, is presented in the paper. The experiments performed with two point levels of different lengths, as well as analytical estimates of other authors, made it possible to estimate the degree of confidence of the model and the boundaries of applicability. The influence of non-uniform density of a liquid on the liquid level in a 4-point hydrostatic level of different topologies is numerically estimated using the model. An estimation of transient processes in the level, caused by an air pressure surge over one of the measuring vessels is carried out.

1. Introduction

Among various methods that allow controlling quasi-static vertical displacements of different objects, a hydrostatic leveling system (HLS) has several advantages due to which it remains relevant and is used to monitor various structures: colliders [1], dams [2], tunnels, buildings [3,4], etc.

The hydrostatic level is a system of communicating measuring vessels filled with a liquid. The liquid level in these vessels determines the plane relative to which quasi-static vertical displacements can be controlled. The use of the multipoint HLS allows one to control vertical displacement fields with the required spatial resolution. In HLS measurements, it is assumed that the liquid level in the measuring vessels is the same. However, its position is influenced by various factors described by Pellissier [5] in 1965: non-uniform liquid density or temperature, air pressure, external mechanical influences and etc. Methods for reducing the influence of these factors are known, however, the structural features of real structures generally do not allow one to implement all of them (minimum level difference, minimum temperature gradient). While under laboratory conditions and in special cases it is possible to obtain an error of the order of 1 μm, for example using the half-filled pipe method, which is less sensitive to temperature variations [6]. However, the half-filled pipe method is difficult to implement in most real designs, that is why in this case the "fully filled pipe system" is used, for which it is necessary to assess the influence of negative factors, as well as proposals to

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reduce their influence or compensation. In the literature, the estimates of the influence of these factors for 2-point longbase levels (consisting of 2 sensors) are described [7]. For the multipoint level in [8], it is suggested to evaluate transient processes based on the electronic-hydraulic analogy, by replacing the measuring and connecting vessels (segments) with an electrical circuit with RLC elements. However, different variants of connection topologies of the multipoint level are not considered, and are not analyzed. Therefore, modeling of liquid flow processes in multipoint levels of various topologies is relevant and significant for assessing the influence of negative factors on level readings and its reduction.

2. Modeling of a Multipoint hydrostatic leveling system

The application of modern three-dimensional finite-element approaches for solving transient problems of liquid motion in hydrostatic level tubes requires considerable computational resources. Since this requires sufficient spatial and temporal discretization, determined by dimensions of real levels, the length of their connecting tubes is much greater than its diameter. Consequently, in order to evaluate liquid flow processes in the levels of different configurations, less resource-intensive approaches should be used.

2.1. Hydraulic Model review

Let's consider an incompressible liquid with density ρ and viscosity μ, which is in a multipoint level with absolutely rigid walls. Figure 1 shows a fragment of the hydrostatic level, consisting of measuring (vertical) and connecting (horizontal) segments. The measuring and connecting segments of the level have a cylindrical shape. Along the segment, with a constant cross-section, the liquid velocity υ and density ρ does not change. Let's consider the liquid pressure *P* at the segment connection point (node).

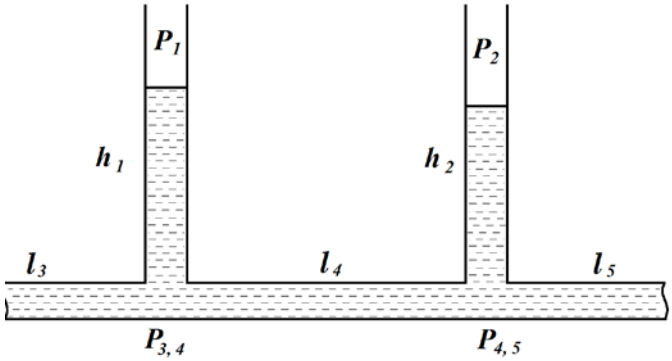


Figure 1. A fragment of the hydrostatic level.

