

# The Operational Risk Assessment for Distribution Network with Distributed Generations

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**Abstract.** Distribution network is an important part of the power system and is connected to the consumers directly. Many distributed generations that have discontinuous output power are connected in the distribution networks, which may cause adverse impact to the distribution network. Therefore, to ensure the security and reliability of distribution network with numerous distributed generations, the risk analysis is necessary for this kind of distribution networks. After study of stochastic load flow algorithm, this paper applies it in the static security risk assessment. The wind and photovoltaic output probabilistic model are built. The voltage over-limit is chosen to calculate the risk indicators. As a case study, the IEEE 33 system is simulated for analyzing impact of distributed generations on system risk in the proposed method.

## 1. Introduction

A lot of distributed generations are access to the distribution system, which changes the operation and structure of the system. Its randomness and discontinuity of output power will likely have a negative impact on the power grid, and may introduce risks to safe operation of the distribution network [1-3]. The distribution network is directly connected to the consumers, so it affects the security and reliability of power system.

The purpose of risk assessment is to assess the potential impact of disturbance events on the system voltage. Distribution network risk assessment model, assessment methods and evaluation indicators constitute the main framework of risk assessment [4-5]. The operational risk assessment of distribution network with distributed generation is carried out by [6]. Based on the probabilistic power flow algorithm, the risk index of the distribution network voltage is obtained when the wind power is connected to different nodes.

In this paper, a stochastic model of wind energy, solar power generation, and load is established. The probability distribution of system node voltage and branch flow is obtained by using stochastic power flow algorithm. The case results show that the operational risk of distribution network with distributed generation is assessed scientifically in this method.

## 2. Probabilistic model of distributed generation



The risk assessment of distribution network with distributed generation should consider all kinds of uncertainties in the system operation, including the uncertainty of the system load and output power of the distributed generation. First the corresponding output probability model is established.

### 2.1. The probabilistic model of wind turbine output power

Weibull distribution is generally used for wind speed model. The probability distribution of wind power is calculated according to the output power characteristics of the wind turbine. The relationship between output power and wind speed is as follows [7].

$$P_{WG} = \begin{cases} 0 & v \leq v_{ci} \\ k_1 v + k_2 & v_{ci} \leq v \leq v_r \\ P_r & v_r \leq v \leq v_{co} \\ 0 & v \geq v_{co} \end{cases} \quad (1)$$

Where  $k_1 = Pr/(v_r - v_{ci})$ ,  $k_2 = -k_1 v_{ci}$ , and  $Pr$  is rated output power of wind turbine.  $v_{ci}$  is the cut in wind speed;  $v_r$  represents the rated speed;  $v_{co}$  is the cut out speed of the wind power generator;  $v$  is the wind speed at the height of the wind turbine hub. The aim of composite energy storage system to stabilize the imbalance of power fluctuation

A large number of measured data show in a region wind speed distribution is approximately Weibull distribution. The probability density function is shown in (2).

$$f(v) = \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (2)$$

Where  $v$  means wind speed,  $c$  and  $k$  are the two parameters of Weibull distribution. By statistics, in most of the time the wind speed is maintained between  $v_{ci}$  and  $v_r$ . By formula (1) the probability density function of wind power output  $P_W$  is shown as follows [7].

$$f(P_W) = \frac{k}{k_1 c} \cdot \left(\frac{P_W - k_2}{k_1 c}\right)^{k-1} \cdot \exp\left[-\left(\frac{P_W - k_2}{k_1 c}\right)^k\right] \quad (3)$$

Generally the power factor is maintained unchanged through parallel compensator, so the wind output is as the PQ node.  $Q$  is reactive power and  $\Phi$  is the angle of power factor, which is generally in the fourth quadrant.

$$Q = P \tan \Phi \quad (4)$$

### 2.2. The probabilistic model of photovoltaic power

The light intensity  $r$  exhibits stochastic characteristics in a certain period of time, which can be approximated as the Beta distribution. Its probability density function is shown as follows [8].

$$f(r) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{r}{r_{\max}}\right)^{\alpha-1} \cdot \left(1 - \frac{r}{r_{\max}}\right)^{\beta-1} \quad (5)$$

Where  $\alpha$  and  $\beta$  are the shape parameter of Beta distribution,  $r$  and  $r_{\max}$  are respectively the real and maximum light intensity,  $W/m^2$ .

$$P_M = r \cdot A \cdot \eta \quad (6)$$

The probability density function of the output power  $P_M$  can be obtained by (5). Its output power of photovoltaic system is  $P_M$ , while  $A$  is total solar cell array and  $\eta$  is photoelectric conversion efficiency [8].

$$f(P_M) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{P_M}{P_{M \max}}\right)^{\alpha-1} \cdot \left(1 - \frac{P_M}{P_{M \max}}\right)^{\beta-1} \quad (7)$$

Photovoltaic generation generally only provides active power to the grid, in order to simplify the process, so the PV power is set as the PQ node with zero reactive power.

