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Active and Reactive Power Optimal Dispatch Associated with Load and DG Uncertainties in Active Distribution Network

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Abstract. In order to reduce the adverse effects of uncertainty on optimal dispatch in active distribution network, an optimal dispatch model based on chance-constrained programming is proposed in this paper. In this model, the active and reactive power of DG can be dispatched at the aim of reducing the operating cost. The effect of operation strategy on the cost can be reflected in the objective which contains the cost of network loss, DG curtailment, DG reactive power ancillary service, and power quality compensation. At the same time, the probabilistic constraints can reflect the operation risk degree. Then the optimal dispatch model is simplified as a series of single stage model which can avoid large variable dimension and improve the convergence speed. And the single stage model is solved using a combination of particle swarm optimization (PSO) and point estimate method (PEM). Finally, the proposed optimal dispatch model and method is verified by the IEEE33 test system.

1. Introduction

With the increasing of energy shortage and environmental pollution, renewable energy power generation has been developed rapidly. As a representative of distributed generation, PV promotes renewable energy and becomes an effective complement to traditional form of power generation, which attracts more application recently [1,2]. With the PV capacity is increasing in distribution network, the operation faces two challenges. PV introduces new uncertainty to the distribution network [3,4], together with the large scale of loads, showing more complicated characteristics. In the other side, after the integration of large-scaled PV to the radial distribution network, the load flow no longer maintains the one-way distribution. So the traditional control mode does not meet the flexible control requirements.

For the first problem, reference [5] proposes a chance-constrained programming based optimal control method of energy storage in view of the randomness of PV generation and wind power generation. Reference [6] proposes a robust interval voltage control strategy considering the uncertainties of PV output and load demand. The optimal reactive power compensation strategies and the allowable active power interval for the PV station are calculated in the strategy. Reference [7] establishes a fuzzy chance-constraints based day-ahead scheduling model considering demand response and its uncertainty. Then according to uncertain programming theory, the creditability change constraints can be turned into their clear equivalence class and the proposed model can be solved by mixed integer programming (MIP) method. For the second problem, in some works only reactive power of DG is used to keep the network voltages at an acceptable level [8,9]. While in the references [6,10,11], the real power also can be used as a control variable.

It is worth noting that reasonable strategy of the DG output to achieve the economic operation in the distribution network system, is important to the dispatch center for the further power generation plan. However, in these researches, the control of DG active and reactive power focuses on the voltage regulation within a day. And the day-ahead dispatch of DG output considering the uncertainties is less involved

According to the power probability model of DG and load, an optimal dispatch model based on chance-constrained programming is proposed in this paper. In this model, the active and reactive power of DG can be scheduled at the aim of reducing the overall operating cost. Then the optimal dispatch model is simplified as a series of single stage model which can avoid large variable dimension and improve the convergence speed. And the single stage model is solved using a combination of particle swarm optimization (PSO) and point estimate. Finally, the proposed optimal dispatch model and method is verified by the IEEE33 test system.

2. Day-ahead optimal dispatch model

In this section, the ability of DG active power curtailment and reactive power auxiliary service are utilized to establish a model for optimal dispatch in active distribution network. This model aims to provide optimized strategy which guarantees the safe operation and reduce the operating cost considering the large scale of DG integration.

2.1. Optimization objective

The optimization objective for operating cost minimization primarily includes four parts, as shown in (1), which contain the cost of network loss, DG dispatch and power quality compensation. In the active distribution network, the DGs represented by photovoltaic generation system should be preferentially used. When DGs affect safety of network, however, it is still necessary to restrict the active power output of DGs. At the same time, the reactive power auxiliary service of DG can provide reactive power support to the system to improve the power quality. So in this paper, the DG curtailment, reactive power auxiliary service and traditional reactive power compensation are taken as the control variables for day-ahead optimal dispatch in distribution network.

$$f = \Delta T \sum_{t=1}^{N_T} \begin{pmatrix} C_{loss}(t) P_{loss}(t) + \sum_{i=1}^{N_{DG}} C_{i,cur}(t) P_{i,DG}(t) \\ + \sum_{i=1}^{N_{DG}} C_{i,anc}(t) Q_{i,DG}(t) + \sum_{i=1}^{N_{node}} C_{i,com}(t) d_i(t) \end{pmatrix} \tag{1}$$

$$d_i = \left(1 - p\left\{U_i^{\min} \le U_i \le U_i^{\max}\right\}\right) S_i, \qquad i \in \Omega_{node} \tag{2}$$

