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Investigation of deformation of elements of three-dimensional reinforced concrete structures located in the soil, interacting with each other through rubber gaskets

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Abstract. In work the technique of calculation of elements of three-dimensional reinforced concrete substructures located in a soil, interacting with each other through rubber linings is realized. To describe the interaction of deformable structures with the ground, special "semi-infinite" finite elements are used. A technique has been implemented that allows one to describe the contact interaction of three-dimensional structures by means of a special contact finite element with specific properties. The obtained numerical results are compared with the experimental data, their good agreement is noted.

1. Introduction

In modern mechanics of a deformable solid, the problems of non-linear deformation of elements of three-dimensional structures with each other and with the surrounding soil are still relevant and in demand in construction, road construction, foundation engineering, etc. Many authors have dealt with the nonlinear problems of mechanics, in particular, papers [1-11] can be noted in which theoretical features of the construction of geometrically and physically nonlinear deformation of structures are noted. Practical problems in various areas of mechanics of a deformed solid are usually solved numerically [12-19]. It is also possible to note realized methods of contact interaction problems [20-24] and hyperelastic behavior of three-dimensional structures [25-29]. This paper develops a mathematical model and realizes a numerical algorithm for studying the stress-strain state of three-dimensional reinforced structures interacting with each other and with the water-saturated soil in which they are located.

2. Formulation of the problem

The purpose of this paper is to construct a technique for calculating the stress-strain state of three-dimensional structures located in the soil, the elements of which interact with each other. The object of calculation is the lining of the subway tunnel from prefabricated reinforced concrete, geometrically representing a cylindrical shell located in the soil mass, under the influence of the weight of the above rocks and the longitudinal compressive force that occurs when this structure is created. The main feature of the design under consideration is the fact that a similar lining of segment blocks is being constructed, which interact with each other through special lining that does not completely cover the interface surfaces, which entirely transmit only compressive forces and partially tangential forces within the frictional forces. Fig. 1 partially shows three rings of lining, in real proportions, with conditionally remote blocks to illustrate their relative location.

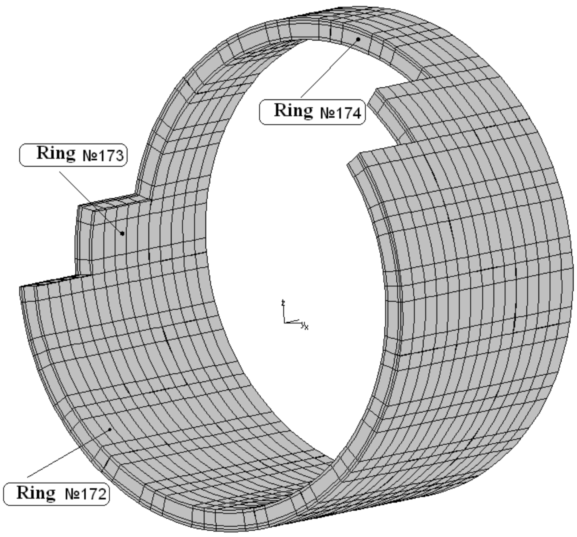

Figure 1. Finite-element partitioning of three rings of the lining of the subway tunnel (some blocks have been removed for clarity).

The calculation was carried out by FEM on the basis of twisted 20-node isoparametric finite elements of the three-dimensional theory of elasticity with quadratic approximation of geometry and displacements. The mesh of similar elements used to calculate the described construction is also shown on Fig.1.

3. Modeling of mechanical contact through gaskets

The mechanism of interaction of blocks through the lining can be illustrated on Fig. 2, where various variants of deformation of the overlays are depicted, depending on the forces exerted by the blocks on each other.