The parameters of real levels and the velocity of fluid motion in them correspond to the numbers Re << 2100 (for cylindrical tubes), i.e. laminar flow. In the hydraulic model approximation, we write the system of equations for a level, consisting of several segments. The Poiseuille law with some correction coefficient *k*, depending on frequency is used. For the Poiseuille law, *k* = 1. In the measuring vessel, the height of the liquid level h_i (the filled part of the vessel) is variable, and in the connecting segment the length l_j is constant. The system of equations consists of the balance of forces (pressures) equations for the measuring (1) and connecting (2) segments:

$$\rho_1 \cdot h_1 \cdot \frac{\mathrm{d}\upsilon_1}{\mathrm{d}t} + P_{3,4} - P_1 - \rho_1 \cdot g \cdot h_1 - \frac{8\mu \cdot h_1 \cdot \upsilon_1}{R_1^2} k_1 = 0 \quad (1)$$

$$\rho_4 \cdot l_4 \cdot \frac{\mathrm{d}\upsilon_4}{\mathrm{d}t} + P_{3,4} - P_{4,5} - \rho_4 \cdot l_4 \cdot g \cdot \delta_4 - \frac{8\mu \cdot l_4 \cdot \upsilon_4}{R_4^2} k_4 = 0 \quad (2)$$

where $P_{4,5}$ is the fluid pressure at the node between the connecting segments *4* and *5*;
P_1, P_2 are the air pressures in the measuring vessel 1 and 2 above the liquid;
υ_i is the liquid velocity in the segment *i*;
R_i is the radius of the segment *i*;
δ_4 is the coefficient of the liquid weight, depending on the direction of the connecting segment 4;
k_i is the frequency-depended friction coefficient in the segment *i*.

The equation of the relationship between the height of the liquid level *h* and the velocity υ of its motion in the measuring vessel 1 is:

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = \upsilon_1 \tag{3}$$

The continuity equations, corresponding to the equality of volumetric flow rates at the node between the connecting segments 3 and 4, have the form:

$$-\upsilon_3 R_3^2 - \upsilon_1 R_1^2 + \upsilon_4 R_4^2 = 0 \tag{4}$$

The system of equations (1) - (4), with respect to the variables *h*, υ with the following initial conditions:

$$h_i = h_i^0, \qquad \upsilon_i = \upsilon_i^0 \tag{5}$$

for every specific topology of the level is solved numerically using the Matlab® software.

2.2. Model verification

In order to verify the presented model, the problem of free damped oscillations of a 2-point level with a constant cross section of the measuring and connecting parts, i.e. a U-tube, was numerically solved. The total length of the liquid column (*L*) for the U-tube consists of two measuring segments heights *h* and one connecting segment *l*. The resulting damped oscillations can be characterized by the values of the damping ratio λ and the period of oscillations *T*. According to [9] for the Poiseuille flow, they are equal:

$$\lambda = \frac{4 \cdot \mu}{\rho \cdot R^2} \tag{6}$$

$$T = 2 \cdot \pi \sqrt{\frac{L}{2g}} \cdot \left(\frac{1}{\sqrt{1 - \lambda^2 L / 2g}} \right) \tag{7}$$

There are other approaches to describe dissipation processes, arising during a laminar non-stationary flow in tubes [10-11]. Comparisons of the calculated values of λ and *T* with analytical results for U-shaped tubes were carried out: with dissipation in the Poiseuille, Ogawa [10], Popov [11] law approximation, with analytical results of Manukin [7] for the longbase level, as well as with own experiments.

For the experiments, a 2-layer PVC fiber-reinforced tube was used as the connecting and measuring vessel. The inner diameter of the tube is 12.5 mm, the outer one is 18.5 mm. The height of each measuring segment in the experiments was 0.3 m, and the length of the connecting section changed. Tubes which total length *L* are 50 m and 20 m, were studied and that corresponded to the real segments size. A short tube, where *L* equaled 1.1 m, was also investigated to estimate the boundaries of applicability of the models. The temperature of the liquid (water) during the experiments was 21÷23 °C, which corresponded to its density of 998 kg/m^3, viscosity - 0.001 Pa·s. Liquid level oscillations, with an initial amplitude of about 20 mm, was recorded by a video camera.

