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On predicting the onset of transient convection in porous media saturated with Non-Newtonian liquid

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Abstract. The onset of transient convection in non-Newtonian liquid immersing porous media was simulated using a Computational Fluid Dynamics (CFD) package for the thermal boundary condition of Fixed Surface Temperature (FST). Most of the simulated values of stability criteria were found to be in good agreement with the predicted and theoretical values of transient critical Rayleigh number for non-Newtonian liquid defined by Tan and Thorpe (1992) for power-law fluids. The critical transient Rayleigh numbers for convection in porous media were found to be in good agreement with theoretical values by using apparent viscosity μapp at zero shear. The critical time and critical depth for transient heat conduction were then determined accurately that

1. Introduction

Heat transfer in porous media has been the subject of many studies due to the increasing interest in chemical catalysis, building insulation, heat exchangers, petroleum reservoirs, geothermal operations, and so on. In recent years, heat transfer in porous media saturated by non-Newtonian liquids has received considerable attention in the literature, notably for power-law fluids. An illustrative example is found in oil reservoir engineering in connection with the production of heavy crude oils which are power law fluids [20], [12] and [6]. This process involves the cyclic injection of steam into the well for the purpose of increasing the temperature of the oil reservoir. The efficiency of this hot process can be increased by studying the combined effects of convective heat transfer and convective flow in a power-law fluid filled porous media. The apparent viscosity for a power-law fluid caused by a shear rate is commonly expressed as

$$\mu_{app} = K\gamma^{n-1} \tag{1}$$

when n is less then unity for shear thinning or pseudo-plastic fluids, while for shear thickening liquids, n is greater than unity, and K is the flow consistency index [2]. The onset of transient convection caused by unsteady-state conduction in an adverse density profile under gravity has been successfully predicted by the transient instability theory of Tan and Thorpe [13], [14], [15] and [16]. Tan and Thorpe [14] derived the maximum transient Rayleigh Number for Newtonian liquid predicted the onset of convection induced by transient heat conduction in a deep fluid. The new theory has successfully predicted the maximum transient Rayleigh number, the maximum penetration depth z $_{max}$, the critical time and critical size of convective plumes for fixed-surface temperature (FST) boundary condition. The theory has been successfully applied to the onset of transient convection in porous media saturated by Newtonian liquids [17], whose equations can be applied to non-Newtonian liquids by assuming an apparent viscosity for zero-shear [13]. The theoretical value of onset of convection between two plates of 39.5 was first calculated by Horton and Rogers [4]. The relevant equations for the onset of transient convection in porous media saturated with non-Newtonian liquids under FST boundary condition summarized as follows.

$$Ra_{\max} = 39.5 = \frac{0.83 K_e g \alpha \Delta T_c}{\nu_{app}} \sqrt{\frac{t_c}{\kappa_m}} \tag{2}$$

$$z_{\max} = 2\sqrt{\kappa_m t_c} \tag{3}$$

$$\lambda_c = 4\sqrt{\kappa_m t_c} \tag{4}$$

$$t_c = 2262 \left(\frac{\nu_{app}\sqrt{\kappa_m}}{K_e g \alpha \Delta T_c} \right)^2 \tag{5}$$

The thermal diffusivity of porous media is defined by convention for heat conduction or energy diffusion, which is before the onset of convection [5]. Tan et al. [17] pointed out the lack of a rational scientific basis for the introduction of the modified thermal diffusivity, which arbitrarily altered the experimental values of Rayleigh number substantially. Nield [9] erroneously believed that the thermal energy equation for flowing fluid should be the only basis of the definition of thermal diffusivity for porous media, which is opposite to the conventional definition of the thermal diffusivity based on a stationary fluid. The thermal diffusivity of any material is defined by convention for heat conduction or energy diffusion, which is used in the linear stability analysis for determining a critical Rayleigh number for predicting the onset of convection from the stationary state. Therefore, the thermal energy equation for a flowing fluid is not the basis for defining thermal diffusivity and Rayleigh number for porous media [20].