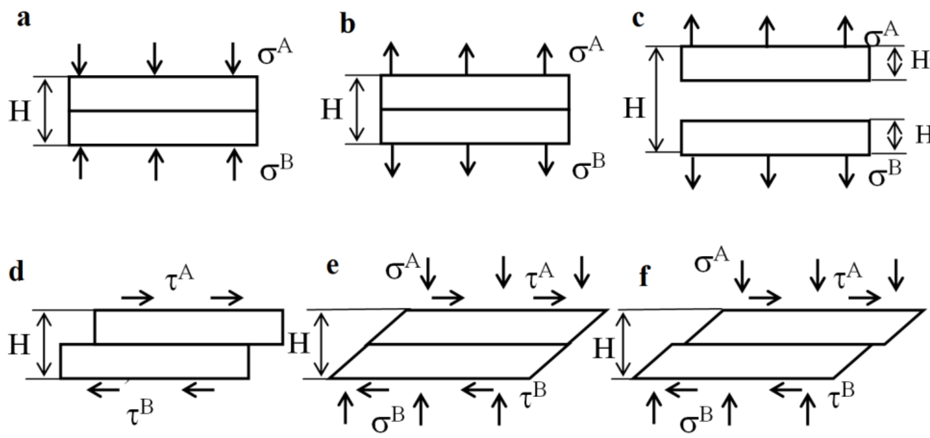

Figure 2. The mechanism of interaction of blocks through gaskets.

For the situation on Fig. 2a, we have that in the overlays there is a stress of compression $\sigma_H = \sigma^A = \sigma^B$ and deformation $\varepsilon_H = \sigma_H / E_H$, where E_H is the modulus of elasticity of the lining material. The geometrical condition for the presence of this situation is $H \le \left(H^A + H^B\right)$, where H^A, H^B is the original thickness of the linings, and H is the distance between the surfaces on which they are fixed.

The situation on Fig. 2b arises in the presence of preliminary reduction, i.e. $H < \left(H^A + H^B\right)$ and, in this case, too, $\sigma_H = \sigma^A = \sigma^B$, $\varepsilon_H = \sigma_H / E_H$ is true.

On Fig. 2c there is no force action and the patches move freely. In this case $H \geq \left(H^A + H^B\right)$, $\sigma_H = 0$.

Fig. 2d shows free slip, in which tangential stresses do not arise, which is realized when $H \geq \left(H^A + H^B\right)$, and in this case $\tau_H = 0$.

Fig. 2e illustrates the elastic interaction with compression and shear without slipping. A similar situation is possible with $H \leq \left(H^A + H^B\right)$ and for stresses and deformations in the overlays can be written: $\sigma_H = \sigma^A = \sigma^B$, $\tau_H = \tau^A = \tau^B$, $\varepsilon_H = \sigma_H / E_H$, $\gamma_H = \tau_H / G_H$. An additional condition here should be the condition $\tau_H \leq f\left|\sigma_H\right|$, where f is the linear coefficient of friction.

If these conditions are not fulfilled, the situation appears on Fig. 2f. In this case $\sigma_H = \sigma^A = \sigma^B$, $\tau_H = f\left|\sigma_H\right|$, $\varepsilon_H = \sigma_H / E_H$ and slip is observed.

All these situations are modeled within the framework of continuum mechanics, i.e. when two overlays are presented in the form of a single material with specific properties. The problem obtained is non-linear and requires the use of special techniques for solving it. A characteristic feature of this nonlinearity is that for normal stresses there are limitations on deformation ($H \leq H^A + H^B$, i.e. the mutual deformation of the linings can't be greater than their total thickness), and for tangential stresses, by their limiting values, that determine the possibility of slippage.

To solve the formulated physically nonlinear problem on the basis of the virtual work equation, an iterative process is constructed, which is a combination of the initial stress method and the additional deformation method. The basic for the determination of the *k*-th iteration is the following variational equation

$$\sum_m \iiint_{\Omega_m} \left\{\sigma^k\right\}^T \left\{\delta\varepsilon\right\} d\Omega + \sum_k \iiint_{\Omega_k} \left(\varepsilon_H^k E_H \delta\varepsilon_H + \gamma_H^k G_H \delta\gamma_H\right) d\Omega_k =$$
$$= \sum_m \left(\iiint_{\Omega m} \rho \left\{g\right\}^T \left(\delta V\right) d\Omega + \iint_{S_m^\sigma} \left\{p\right\}^T \left\{\delta V\right\} ds \right) + \sum_k \iiint_{\Omega_k} \left(\hat{\sigma}_H^k \delta\varepsilon_H + \hat{\tau}_H^k \delta\gamma_H\right) d\Omega_k, \tag{1}$$

where $\hat{\sigma}_H^k$ and $\hat{\tau}_H^k$ are determined as the differences between the corresponding values obtained from linear relationships of the theory of elasticity and their real values, which depend on the interaction mechanism (Fig. 2).