Further, for each frame, the position of the liquid level was determined using the image recognition algorithm described in [12]. As a result, evolutions of the liquid level height were obtained. It should be noted that during each experiment the liquid had time to make several complete oscillations, aperiodic process was not observed. Based on the processing of the experimental data, T and λ were evaluated. For a better understanding, the results are shown in Table 1.

Table 1. Comparison of the results for a 2-point HLS (U-tube) on different models.

	L = 1.1m		L = 20m		L = 50m	
Type of Model	T (s)	λ (s^{-1})	T (s)	λ (s^{-1})	T (s)	λ (s^{-1})
Classic (Poiseuille)	1.49	0.103	6.38	0.103	10.165	0.103
Popov	1.49	0.273	6.38	0.154	10.26	0.131
Ogawa	1.49	0.397	6.37	0.369	10.17	0.351
Manukin	1.16	0.077	7.21	0.077	11.51	0.077
Numerical (k = 1)	1.49	0.103	6.37	0.103	10.165	0.103
Experiment	1.46±0.05	0.49±0.01	7.0±0.12	0.142±0.005	11.45±0.15	0.120±0.005

The presented results show that:

- for k = 1, the numerical results coincide with the analytical ones (Poiseuille law), which indicates the correct numerical implementation of the model;
- the considered models, except Ogawa, do not accurately describe dissipative processes in the short levels (for the conditions under consideration);
- for the long levels (about hundreds of meters), the oscillation period should be estimated by the Manukin model;
- for the average range levels (about tens meters) with an error of 10%, it is possible to estimate the oscillation period according to the proposed model (Numerical), and to estimate damping ratio, the correction coefficient should be used, depending on the oscillation frequency ω in the Poiseuille law according to the Popov model:

$$k = 0.4 + \frac{R}{4}\sqrt{\frac{\omega \cdot \rho}{2 \cdot \mu}} \tag{8}$$

The coefficient can be used to describe more accurately damping processes in equations (1) – (2).

3. The influence of an air pressure surge on the liquid level in HLS

Let's consider a level consisting of 4 measuring vessels (with the heights of the liquid levels h_1, h_2, h_3 and h_4), connected together by the connecting segments. The diameter of the connecting segment is 12.5 mm, the measuring one is 50 mm. The lengths of the segments between the sensors No. 1 and No. 2; No. 2 and No. 3; No. 3 and No. 4; No. 4 and No. 1 are 10, 20, 15, 25 m, respectively.



Figure 2. The location of the level on the foundation with a sequential connection circuit.



Figure 3. The location of the level on the foundation with a closed circuit.

For instance, let us consider 2 options: serial connection from sensor No. 2 through sensors No. 3, No. 4 and up to No. 1 (without any connection from No. 1 to No. 2), shown in figure 2, as well as the connection diagram where all 4 sensors are closed in a circuit shown in figure 3. The initial conditions of the problem, as well as the characteristics of the level are presented in Table 2.

Table 2. Task parameters.

υ^0 (m/s)	$h^0_{1\text{-}4}$ (m)	R_l (m)	R_h (m)	ρ (kg/m^3)	μ (Pa·s)
0	0.1	0.006125	0.025	999	0.001

Let the excess air pressure $P_{h,2}$ over sensor No. 2 change abruptly at t = 5 s by +100 Pa relative to the air pressure over the other sensors. Consequently, a transient process occurs, after which the values of h are set on new levels. The results of the change in the liquid levels in the sensors are shown in Figures 4 and 5.