Where, ΔT is the time period. N_T is the number of dispatch periods. $C_{loss}(t)$ and $P_{loss}(t)$ are the unit loss cost and total loss during period t. N_{DG} is the number of DGs. $C_{i,cur}(t)$ and $P_{i,DG}(t)$ are the unit power cost and restricted power of i^{th} DG during period t. $C_{i,anc}(t)$ and $Q_{i,DG}(t)$ are the unit reactive power cost and reactive power of i^{th} DG during period t. $C_{i,com}(t)$ and $d_i(t)$ are the compensation cost for substandard power quality and compensation power. U_i, $U_I^{\min}$ and $U_I^{\max}$ are the voltage amplitude, and its lower limit and upper limit of node i. S_i is the apparent power of the node i. Ω_{node} is the set of nodes.

2.2. Constrains

2.2.1. Power balance

$$P_i - U_i \sum_{j=1}^{Nnode} U_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}) = 0 \tag{3}$$

$$Q_i - U_i \sum_{j=1}^{Nnode} U_j (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}) = 0 \tag{4}$$

Where, N_{node} is the number of nodes; P_i and Q_i are the active and reactive power input at node i. G_{ij}, B_{ij} and δ_{ij} are the conductance, susceptance and phase angle difference between nodes i and j.

2.2.2. Safety. Safety restriction here primarily refers to the probabilistic constraints of node voltage and branch flow.

$$p\left\{U_i^{\min} \le U_i \le U_i^{\max}\right\} \ge p_{U_i}, \quad i \in \Omega_{node} \tag{5}$$

$$p\left\{S_l \le S_l^{\max}\right\} \ge p_{S_l}, \quad l \in \Omega_l \tag{6}$$

Where, $p\{\bullet\}$ indicates the probability of the event in $\{\bullet\}$ comes into existence, S_l and S_l^{max} are the apparent power and its upper limit of the branch l, p_{U_i} and $p_{S_{ij}}$ are the probability level of U_i and S_l, Ω_l is the set of branches.

2.2.3. DG output. The photovoltaic generation represents DG in this paper. The control variables contain active power curtailment and reactive power compensation. Considering the uncertainties, the control variable in this optimal dispatch model is set as the ratio of the practical output to the maximum output, following the constraint shown below:

$$\forall\left(i \in \Omega_{DG}\right) : \exists\left(0 \le \alpha_i \le 1 \,\&\, 0 \le \beta_i \le 1\right) \tag{7}$$

Where, α_i and β_i are the ratios of the practical active and reactive power output to the maximum output for DG i.

2.2.4. reactive power compensation

$$Q_{i,C}^{\min} \le Q_{i,C} \le Q_{i,C}^{\max}, \quad i \in \Omega_C \tag{8}$$

Where, $Q_{i,C}^{\max}$ and $Q_{i,C}^{\min}$ are the capacity upper and lower limit of reactive power compensation devices respectively. Ω_C is the set of reactive power compensation devices.

This optimal dispatch model is a dynamic optimization model with multi-period coupling. It is should be noted that the reactive power compensation devices represented by capacitors, should limit the actions numbers within a dispatch cycle to extend the use life. In this paper, in order to highlight the chance-constrained model and the solving method, the capacitor problem of multi-period coupling is simplified as a reasonable set of N_T based on action number constraint. In this paper, a N_T is set to be 4, then dynamic optimization problem is simplified to several static chance-constrained programming problems.

3. Optimization method

3.1. Stochastic load flow based on point estimate method

Based on point estimate method (PEM)[12,13], the moment of relative variables can be calculated using the probability density function of random variables. According to the moments, the estimated point set and the corresponding probability of each point are obtained. Then the probability distribution of the nonlinear function can be obtained.

It is assumed that X is an n-dimension random variable, the probability density function is $f(x)$, and $Y=h(X)$ is a nonlinear function taking X as the variable. PEM can replace the joint probability density function with probability set. In this process, the j^{th} order center moment of the random variable X_i is calculated firstly:

$$\lambda_{i,j} = \frac{\int_{-\infty}^{\infty}\left(x-\mu_i\right)^j f_i\left(x\right)dx}{\left(\sigma_i\right)^j} \tag{9}$$

