IOP Conf. Series: Materials Science and Engineering **199** (2017) 012053 doi:10.1088/1757-899X/199/1/012053

Optimal Control of Micro Grid Operation Mode Seamless Switching Based on Radau Allocation Method

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Abstract. The seamless switching process of micro grid operation mode directly affects the safety and stability of its operation. According to the switching process from island mode to grid-connected mode of micro grid, we establish a dynamic optimization model based on two grid-connected inverters. We use Radau allocation method to discretize the model, and use Newton iteration method to obtain the optimal solution. Finally, we implement the optimization mode in MATLAB and get the optimal control trajectory of the inverters.

1. Introduction

With the development of distributed generation, micro grid has been widely used as the main platform for its application. The micro grid is composed of distributed power, load, monitoring, energy conversion and protection devices, which can realize self-management and control. The operation mode of micro grid is divided into island, grid-connected and seamless switching mode. The seamless switching mode refers to the micro grid changes the operation mode between island and grid-connected, which is an important factor that influences the safety and stability of the micro grid[1-3].

According to the research of the control strategy of seamless switching, Wang Chengshan proposed a seamless switching strategy which has a certain degree of inhibition on the impact of dynamic current on the micro grid in [4], but the experimental results show that in 30ms, the voltage of micro grid has a large extent of fluctuation and reaches islanding protection conditions of other distributed generators in the area, leading to the system collapse. In [5], based on the analysis of the operation characteristics of the micro grid, the traditional voltage loop is improved by the principle of energy conservation, and the output of the voltage loop is set in advance, and the fluctuation of the voltage in the regulation process is reduced by using the switch in advance. Based on the energy storage of seamless switching strategy, the control mode of energy storage in different modes is changed according to the literature [6]. In the situation of active off grid, the inverter gradually assumed the full load of the area before the PCC disconnection, shorten the transition time; for the passive off grid, the hysteresis current control method is adopted to accelerate the current at the PCC point down to zero, and the transition process is shortened. However, there is still a large fluctuation in the current and voltage during the switching process. In [7], proposed a strategy based on the three loop control of voltage, current and power, utilizing the characteristics of the current of inductor and the voltage of capacitor which cannot mutation in the whole process, and the voltage and current in the switching process did not appear large fluctuations. In [8], the multi loop controller is used to achieve seamless switching from grid-connected mode to island mode, but the change of voltage and frequency of the island mode is not mentioned. In [9], proposed a voltage and current weighted control method based

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on the analysis of seamless switching process, suppressing current shock during the switching process, but the switching time is too long. In [10], proposed a four stages segment control based on abc static coordinate, although the fluctuation in the process of switching is reduced, but the excessive control stages increase difficulty of controlling and cannot guarantee the speed of micro grid off the grid. It can be seen that the seamless switching process of micro grid has made some achievements but there are still some problems, and there are still some space for further research and exploration.

In this paper, a control strategy for dynamic process of seamless switching is proposed, controlling voltage and frequency of grid-connected inverters in the dynamic process of seamless switching of micro grid by Radau allocation method， and achieve the goal of smooth and stable switching. Establishing a seamless switching model in MATLAB, the simulation results show that the method is effective and feasible.

2. Seamless switching control model

2.1. Inverter mathematical mode

The structure of the voltage source inverter [11] is shown in figure 1.

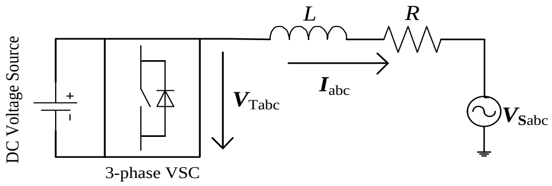

Figure 1. Structure of voltage source inverter

Kirchhoff's voltage law can be obtained from the output of the inverter:

$$\frac{\mathrm{d}\boldsymbol{I}_{\text{abc}}}{\mathrm{d}t}=\frac{1}{L}(-R\boldsymbol{I}_{\text{abc}}+\boldsymbol{V}_{\text{Sabc}}-\boldsymbol{V}_{\text{Tabc}}) \tag{1}$$