4. Contact finite element

To implement the previously described mathematical model of the interaction of overlays within the framework of the FEM, the so-called contact element is determined. Geometrically it is a shell quadrangular element with 16 nodes. As initial information for it, the radii-vectors of the points defining the lower (odd numbers) and the upper (even numbers) of the surface are given, and the initial thickness $H = H^A + H^B$, which can be constant on the element, may vary (in this case, their nodal values are specified).

Approximations of the face surfaces are introduced

$$\vec{r}^{(-)}\left(\xi,\eta\right) = \sum_{i=1}^{8} \vec{r}_{2i-1} N_i\left(\xi,\eta\right), \quad \vec{r}^{(+)}\left(\xi,\eta\right) = \sum_{i=1}^{8} \vec{r}_{2i} N_i\left(\xi,\eta,\zeta\right), \tag{2}$$

where $N_i\left(\xi,\eta\right)$ - are the known form functions for the two-dimensional quadratic approximation of the Sirendip family. To approximate the displacement vector, we use a similar representation

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$$\vec{V}^{(-)}(\xi,\eta)=\sum_{i=1}^{8}\vec{V}_{2i-1}N_i(\xi,\eta),\quad \vec{V}^{(+)}(\xi,\eta)=\sum_{i=1}^{8}\vec{V}_{2i}N_i(\xi,\eta). \tag{3}$$

In the process of deformation, initially parallel facial surfaces $\vec{r}^{(-)}$ and $\vec{r}^{(+)}$ cease to be so. An illustration of this situation is given on Fig. 3, where it is clear that the unit vectors of the normals $\vec{P}_3^{(-)}$, $\vec{P}_3^{(+)}$ are essentially different. Therefore, we will determine all geometric, kinematic and power characteristics on both face surfaces independently. In other words, the stress-strain state will be determined independently in each overlay (adjacent, respectively, to the surfaces $\vec{r}^{(-)}$ and $\vec{r}^{(+)}$), which will allow more correctly simulate their state when slipping relative to each other.

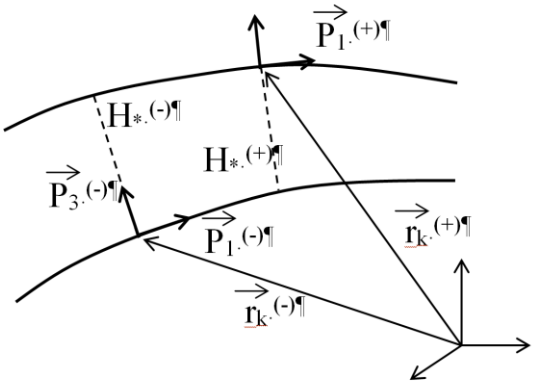

Figure 3. Schematic representation of the face surfaces of the contact elements.

The distances $H_*^{(-)}$ and $H_*^{(+)}$ from one face to the other, along the normal to it, are determined in the form

$$H_*^{(-)}=\vec{P}_3^{(-)}\cdot\left[\vec{r}^{(+)}+\vec{V}^{(+)}-\vec{r}^{(-)}-\vec{V}^{(-)}\right],\quad H_*^{(+)}=\vec{P}_3^{(+)}\cdot\left[\vec{r}^{(+)}+\vec{V}^{(+)}-\vec{r}^{(-)}-\vec{V}^{(-)}\right]. \tag{4}$$

These values are basic in judging the nature of the interaction between the overlays (the classification is shown in Fig. 2). The compression deformation is defined as

$$\varepsilon_H^{(+)}=\frac{H_*^{(+)}-H}{H},\varepsilon_H^{(-)}=\frac{H_*^{(-)}-H}{H}. \tag{5}$$

The definition of tangential deformations and stresses (for the estimation of frictional forces) requires the introduction of local coordinate systems x', y', z' oriented along the unit vectors $\vec{P}_1, \vec{P}_2, \vec{P}_3$ for each of the face surfaces. The calculation of the derivatives along these directions is performed in a known manner and the relations for the shear deformation are taken in vector form

$$\gamma_{z'y'}=\vec{P}_1\cdot\frac{\partial\vec{V}}{\partial z'}+\vec{P}_3\cdot\frac{\partial\vec{V}}{\partial x'},\quad \gamma_{y'z'}=\vec{P}_2\cdot\frac{\partial\vec{V}}{\partial z'}+\vec{P}_3\cdot\frac{\partial\vec{V}}{\partial y'}.. \tag{6}$$

5. Modeling the interaction of the lining with the soil

To reproduce the interaction of the lining blocks with the soil, without increasing the dimension of the problem, so-called "semi-infinite finite elements" or "finite elements with remote nodes" were used. They are 16-node crooked parallelepipeds, in which the inner curved surface mates with the upper surface of the lining blocks, and the outer surface is removed from the inner surface for considerable distances.