Where, $\lambda_{i,j}$ is the j^{th} order center moment of the i^{th} dimension in X. μ_i, σ_i and $f_i(x)$ are the mathematical expectation, standard deviation and the probability density function of X_i respectively.
The position coefficient and the corresponding probability of estimated points can be obtained using the parameters above based on the 2-point estimate method:

$$\xi_{i,k} = \frac{\lambda_{i,3}}{2} + \left(-1\right)^{3-k}\sqrt{n+\left(\frac{\lambda_{i,3}}{2}\right)^2} \tag{10}$$

$$w_{i,k} = \frac{\left(-1\right)^k \xi_{i,3-k}}{2n\sqrt{n+\left(\frac{\lambda_{i,3}}{2}\right)^2}} \tag{11}$$

Where, k is the index of estimated points, and in the 2-point estimate method, k=1,2. $\xi_{i,k}$ and $w_{i,k}$ are the position coefficient and the corresponding probability of the k^{th} estimated point for X_i.
Based on the position coefficient and probability, the estimated points of X_i can be obtained:

$$x_{i,k} = \mu_i + \xi_{i,k}\sigma_i \tag{12}$$

According to the nonlinear function between Y and X, the function value of Y is calculated based on each estimated point $x_{i,k}$. Then the moments of Y can be obtained:

$$E\left(Y_j\right) = \sum_{k=1}^{2}\sum_{i=1}^{n} w_{i,k}\left[h\left(\mu_1,\dots,x_{i,k},\dots\mu_n\right)\right]^j \tag{13}$$

Finally, the probability density of Y can be obtained according to the Cornish-Fisher series expansion method.

In this paper, the optimal dispatch variables contain α_i. If the probability density function of the maximum active power output $P_{i,DG}^{\max}$ is $f_i(P^{\max})$, the central moments of $\alpha_i P_{i,DG}$, which represents the actual output of the i^{th} DG after dispatch is:

$$\begin{aligned}\lambda_{i,j} &= \frac{\int_{-\infty}^{\infty}\left(\alpha_i P_{i,DG} - \alpha_i\mu_i\right)^j f_i\left(P_{i,DG}^{\max}\right)dP_{i,DG}}{\left(\alpha_i\sigma_i\right)^j} \\ &= \frac{\int_{-\infty}^{\infty}\left(P_{i,DG} - \mu_i\right)^j f_i\left(P_{i,DG}^{\max}\right)dP_{i,DG}}{\left(\sigma_i\right)^j}\end{aligned} \tag{14}$$

It can be seen that the central moments of $\alpha_i P_{i,DG}$ and $P_{i,DG}^{\max}$ are the same, which leads to the same $\xi_{i,k}$ and $w_{i,k}$, so the estimated point is:

$$\alpha_i P_{(i,DG),k} = \alpha_i\mu_i + \xi_{i,k}\alpha_i\sigma_i = \alpha_i\left(\mu_i + \xi_{i,k}\sigma_i\right) \tag{15}$$

It shows that the ratio of the actual output estimated points and the maximum output estimated points are all α_i. Then the estimated points of the reactive power output can be obtained:

$$\beta_i Q_{(i,DG),k} = \beta_i\sqrt{\left(S_{i,DG}^{\max}\right)^2 - \left(P_{(i,DG),k}\right)^2} \tag{16}$$

Based on the above deduction, the estimated points of DG actual output can be calculated by the proportion and the maximum output estimated points. The calculation efficiency has been greatly improved.

3.2. Method for chance-constrained programming model

The chance-constrained programming model can be solved by the stochastic sampling and intelligent optimization algorithms combined method. But when the dimentions of random variable is large, the sampling scale will also increase greatly in order to ensure full coverage of the sample space. Using 2-point estimate method, only two estimated points should be calculated for one dimention of random variable, so a probabilistic load flow can be completed with $2n$ times of load flow calculation. The 2-point estimate and particle swarm optimization(PSO) combined method can greatly reduce the computation burden. The specific solution procedure is as follows:

- Step1: Carry out 2-point estimate to the random variable in a period of time, as (9)-(12) shown;
- Step2: Initialize the group of particles with random assignment of velocity and position;
- Step3: Carry out the load flow according to the estimated points and the particles as shown in (13), and obtain the moments of the parameters required by the objective function and the constraints;
- Step4: Calculate the fitness value of each particle, using the moments of the parameters and the corrspongding probability density function;
- Step5: Compare the fitness value with the personal best position *pbest* for each particle, and chose the better one as the new *pbest*;
- Step6: Compare the fitness value with the global best position *gbest* for each particle, and chose the better one as the new *gbest*;
- Step7: Update the velocity and position of particles;
- Step8: End the process if meet the end condition, else go to Step3.