The expression in the XY coordinate system is:

$$\frac{\mathrm{d}I_{\text{x}}}{\mathrm{d}t}=\frac{1}{L}(-RI_{\text{x}}+V_{\text{Sx}}-V_{\text{Tx}}) \quad \frac{\mathrm{d}I_{\text{y}}}{\mathrm{d}t}=\frac{1}{L}(-RI_{\text{y}}+V_{\text{Sy}}-V_{\text{Ty}}) \tag{2}$$

Here, $I_{\text{x}}=\text{I}_{\text{m}}\cos\theta_{\text{I}}$, $I_{\text{y}}=I_{\text{m}}\sin\theta_{\text{I}}$, $V_{\text{Sx}}=V_{\text{Sm}}\cos\theta_{\text{VS}}$, $V_{\text{Sy}}=V_{\text{Sm}}\sin\theta_{\text{VS}}$, $V_{\text{Tx}}=V_{\text{Tm}}\cos\theta_{\text{VT}}$, $V_{\text{Ty}}=V_{\text{Tm}}\sin\theta_{\text{VT}}$. I_{m}, V_{Sm}, V_{Tm} is the amplitude of a phase of $\boldsymbol{I}_{\text{abc}}$, $\boldsymbol{V}_{\text{Sabc}}$, $\boldsymbol{V}_{\text{Tabc.}}$ $\boldsymbol{I}_{\text{abc}}$, $\boldsymbol{V}_{\text{Sabc}}$, $\boldsymbol{V}_{\text{Tabc}}$ are 3-phase vectors of the current of inverter, the voltage of load and the output voltage of inverter. $\theta_{\text{I}}(t)=\int_0^t\omega\mathrm{d}t+\alpha_{\text{I0}}$, $\theta_{\text{V}_\text{T}}(t)=\int_0^t\omega\mathrm{d}t+\alpha_{\text{V}_{\text{T0}}}$, $\theta_{\text{V}_\text{S}}(t)=\int_0^t\omega\mathrm{d}t+\alpha_{\text{V}_{\text{S0}}}$ are the instantaneous angle value of a phase of $\boldsymbol{I}_{\text{abc}}$, $\boldsymbol{V}_{\text{Sabc}}$, $\boldsymbol{V}_{\text{Tabc}}$. α_{I0}, α_{VT0}, α_{VS0} are the initial angle value of a phase of $\boldsymbol{I}_{\text{abc}}$, $\boldsymbol{V}_{\text{Sabc}}$, $\boldsymbol{V}_{\text{Tabc}}$. The expression for the XY coordinate system is:

$$\frac{\mathrm{d}I_{\text{m}}}{\mathrm{d}t}=I_{\text{m}}\omega\tan\theta_{\text{I}}-\frac{R}{L}I_{\text{m}}+\frac{\left(V_{\text{Sm}}\cos\theta_{\text{Vs}}-V_{\text{Tm}}\cos\theta_{\text{V}_\text{T}}\right)}{L\times\cos\theta_{\text{I}}} \tag{3}$$

$$\frac{\mathrm{d}I_{\text{m}}}{\mathrm{d}t}=I_{\text{m}}\omega\frac{1}{\tan\theta_{\text{I}}}-\frac{R}{L}I_{\text{m}}+\frac{\left(V_{\text{Sm}}\sin\theta_{\text{V}_\text{S}}-V_{\text{Tm}}\sin\theta_{\text{V}_T}\right)}{L\times\sin\theta_{\text{I}}} \tag{4}$$

Combine (3) and (4), then we can get equation (5):

$$\omega=\frac{V_{\text{Tm}}\sin(\theta_{\text{I}}-\theta_{\text{V}_\text{T}})-V_{\text{Sm}}\sin(\theta_{\text{I}}-\theta_{\text{V}_\text{S}})}{I_{\text{m}}L} \tag{5}$$

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2.2. seamless switching control model of micro grid

In figure 2, two inverters represent two grid-connected inverters of two micro grids, they are connected with fixed load, and the output voltage of the inverter is three-phase symmetrical voltage. This model is used to simulate the switching process of two micro grids from island mode to grid-connected mode. The goal of optimizing is to decrease the fluctuation of the dynamic voltage and frequency at the point of load before two micro grids connected. The dynamic optimization model can be constructed as follows:

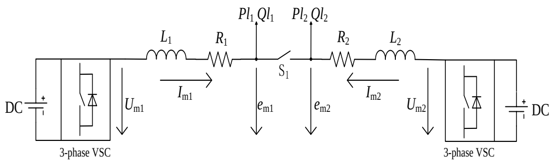

Figure 2. Seamless switching control structure for grid-connected inverters

Objective function:

$$\min\ J=\int_{t_0}^{t_f}\begin{array}{l}(e_{m1}-e_{mN})^2+(e_{m2}-e_{mN})^2+(\omega_1-\omega_N)^2\\ +(\omega_2-\omega_N)^2\end{array}\mathrm{d}t \tag{6}$$

Differential-algebraic equations:

$$\frac{\mathrm{d}\,\mathrm{I}_{m1}}{\mathrm{d}t}=\tan\theta_{\mathrm{I}_{m1}}I_{m1}\omega_1-\frac{R_1}{L_1}I_{m1}+\frac{U_{m1}\cos\theta_{\mathrm{U}_{m1}}-e_{m1}\cos\theta_{e_{m1}}}{L_1\cos\theta_{\mathrm{I}_{m1}}}\quad \frac{\mathrm{d}\theta_1}{\mathrm{d}t}=\omega_1 \tag{7}$$

$$\frac{\mathrm{d}\,\mathrm{I}_{m2}}{\mathrm{d}t}=\tan\theta_{\mathrm{I}_{m2}}I_{m2}\omega_2-\frac{R_2}{L_2}I_{m2}+\frac{U_{m2}\cos\theta_{\mathrm{U}_{m2}}-e_{m2}\cos\theta_{e_{m2}}}{L_2\cos\theta_{\mathrm{I}_{m2}}}\quad \frac{\mathrm{d}\theta_2}{\mathrm{d}t}=\omega_2 \tag{8}$$

Equality constraints:

$$\omega_1=\frac{U_{m1}\sin(\theta_{\mathrm{I}_{m1}}-\theta_{\mathrm{U}_{m1}})-e_{m1}\sin(\theta_{\mathrm{I}_{m1}}-\theta_{e_{m1}})}{I_{m1}L_1}\quad \omega_2=\frac{U_{m2}\sin(\theta_{\mathrm{I}_{m2}}-\theta_{\mathrm{U}_{m2}})-e_{m2}\sin(\theta_{\mathrm{I}_{m2}}-\theta_{e_{m2}})}{I_{m2}L_2} \tag{9}$$

Inequality constraints:

$$0.95e_{mN}\le e_{m1},e_{m2}\le 1.05e_{mN}\ \ 0.95\omega_N\le\omega_1,\omega_2\le 1.05\omega_N\ \left(t\in\left[t_0,t_f\right]\right) \tag{10}$$

Terminal constraint:

$$\omega_1=\omega_2\ \ \theta_{e_{m1}}=\theta_{e_{m2}}\ \ e_{m1}=e_{m2}\ \ t_f=1.2\ \ \frac{\mathrm{d}\,\mathrm{I}_{m1}}{\mathrm{d}t}=0\ \ \frac{\mathrm{d}\,\mathrm{I}_{m2}}{\mathrm{d}t}=0 \tag{11}$$

ω_1, ω_2 are angular frequency of two inverters, U_{m1}, U_{m2} are amplitude of output phase voltage of two inverters, e_{m1}, e_{m2} are magnitude phase voltage of the load, I_{m1}, I_{m2} are amplitude of phase current of two inverters, $\theta_{I_{m1}}$, $\theta_{I_{m2}}$ are the instantaneous value of current phase angle of two inverters, $\theta_{e_{m1}}$, $\theta_{e_{m2}}$ are the instantaneous value of voltage phase angle of loads, $\theta_{U_{m1}}$, $\theta_{U_{m2}}$ are the instantaneous value of output voltage phase angle of inverters, e_{mN} and ω_N are the standard value of angular frequency and voltage. Pl_1, Pl_2, Ql_1, Ql_2 are the active and reactive loads connected to two inverters.