The simulations of the onset of transient convection in deep fluid and porous media have been successfully conducted by Tan et al. [19] and Tan et al. [17] respectively, with simple Boussinesq's approximation. In this study, we employed a commercial Computational Fluid Dynamics (CFD) package FLUENT to simulate accurately the onset of convection caused by unsteady-state heat conduction and the transport phenomena in porous media saturated with non-Newtonian liquids. The results of simulations will be compared with theoretical predictions.

2. Computational methods

2.1. The simulation model

In this study, the onset of convection in porous medium saturated by non-Newtonian liquid (CMC-5) was investigated. The deep layer was bounded above and below by two rigid impermeable conducting boundaries. 2-D unsteady-state simulations were carried out for the Fixed Surface Temperature (FST) boundary condition. The top of the deep layer was cooled instantaneously under unsteady-state, while its bottom surface was maintained at FST. The non-Newtonian liquid filling the porous medium initially with higher temperature T_0. In the simulation the top surface was suddenly cooled down by a temperature difference of $\Delta T_s = (T_0 - T_s)$.

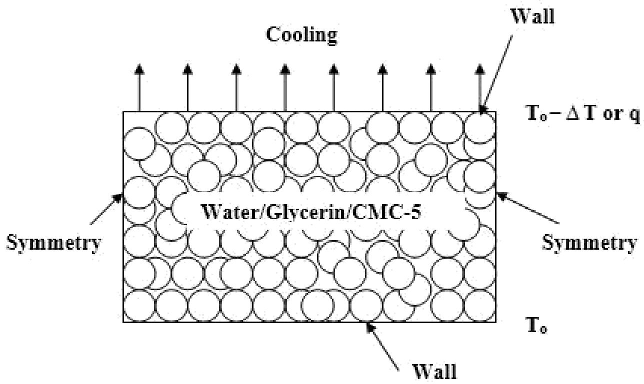

Figure 1. Schematic of the boundary layer setting of the domains used in the simulation.

The physical properties of the porous medium applied in this simulation were based on the glass beads and CMC-5. The density, thermal conductivity and specific heat capacity of the glass beads was assumed to be a constant. However, density, thermal conductivity and specific heat capacity of the CMC-5 were based on the mean temperature of the top and bottom surface temperature. Since CMC-5 is non-Newtonian fluid thus its viscosity depends strongly on shear rate. FLUENT will calculate the shear rate during the flow of fluid and compute the apparent viscosity from the power law. The physical properties of the porous media were presented in Table 1. The permeability of different sizes of glass beads were obtained from the Kozeny-Carmen relation [3].

Table 1. Properties of porous media consisted of glass beads in case1 in CHF boundary condition.

Properties	Unit	Glass bead (27^{o}C)	CMC-5 (45^{o}C)	Porous media
Density, ρ	kg/m^3	2500	988.7	1978.6
Diameter, d_s	m	n/a	n/a	0.05
Porosity, ϕ		n/a	n/a	0.345
viscosity, μ	Pa.s	n/a	3.5	3.5
kinematic viscosity, ν	m^2/s	n/a	3.54×10^{-3}	3.54×10^{-3}
Specific heat, c_p	J/kg.K	750	4217	n/a
Expansion coeff., α	K^{-1}	n/a	5.51×10^{-4}	5.51×10^{-4}
Permeability, K_e	m^2	n/a	n/a	1.385×10^{-6}
Thermal conductivity, k_m	W/m.K	1.4	6.15×10^{-1}	1.1293
Thermal diffusivity, κ_m	m^2/s	7.467×10^{-7}	1.476×10^{-7}	4.235×10^{-7}
Darcy number, Da		n/a	n/a	4.222×10^{-6}

$$K_e = \frac{{d_S}^2 \phi^3}{172.8(1-\phi)^2} \tag{6}$$

The Darcy number *Da [11]* should be infinitesimally small with less than 10^{-5} in porous media condition. This mean critical depth layer has to exceed the glass bead size before convection occurring.

$$Da = \frac{Ke}{Z_{\max}{}^{2}} \tag{7}$$

2.2. Setup for Simulation

The design of the computation cells is vital to the numerical convergence of the solution. The boundary layer occurring at the cooled walls temperature and velocities have their largest variation in this region. Therefore, the mesh should be optimized to adequately solve this region to provide more accurate results. For unsteady-state simulation, the critical time and size of plumes predicted by the theory of Tan et al. [17] can be used to determine the computation cell size, their detailed set-up were adopted in this study. The readers can refer to their paper for details.