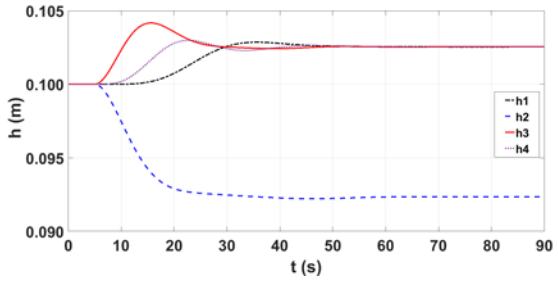


Figure 4. The change in the liquid level heights on the sensors with a sequential circuit.

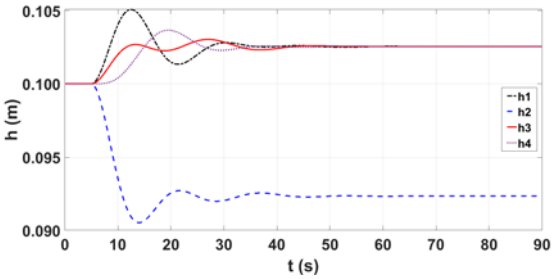


Figure 5. The change in the liquid level heights on the sensors with a closed circuit.

As expected, the change in air pressure over one of the sensors causes a measurement error of 10 mm. The presented graphs demonstrate different nature and frequencies of the transient processes that occur when the gas pressure changes over one of the sensors with different connection circuits. In the case of the sequential circuit, after an air pressure change, the liquid levels are set to new values with an error of ± 0.1 mm in 44 s from the time of the onset of the transient process, and in the presence of a closed circuit in a somewhat shorter time, which is equal to 36 s.

4. The influence of non-uniform density of a liquid on the liquid level in HLS

One of the main factors, affecting the position of the liquid level in the level is non-uniformity of liquid density of its different parts. It can be caused by a non-uniform temperature of the environment. Within the framework of the suggested model, the levels, for which liquid density is constant for each segment, and can be different in various segments, are considered. For example, firstly let us consider a simple case - a 2-point-level, and after that a multi-point one with different topologies.

4.1. 2-point HLS case

On the monitored objects due to structural features, situations, where the connecting tube between the sensors is not located on the same level, but goes on two levels - with periodic ascents and descents, are possible. This case corresponds to a 2-point level of the following configuration (Figure 6).

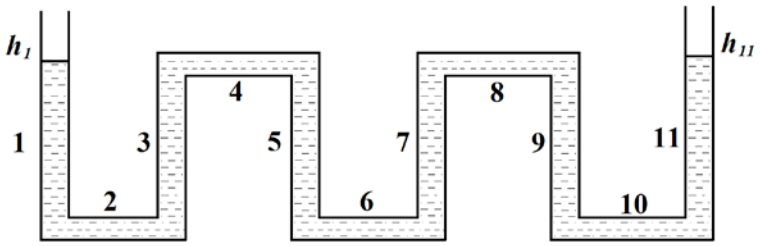


Figure 6. The scheme of a 2-point level of 11 segments.

The level, presented in figure 6, consists of segments of the same length of 1 m. The diameter of all the vessels is 0.0125 m, the initial liquid velocities in all the segments are zero. As a liquid, we choose water, whose viscosity is 0.001 Pa s and the Poiseuille law of friction (k = 1). Let us investigate the influence of the change in the density of each segment on the difference of the liquid level heights Δh. Let ρ_1 = 999 kg/m^3, and ρ_2 = 998 kg/m^3, which corresponds to temperatures of 16 °C and 21 °C. For each density change, the problem (1) - (5) is solved and on the basis of the obtained results the difference of the steady-state values of the liquid levels in the measuring vessels is calculated. Results for a few cases are presented in Table 3.

Table 3. The difference between liquid level heights depending on density in the segments.