3. Solving algorithm of seamless switching optimization control problem

3.1. Radau collocation method

Radau collocation method is a kind of implicit Runge Kutta method[12], the collocation points are selected based on the root of the orthogonal polynomials in the interval, the linear representation of control variables and state variables by Lagrange method in dynamic model, and the variable is represented by a polynomial at each collocation point, the model of differential equations into a series of algebraic equations, so the solution of algebraic equations is equal to the solution of differential equations, and equality and inequality constraints in the model is also discretized by this method, eventually the infinite dimensional problem is translated into finite dimensional DAE problem.

The dynamic optimization model can be changed into the following form:

Objective function:

$$\min\ J = C(\boldsymbol{u}, \boldsymbol{x}) \tag{12}$$

Differential-algebraic equations:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \quad g(\boldsymbol{x}, \boldsymbol{u}) = 0 \tag{13}$$

Control and trajectory constraints:

$$c(\boldsymbol{x}, \boldsymbol{u}) \le 0 \tag{14}$$

Terminal equation:

$$b(\boldsymbol{x}_{\mathrm{tf}}, \boldsymbol{u}_{\mathrm{tf}}) = 0 \tag{15}$$

Variable range limit:

$$\boldsymbol{x}^{\mathrm{L}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{U}} \qquad \boldsymbol{u}^{\mathrm{L}} \le \boldsymbol{u} \le \boldsymbol{u}^{\mathrm{U}} \tag{16}$$

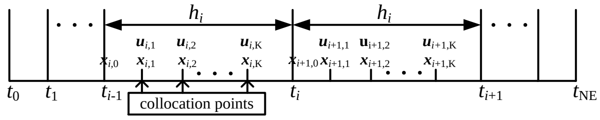

Figure 3. Schematic diagram of Radau allocation method

As shown in figure 3, we first divide the time *t* into NE intervals: [t_0, t_1], [t_1, t_2]... [t_{NE-1}, t_{NE}], where t_{NE} is the t_f at the end of interval, h_i is the length of interval, the root of the K order Legendre orthogonal polynomial is taken as the position of collocation points, discretizing the state variables $\boldsymbol{x}(t)$ and control variables $\boldsymbol{u}(t)$ by Lagrange interpolation in the interval as (17) and (18),

$$\boldsymbol{x}(t) = \sum_{j=0}^{\mathrm{K}} l_j(\tau)\boldsymbol{x}_{ij}, l_j(\tau) = \prod_{k=0,j}^{\mathrm{K}} \frac{\tau - \tau_k}{\tau_j - \tau_k} (i = 1,2,...,\mathrm{NE}) \tag{17}$$

$$\boldsymbol{u}(t) = \sum_{j=0}^{\mathrm{K}} \theta_j(\tau)\boldsymbol{u}_{ij}, \ell_j(\tau) = \prod_{k=1,\neq j}^{\mathrm{K}} \frac{\tau - \tau_k}{\tau_j - \tau_k} (i = 1,2,...,\mathrm{NE}) \tag{18}$$

Here, $t = t_{i\text{-}1} + h_i\tau, \tau \in [0,1], \tau_0 = 0, \tau_j (j = 1,2,...,\mathrm{K})$ is the root of the K order Legendre orthogonal polynomials, $\ell_j(\tau), \theta_j(\tau)$ are the Lagrange basis function, k=0, j shows the value of k start from 0 and $k \ne j$, and $\boldsymbol{x}_{ij}$,, $\boldsymbol{u}_{ij}$ are the value of state variables, control variables in the collocation points. At the same time by the Lagrange condition $\ell_k(\tau_j) = \delta_{kj}$ (δ_{kj} =Kronecker delta), we can get equation (19):

$$\boldsymbol{x}(t_{ij}) = \boldsymbol{x}_{ij}, t_{ij} = t_{i-1} + h_i \cdot \tau_j (j = 0,1,...\mathrm{K}) \tag{19}$$

Put equation (19) into equation (13) and get residual equation:

$$h_i R(t_{ik}) = \sum_{k=0}^{\mathrm{K}} \dot{\ell}_k(\tau_j)\boldsymbol{x}_{ik} - h_i F(\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}) = 0 \ (i = 1,2,...,\mathrm{NE}, j = 1,2,...,\mathrm{K}) \tag{20}$$

$\ell_k(\tau_j) = \left.\frac{dl_k(\tau)}{d\tau}\right|_{\tau=\tau_j}$ can be calculated in advance, and $t_{ik} = t_{i-1} + h_i\tau, \tau \in [0,1]$, When time t is the

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optimization variable, it is beneficial to solve this kind of expression. The optimization process of interval length can be used to find the breakpoints in the trajectory to ensure that the error is within the allowable range.