3. Results and discussions

The CFD simulations were first conducted for water, glycerin and non-Newtonian liquids to ascertain the validity of zero-shear model for predicting the onset of transient convection before we proceed to the case of porous media.

3.1. Unsteady-state convection in porous media saturated by Newtonian liquids

Simulations were first carried out to model the onset of convection in porous media saturated by Newtonian liquids. The top cooling of water and glycerin under FST condition bounded by two rigid surfaces are simulated with a surface temperature difference, ΔT of 10ºC and 27ºC. The permeability of water in FST condition is 1.99 x 10^{-8} m^2, while glycerin is saturated for permeability 2.22 x 10^{-7} m^2 in both with porosity of 0.345.

The simulated critical times and maximum Rayleigh numbers at the onset of convection are found to be accurate when compared to those values calculated from equations (5) and (2) of Tan et al.[17], as shown in table 2. The close agreement of theoretical and simulated critical times validated the setup of the simulation. Consequently, more simulations were conducted for porous media saturated with non-Newtonian liquids.

3.2. The Formation and Development of the Thermal Plume

The formation and development of the fingering thermal plumes in porous media saturated by non-Newtonian were clearly seen from the temperature contour and velocity vector profile at various time steps, as shown in figure 2. They were the mirror image of rising fingering plumes generated by bottom heating observed by Tan, Sam, and Jamaludin [17]. The various stages of formation and development of the transient falling fingering plume were described table 3. At the onset of convection, the non-Newtonian liquid displayed the same development with Newtonian liquid in porous media as table 3. The main difference between the plumes in porous media and continuous fluid is the shape of the plumes, fingering plumes were formed in the porous media (figure 3) and mushroom shaped plumes for normal liquids. The high viscosity and porous media contributed to the long critical times of convection. The small bead size also extended the long critical times owing to the low permeability.

Table 2. Comparison between the critical time and critical Rayleigh number for Newtonian and non-Newtonian liquid at FST boundary condition saturate porous media.

liquid	Bead size, d(cm)	ΔT (ºC)	t_c (s)		Ra_c		
			Predicted	Simulation	Theory	Simulation	Deviation (%)
Water	0.6	10	2899	2500	39.5	36.7	7.2
Glycerin	2	27	151000	110000	39.5	33.7	14.6
CMC-5	5	27	246000	220000	39.5	37.4	5.4

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Table 3. The stages of plumes formation of non-fingering plumes in non-Newtonian liquids in porous media induced by transient top cooling.

Stage	Description
I	(a)Surface contraction due to cooling and draining down of cooled water from the top layers. Thermal boundary layer starts to form and thickening but no fluid motion is observed, figure 2a. (b)The mechanism of heat transfer is purely heat conduction.
II	(a) The local temperature gradient exceeds the point of stability, thus begins the onset of convection, velocity of fluid increases rapidly, figure 2b. The fluid starts to accelerate. (b) Formation of the tail side of the plumes, figure 2c.
III	(a) Filamentous plumes extend deep in bulk fluid like ‘finger’, figure 2d. (b)The scale of plumes size and the amplitude of plumes are dependent on the rate of the heat conduction. (c) Thermal plumes continuous moving downwards, and begin to coalesce with adjacent plumes. The bulk fluid. Occasionally growing plumes (d)Thermal plumes dissipate heat to the surrounding and disappear.