The main task of this element is to reproduce the elastic repulsion of the soil massif when deforming the rings of the lining. Therefore, the movements of remote nodes are considered to be zero. An additional argument in the validity of this assumption may be the following statement: the construction of the tunnel is conducted in a steady mass of soil and must not be accompanied by its deformations at a considerable distance from the rings of the lining of the tunnel.

The technology of constructing the stiffness matrix of such an element practically coincides with that described in [30-32] for a 20-node EM. The only difference is that in the mailing the component of the rigidity matrixes of these EMs only the stiffnesses that relate to the adjacent nodes are involved, and the nodes with zero displacements are not assigned global numbers. This allows us to limit ourselves only to the degrees of freedom defined on the outer surface of the lining blocks, which have already been introduced in modeling the corresponding concrete arrays.

6. Comparison of the results of calculations with experimental studies

To assess the reliability of the calculations and evaluate the developed methodology, their results were compared with the results of monitoring studies of the lining of the tunnel being built. In particular, special devices were installed on the ring blocks, measuring their deformations (strain gauges) before placing these blocks in the lining of the tunnel. Then, after sufficient time necessary to stabilize the stress state, both in the lining and in the soil massif surrounding it, the readings of circumferential and longitudinal deformations were taken. The time of this stabilization was determined by systematic observation of the instrument readings and evaluation of their variation over time.

A comparison of the results of experimental and numerical studies of these rings on the example of circumferential stresses for 172-th and 173-th lining rings is given on Fig. 4-5. The dashed line corresponds to experimental data, solid line - calculation without taking into account the water saturation of the soil, dash-dotted line - calculation taking into account the hydrostatic pressure of groundwater.

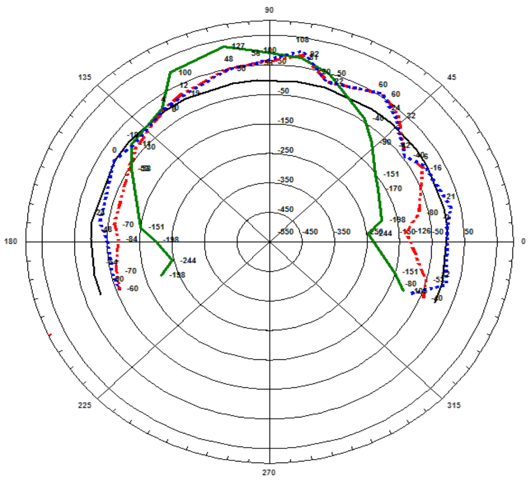

Figure 4. Circumferential stresses in one of the rings of the lining of the subway tunnel (numerical calculation and experimental data).

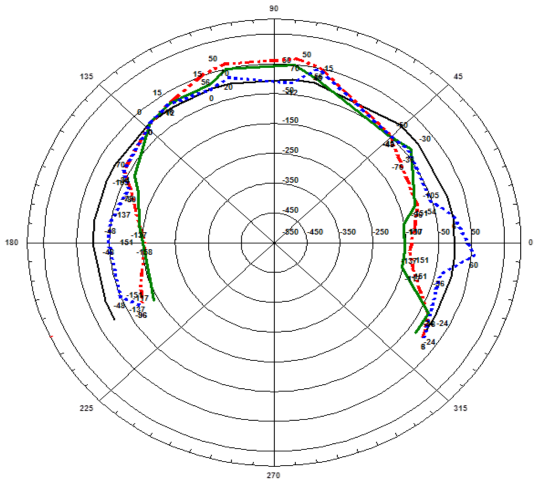

Figure 5. Longitudinal stresses in one of the rings of the lining of the subway tunnel (numerical calculation and experimental data).

7. Conclusion

Proceeding from the obtained results, it should be noted that the developed numerical method for studying the stressed-deformed state of the lining rings of the subway tunnel in a three-dimensional setting, with modeling of the contact interaction of the blocks with each other and with the introduction of "semi-infinite" finite elements to reproduce the elastic backing of the soil, gives results that are in good agreement with Data of full-scale tests. Consequently, it is possible to calculate such designs on its basis and obtain reliable results.

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