4. Case verification

In this paper, IEEE33 node test system is adopted to verify the proposed optimal dispatch model and method, with the specific parameters of the line and load can be found in [14]. Three photovoltaic generation units with the same parameters is integrated at nodes 13, 18, 22 25 and 29, with the parameters shown in table 1. Two three-phase shunt capacitor banks are respectively connected in node 14 and 30, with the capacity both of 5*120kVar. At the meanwhile, a three-phase SVC is connected in node 7, with the continuous adjustable capacity from -600kVar to 600kVar. In test region, the maximum irradiance is set as 1000W/m^2. The upper limit of voltage is 1.05 pu and the lower limit is 0.95 pu. The $C_{i,cur}$, $C_{i,anc}$ and $C_{i,com}$ are all set to be 0.5 yuan/kWh.

Table 1. Parameters of PV systems

node	Single System Area/(m^2)	Photoelectric Conversion Efficiency /%	Maximum Irradiance /(W/m^2)
13,18,22,25,29	3000	14%	1000

The case is set up two scenarios of light load and heavy load to verify the effectiveness of the proposed optimal dispatch model and algorithm. The load in light load scenario is set to 1/4 of the base load. While the load in heavy load scenario is 1.6 times. And the irradiance in the two scenarios are both 540 W/m^2.

4.1. Light Load scenario

Due to the day-ahead optimal dispatch has achieved decoupled, the verification here focuses on the results within a period of time. The optimal strategies obtained by model of chance-constraint programming optimization (CCPO) and traditional optimization (TO) under the different standard deviation are shown in table 2.

Table 2. optimal strategies in light load scenario

Deviation	Model	C1	C2	C3	PV1	PV2	PV3	PV4	PV5
5%	CCPO	0	0	-324.1	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0

	TO	0	2	-306.5	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0
10%	CCPO	0	1	-600	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0
	TO	0	2	-306.5	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0

In order to verify the effect of strategy considering the uncertainty, two scenarios are chosen from the multiple possible scenarios. In the normal scenario, the irradiance and load are set to be the mathematical expectation. In the extreme scenario, the irradiance is set to be a large one while the load to be the small one in the value range based on the probability density functions. Figure 1 and 2 shows the effect of the dispatch strategies on the voltage under two scenarios. And the operating cost comparison is given in the table 3.

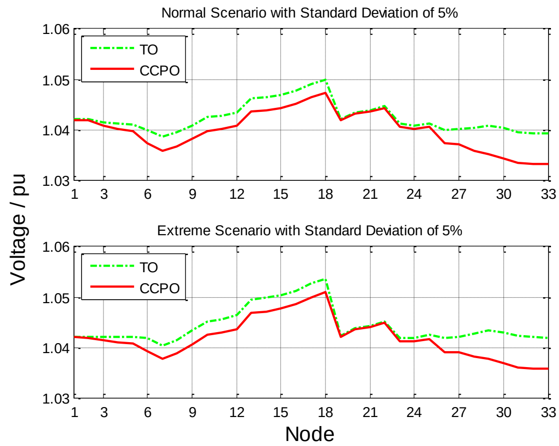

Figure 1. Voltage distribution with standard deviation of 5% in light load scenario.

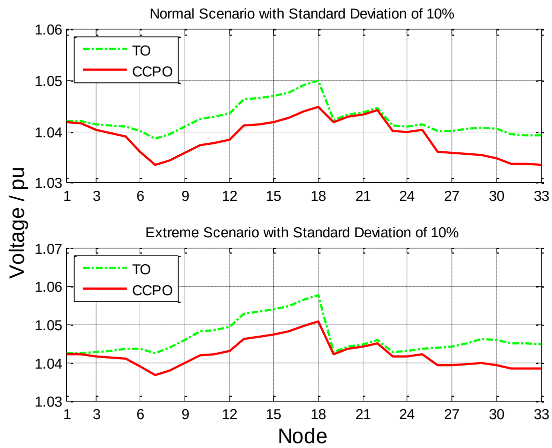

Figure 2. Voltage distribution with standard deviation of 10% in light load scenario.

Table 3. operating cost comparison in light load scenario

Deviation	Model	Cost in Normal Scenario / RMB		Cost in Extreme Scenario/ RMB	
		no compensation	Compensation	no compensation	Compensation
5%	CCPO	26.1	26.1	27.1	99.9
	TO	15.6	15.6	17.7	146.3
10%	CCPO	31.9	31.9	32.3	71.6
	TO	15.6	15.6	20.9	220.7

It can be seen that the power quality is improved by the strategies of CCPO. Although compared to the CCPO, the operating cost of TO is less in normal scenario, once an extreme scenario occurs, the operating cost sharply increases due to the power quality compensation brought by the voltage beyond limit. And the operating cost of CCPO is little higher in normal scenario, but it also presents a better voltage stability at the same time. Although the cost will increase due to the power quality compensation in extreme scenario, the overall level of operating cost is relatively stable, making the operation safety and economy balanced.