The value of liquid density in the segment											Δh (m)
1	2	3	4	5	6	7	8	9	10	11	
ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	0
ρ_2	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	0.001
ρ_2	ρ_1	ρ_2	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	0
ρ_1	ρ_1	ρ_2	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	ρ_1	0.001
ρ_2	ρ_2	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_1	ρ_1	ρ_1	0
ρ_2	ρ_2	ρ_2	ρ_1	ρ_2	ρ_1	ρ_1	ρ_1	ρ_2	ρ_1	ρ_1	0.002
ρ_2	ρ_1	ρ_1	ρ_1	ρ_2	ρ_1	ρ_1	ρ_1	ρ_2	ρ_1	ρ_1	0.003
ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	ρ_1	ρ_2	0

The presented results show that the change in liquid density in the horizontal segments does not influence on the liquid level, as distinct from liquid density in the vertical ones. However, the effect of different temperature (density) of the vertical segments can be summed up or compensated, depending on their mutual position. This should be taken into account when mounting hydrostatic leveling systems and multiple ascents and descents of the connecting tube in the presence of a temperature gradient along it should be avoided. Or measuring the temperature of each vertical section and implementing software compensation should be performed.

4.2. 4-point HLS case

Let us consider the levels of Section 3, the schemes of which are shown in figure 2 and figure 3. They consist of 4 measuring vessels (sensors): connected between each other in series and closed in a circuit. The segment between sensors 2 and 3 is supplemented by a single level difference of 1 m, similar to that shown in figure 6. Liquid density in all the segments is equal to ρ_1. At time t = 0, liquid

density on the left side of the level difference changes from ρ_1 to ρ_2. The parameters of this task are given in Table 4.

Table 4. Task parameters.

υ^0 (m/s)	$h^0_{1\text{-}4}$ (m)	R_l (m)	R_h (m)	ρ_1 (kg/m^3)	ρ_2 (kg/m^3)	μ (Pa·s)
0	0.1	0.006125	0.025	998	999	0.001

The results of the change in the liquid level heights in the measuring segments, caused by the change in density, are presented in Figures 7 and 8.

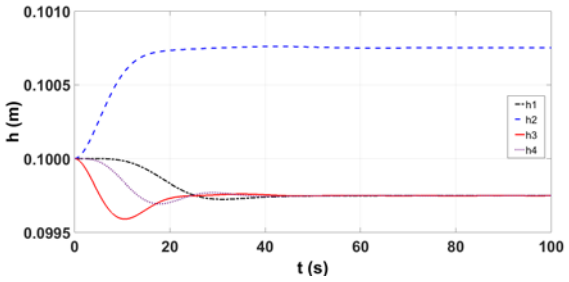


Figure 7. The change in the liquid level heights on the sensors with a sequential circuit.

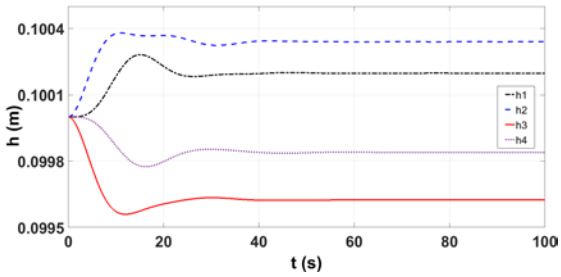


Figure 8. The change in the liquid level heights on the sensors of the level with a closed circuit.

Figure 7 and figure 8 demonstrate that the considered topologies of the level affect both nature of the transient processes and the values of the liquid levels of in the measuring vessels. In the first case (figure 7), the height of the liquid level on sensor No. 2 changed by 1 mm from the values of the other sensors, as in the problem, considered in § 4.1 (line 4 in Table 3). In the second case, the liquid level heights on all the sensors took different values, but the maximum level difference was only 0.68 mm (between sensors No. 2 and No. 3). This should be taken into account when designing HLS and, if possible, to close the hydraulic circuit.

5. Conclusions

The article presents a hydraulic model for a multipoint hydrostatic level. The conducted experiments with two point levels allowed estimating the degree of confidence of the model and the boundaries of its applicability. With the help of this model, it is possible to perform numerical estimations of transient processes in multipoint levels, with a number of sensors and topologies and various sizes. The proposed model also makes it possible to evaluate the influence of non-uniform density of liquid and air pressure on the level readings.

Acknowledgments

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