The initial value of the state variable $\boldsymbol{x}(t_0)$ can be expressed as:

$$\boldsymbol{x}_{1,0} = \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{21}$$

The final value can be represented by the interval extrapolation expression:

$$\boldsymbol{x}_{\mathrm{f}} = \boldsymbol{x}(t_{\mathrm{f}}) = \sum_{j=0}^{\mathrm{K}} \ell_j(\tau=1)\boldsymbol{x}_{\mathrm{NE},j} \tag{22}$$

Terminal constraint can be expressed as:

$$G_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}) = G_{\mathrm{f}}[\sum_{j=0}^{\mathrm{K}} \ell_j(\tau=1)\boldsymbol{x}_{\mathrm{NE},j}] = 0 \tag{23}$$

In order to ensure the continuity of the state variables at the end of interval:

$$\boldsymbol{x}_{i0} = \sum_{j=0}^{\mathrm{K}} \ell_j(\tau=1)\boldsymbol{x}_{\mathrm{NE},j} \; (i = 2,3,...,\mathrm{NE}) \tag{24}$$

In order to satisfy the constraint of the control variables at the end of interval, at the end of interval $[t_{i-1}, t_i]$:

$$\left.\begin{array}{l} \boldsymbol{u}^{\mathrm{L}} \le \sum_{j=1}^{\mathrm{K}} \theta_j(\tau=0)\boldsymbol{u}_{ij} \le \boldsymbol{u}^{\mathrm{U}} \\ \boldsymbol{u}^{\mathrm{L}} \le \sum_{j=1}^{\mathrm{K}} \theta_j(\tau=1)\boldsymbol{u}_{ij} \le \boldsymbol{u}^{\mathrm{U}} \end{array}\right\} (i = 1,...\mathrm{NE}) \tag{25}$$

The original dynamic optimization problem is transformed into:

$$\min_{x_{ij},u_{ij}} J = \psi[\boldsymbol{x}(\mathrm{f})] + \sum_{i=1}^{\mathrm{NE}} \sum_{j=1}^{\mathrm{K}} \omega_{ij}\varphi[\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}] \; (t \in [t_0, t_{\mathrm{f}}]) \tag{26}$$

constraint conditions：

$$\sum_{k=0}^{\mathrm{K}} \dot{\ell}_k(\tau_j)\boldsymbol{x}_{ik} - h_i F(\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}) = 0 \; (i = 1,...,\mathrm{NE}, j = 1,...,\mathrm{K}) \tag{27}$$

$$G[\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}] = 0 \; (i = 1,...,\mathrm{NE}, j = 1,...,\mathrm{K}) \tag{28}$$

$$C[\boldsymbol{x}_{ij}, \boldsymbol{u}_{ij}] = 0 \; (i = 1,...,\mathrm{NE}, j = 1,...,\mathrm{K}) \tag{29}$$

$$\boldsymbol{x}_{i0} = \sum_{j=0}^{\mathrm{K}} l_j(\tau=1)\boldsymbol{x}_{\mathrm{NE},j} \;\; (i = 2,3,...,\mathrm{NE}) \tag{30}$$

$$\boldsymbol{x}_{1,0} = \boldsymbol{x}_0 \;\; \boldsymbol{x}_{\mathrm{f}} = \sum_{j=0}^{\mathrm{K}} \ell_j(\tau=1)\boldsymbol{x}_{\mathrm{NE},j} \tag{31}$$

$$G_{\mathrm{f}}[\boldsymbol{x}(\mathrm{f})] = 0 \tag{32}$$

$$\boldsymbol{x}_{1,0} = \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{33}$$