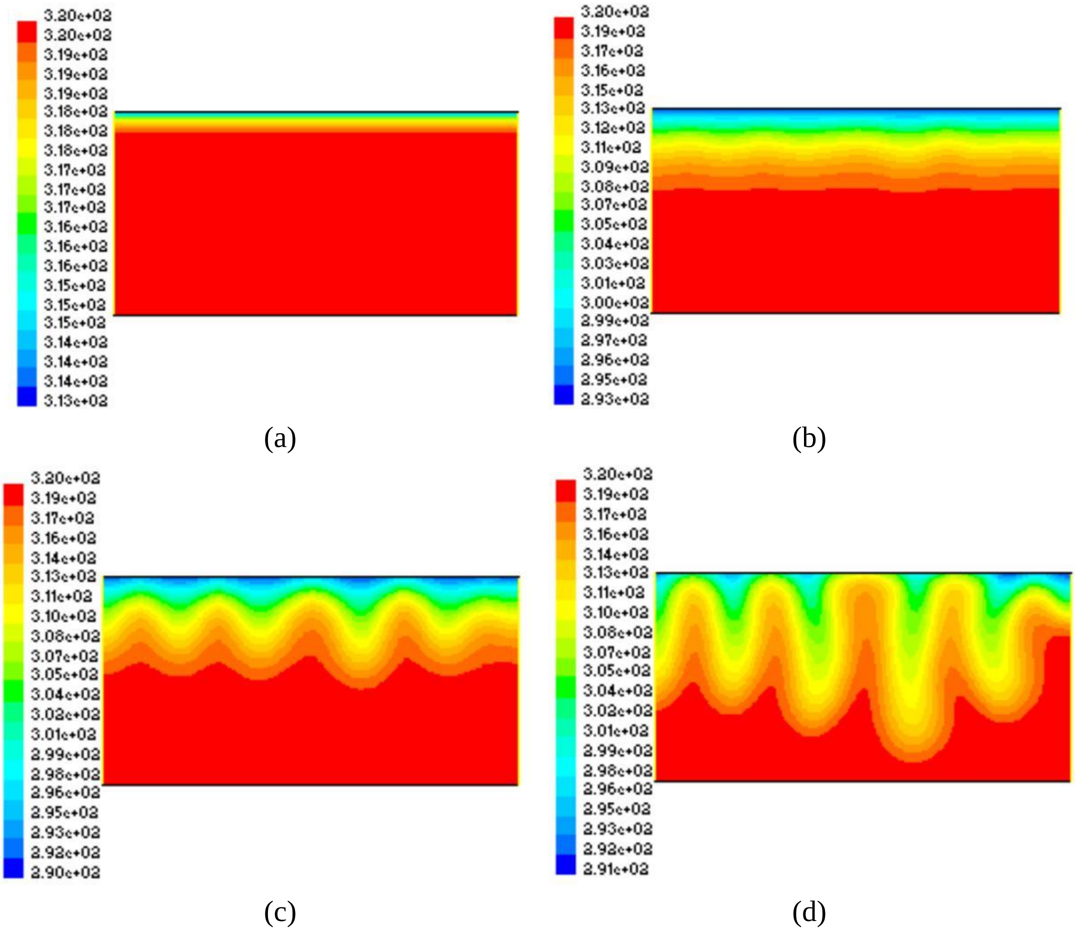

Figure 2. The development of thermal plumes for 5-cm glass beads saturated with CMC-5 for FST top cooling (ΔT = 27ºC) with t_c = 220,000 s.

3.3. Critical Times and Critical Rayleigh Number

The critical times of onset of transient convection in the simulations were determined from the temperature contour and velocity magnitude (figure 3). They are compared to the values predicted from equation (5) for various temperature difference T, as shown in table 4 and figure 5, which show very good agreement. The critical times of onset of convection is inversely proportional to the temperature difference and permeability.