4.2. heavy load scenario

Table 4 shows the optimal strategies obtained by model of CCPO and TO under the different standard deviation in heavy load scenario.

Table 4. optimal strategies in heavy load scenario

Deviation	Model	C1	C2	C3	PV1	PV2	PV3	PV4	PV5
5%	CCPO	5	5	-600	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0.9
	TO	5	5	-600	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0.6
10%	CCPO	5	5	-600	α=1 β=0.6	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=1
	TO	5	5	-600	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0	α=1 β=0.6

Figure 3 and 4 show the effect of the dispatch strategies on the voltage under two scenarios. And the operating cost comparison is given in the table 5. Because the reactive power compensation ability of DG is limited, the effect differences of two strategies on the voltage is smaller than that in the light load scenario. But the strategy based on CCPO still presents the adaptability to guarantee the safe and stable operation in the extreme scenario.

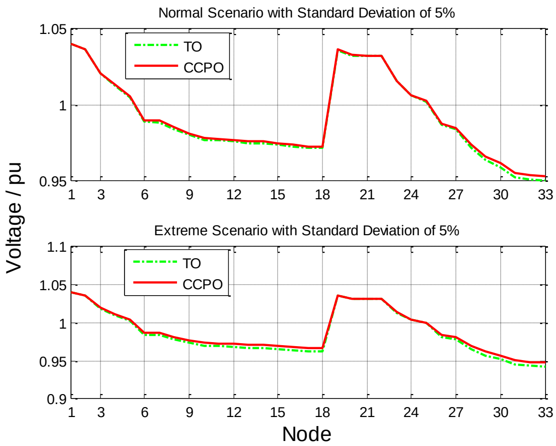

Figure 3. Voltage distribution with standard deviation of 5% in heavy load scenario.

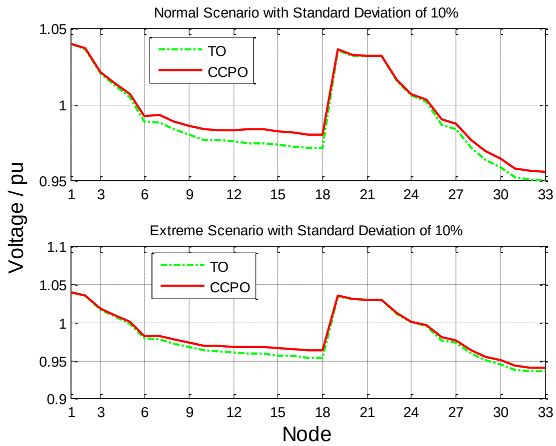

Figure 4. Voltage distribution with standard deviation of 10% in heavy load scenario.

Table 5. operating cost comparison in heavy load scenario

Deviation	Model	Cost in Normal Scenario / RMB		Cost in Extreme Scenario/ RMB	
		no compensation	Compensation	no compensation	Compensation
5%	CCPO	1102.5	1102.5	1158.0	2162.3
	TO	901.1	901.1	982.9	2562.9
10%	CCPO	1593.6	1593.6	1770.5	3425.8
	TO	901.1	901.1	1068.3	4949.8

5. Conclusion

Considering the uncertainties of DG and load in the day-ahead optimal dispatch, a chance-constrained programming is proposed in this paper. This model, using the control ability of DG output and reactive power compensation device, aims at the minimizing the overall operating cost. Then based on the simplification to a static optimization model, the chance-constrained programming is solved using a combination of particle swarm optimization (PSO) and point estimate.

The verification in the IEEE 33 test system shows that the chance-constraint programming model presents better adaptability in normal and extreme scenarios, because of the consideration of uncertainties. Compared to the TO, although the operating cost of CCPO is little higher in normal scenario due to the control margin, it can guarantee the operation safety in extreme scenario and reduce the compensation cost caused by the voltage beyond limit. In the active distribution network with large scale DG integration and complicated uncertain characteristics, the day-ahead optimal dispatch model and method based on the chance-constrained programming can effectively reduce the adverse effects of uncertainty on optimal dispatch.

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