$$\boldsymbol{x}^{\mathrm{L}} \le \boldsymbol{x}_{ij} \le \boldsymbol{x}^{\mathrm{U}} \; \boldsymbol{u}^{\mathrm{L}} \le \boldsymbol{u}_{ij} \le \boldsymbol{u}^{\mathrm{U}} \; (i = 1,...,\mathrm{NE}, j = 1,...,\mathrm{K}) \tag{34}$$

$$\boldsymbol{u}^{\mathrm{L}} \le \sum_{j=1}^{\mathrm{K}} \theta_j(\tau=0)\boldsymbol{u}_{ij} \le \boldsymbol{u}^{\mathrm{U}} \;\; \boldsymbol{u}^{\mathrm{L}} \le \sum_{j=1}^{\mathrm{K}} \theta_j(\tau=1)\boldsymbol{u}_{ij} \le \boldsymbol{u}^{\mathrm{U}} \; (i = 1,...,\mathrm{NE}) \tag{35}$$

Here, $\ell_j(\tau), \theta_j(\tau), \ell_k(\tau_j)$ (Lagrange basis functions and their derivatives) are determined by the position of orthogonal polynomials which can be calculated in advance. ω_{ij} is positive integration factor. The discretized model (27) ~ (35) can be solved by Newton iteration method.

4. Example analysis

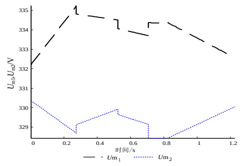
4.1. Optimization results of seamless switching process model

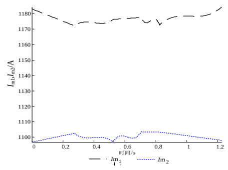
By using model raised in 2.2, the switching process of two micro grids from island mode to grid-connected mode is simulated and the optimal control trajectory is solved according to the Radau collocation method in MATLAB.

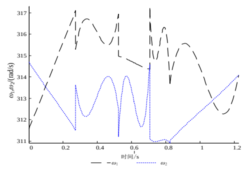
Table 1. Initial value of variables

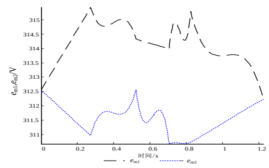
Variable / inverter	ω /rad/s	e_m /V	Pl /MW	Ql /MVar	θ_{Im} /rad	θ_{Um} /rad	θ_{em} /rad	R /Ω	L /H
Inverter 1	311.86	312.54	0.5	0.24	0.0525	0.5971	0.5	0.01	0.0001
Inverter 2	314.63	312.54	0.46	0.23	2.5364	3.0921	3	0.001	0.0001

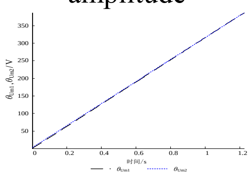
Model optimization results are as shown in figure 4:

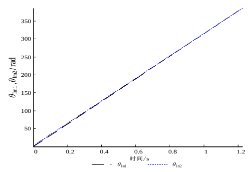

Inverters output phase voltage amplitude

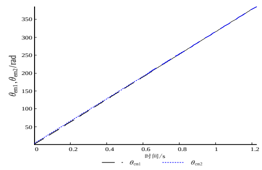

Inverters output phase current amplitude


Inverters angular frequency


load phase voltage amplitude


Inverters output voltage phase angle


load current phase angle


load voltage phase angle

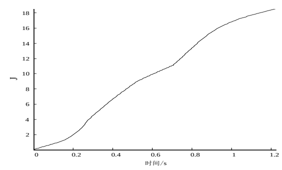

Objective function J

Figure 4. Objective function *J*

The result of running the program can be seen, through the method of dynamic optimization, the optimal control trajectory were found, the optimization results is 18.57, the voltage of loads , phase angle and frequency of two inverters are equal at the end of the time interval, which reached the grid-connected conditions.

5. Conclusion

According to the switching process of micro grid from the island mode to grid-connected mode, a dynamic optimization model was established, and the discrete solution method of dynamic model based on Radau collocation method is proposed. By using this method, the optimality conditions are solved by newton iterative method after discretizing state variables and control variables at the Radau collocation points. Two grid-connected inverters models are constructed, and the optimal control trajectory is obtained by using this method. The next step should be to consider how to build a more realistic model of the micro grid to verify.

Acknowledgments

Project supported by National Natural Science Foundation of China(No. 51307063)

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