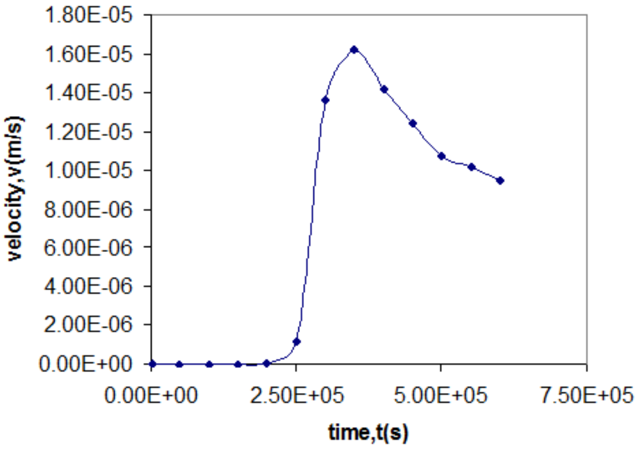

Figure 3. Maximum magnitude of velocity at various times for top cooling of non-Newtonian in FST boundary condition. (case1, 5cm glass beads, $t_c = 220,000$ s).

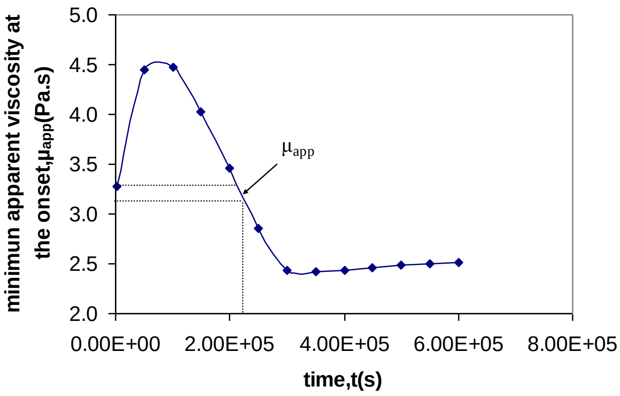

Figure 4. Minimum viscosity versus time for 5-cm glass beads saturated with CMC-5 FST top cooling at, $\Delta T = 27^oC$, $t_c = 220,000$ s. The average viscosity at initial and critical time is the apparent viscosity.

The maximum transient Rayleigh number is calculated from the critical time and the corresponding apparent viscosity (figure 4) in simulation for top cooling of CMC-5. Its average value is 38.4, which is close to the theoretical value 39.5 for FST (solid-solid) boundary condition.

Table 4. Comparison of the critical times and Rayleigh number from the simulations and redictions for non-Newtonian liquids in porous media for FST boundary condition.

bead size, *d* (cm)	ΔT, (^{0}C)	Permeability Ke (x10^{-6} m^{2})	t_c (s) Predicted	t_c (s) Simulation	Ra Simulation	z_{max} (m) Predicted	z_{max} (m) Simulation
5	27	1.38	2.5E+05	2.2E+05	37.4	0.7	0.6
5	24	1.38	3.4E+05	2.9E+05	36.4	0.8	0.7
5	21	1.38	4.6E+05	4.0E+05	36.9	0.9	0.8
5	18	1.38	6.4E+05	6.2E+05	38.9	1.0	1.0
5.5	27	1.68	1.6E+05	1.4E+05	37.2	0.5	0.5
5.5	24	1.68	2.1E+05	2.0E+05	38.3	0.6	0.6
5.5	21	1.68	2.9E+05	2.7E+05	37.8	0.7	0.7
5.5	18	1.68	4.0E+05	4.2E+05	40.5	0.8	0.8
6	27	1.99	9.1E+04	1.0E+05	41.4	0.4	0.4
6	24	1.99	1.3E+05	1.2E+05	37.7	0.5	0.5
6	21	1.99	1.8E+05	2.0E+05	41.2	0.6	0.6
6	18	1.99	2.7E+05	2.4E+05	37.6	0.7	0.6

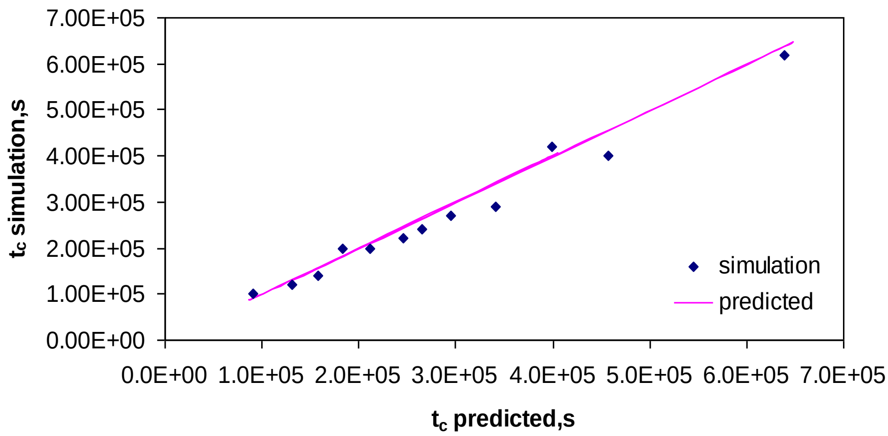

Figure 5. The critical times from the simulations and predictions for non-Newtonian liquids in porous media.

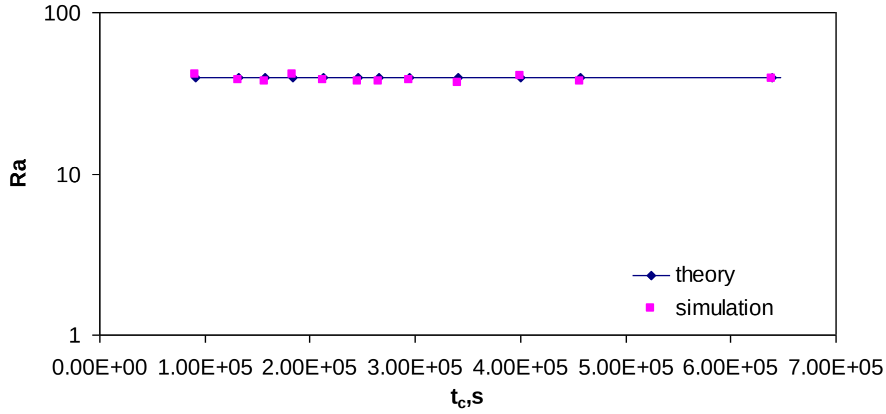

Figure 6. Maximum transient Rayleigh number for various simulations run for FST boundary condition.

3.4. Critical Thermal Depth

The critical depth of penetration just before the onset of convection in simulations were determined by measuring the distance from the top surface. They compare well with predictions by equation (3) as shown in table 4 and figure 7. The results of simulations showed that the domain size for the formation of fingering plumes are sufficient for them to form and develop freely without hindrance.

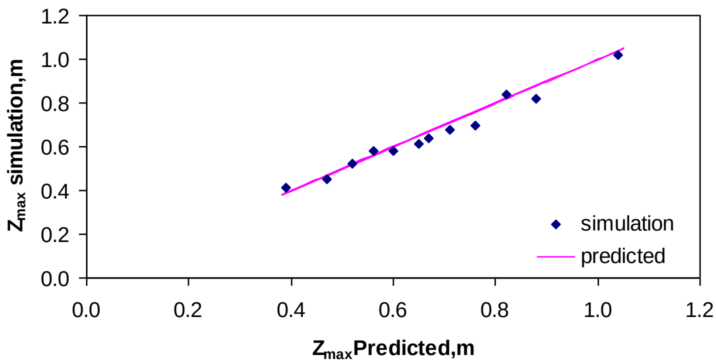

Figure 7. Comparison of the critical thermal depth, Z_{max} from the simulations and predicted for top cooling of FST boundary condition

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4. Conclusions

The CFD simulations of the onset of transient convection caused by top cooling of non-Newtonian liquids in porous media have been successfully conducted. They provide good verification of the key stability parameters predicted by transient instability theory advanced by Tan and Thorpe [15] and [16].

The critical times of simulations were accurately predicted with equation (5), which allow the computation of the maximum transient Rayleigh number. The average value of 38.4 was close to that of theory 39.5 [4].

The maximum critical thermal depths were predicted accurately for various bead diameters and permeability.

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