### 2.3. Load model

The normal distribution can be used to approximately reflect the uncertainty of the load power. Suppose the expected value and variance of active and reactive power respectively as  $\mu_p, \sigma_p^2, \mu_Q, \sigma_Q^2$ . The probability density function of the active and reactive power are as follows [9].

$$f(P) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{(P - \mu_p)^2}{2\sigma_p^2}\right) \quad (8)$$

$$f(Q) = \frac{1}{\sqrt{2\pi}\sigma_Q} \exp\left(-\frac{(Q - \mu_Q)^2}{2\sigma_Q^2}\right) \quad (9)$$

### 3. Risk assessment model

#### 3.1. Evaluation methods

According to the different ideas and modeling methods, the existing stochastic power flow algorithm is mainly divided into analytical and simulation method. The analytical method of stochastic power flow calculation is based on the linear equation of the AC power flow. Assuming that the network structure is constant and the distribution of the injected power is independent of each other, the distribution function of the injected power is obtained by convolution technique. The output random variables (node voltage and branch power) have linear relation with the input random variables (the injected power). AC power flow equation is generally expressed as (10).

$$S = f(X) \quad (10)$$

According to Taylor series expansion, high order terms are ignored.  $X_0$  is calculated by Newton Raphson flow calculation, and  $J_0$  is the Jacobi matrix of the last iteration [10].

$$\begin{aligned} S_0 + \Delta S &= f(X_0 + \Delta X) = f(X_0) + J_0 \Delta X + \dots \\ S_0 &= f(X_0) \\ \Delta X &= J_0^{-1} \Delta S \end{aligned} \quad (11)$$

The Cornish-Fisher series expansions and cumulant are used to calculate the probability distributions. This method transforms complex convolution operations into arithmetic operations, which greatly improves computational speed. Suppose  $f(x)$  is the probability density function of the continuous random variable. Its center distance expression  $M_v$  and the relationship between cumulant  $K$  and  $M_v$  are as follows.  $K_r$  is the  $r$ -order cumulant of the random variable  $x$  [11].

$$\begin{cases} M_v = \int_{-\infty}^{+\infty} (x - E(X))^v f(x) dx \\ K_1 = M_1 \\ K_{r+1} = M_{r+1} - \sum_{j=1}^r C_r^j M_j K_{r-j+1} \end{cases} \quad (12)$$

Suppose there are  $n$  independent random variables  $x_1, x_2, \dots, x_n$ , the random variable  $y$  is its linear combination. Then the  $r$ -order semi-invariant of  $y$  is as follows [11].

$$y = a_1 x_1 + a_2 x_2 + \dots + a_n x_n \quad (13)$$

$$K_y^{(r)} = a_1^r K_{x_1}^{(r)} + a_2^r K_{x_2}^{(r)} + \dots + a_n^r K_{x_n}^{(r)} \quad (14)$$

$K_{x_i}^{(r)}$  is the  $r$ -order cumulant of the random variable  $x_i$ .

The mean and variance of a continuous random variable  $x$  respectively are  $\mu$  and  $\sigma$ .  $\varepsilon = (x - \mu)/\sigma$ . Its probability density function is as follows according to Cornish-Fisher series expansion [12].

$$\begin{aligned} f(\varepsilon) &= \varphi(\varepsilon) + \frac{1}{6} [\varphi^2(\varepsilon) - 1] K_3 + \frac{1}{24} [\varphi^3(\varepsilon) - 3\varphi(\varepsilon)] K_4 \\ &\quad - \frac{1}{36} [2\varphi^3(\varepsilon) - 5\varphi(\varepsilon)] K_3^2 + \frac{1}{120} [\varphi^4(\varepsilon) - 6\varphi^2(\varepsilon) + 3] K_5 + \dots \end{aligned} \quad (15)$$

Where  $\varphi(\varepsilon)$  is probability density function of standard normal distribution, and  $K_r$  is the  $r$ -order cumulant of the random variable  $\varepsilon$ . The probability distribution function is given by integration [13].

$$F(\varepsilon) = \int_{-\infty}^{\varepsilon} f(\varepsilon) d\varepsilon \quad (16)$$

The specific steps in the risk assessment are as follows.

1) According to the probability distribution of node injection power  $W$  (distributed power output and load), the first order cumulant of  $W$  are calculated.

2) According to the formula (11), (12), (13) and (14), the first order cumulant of the state variable node voltage are calculated.

3) The probability distribution of node voltage can be obtained by the formula (15) and (16).

### 3.2. Risk assessment index

The static security of distribution network includes the overload of branch power and the over-limit of node voltage. This paper only studies the voltage over-limit risk. Based on the basic definition of power system risk assessment, for the uncertainty of the power system, a comprehensive measure of the possibility and severity is given. The basic formula of risk indicator is shown in (17) [13].

$$Risk = P_r \times Sev \quad (17)$$

Stochastic power flow calculation is used to get the probabilistic density function and the probabilistic distribution function of the node voltage. According to the cumulative distribution function, the node voltage over-limit probability is obtained.

$$P_r(V_i) = \max\{P_r(V_i > V_{i\max}), P_r(V_i < V_{i\min})\} \quad (18)$$

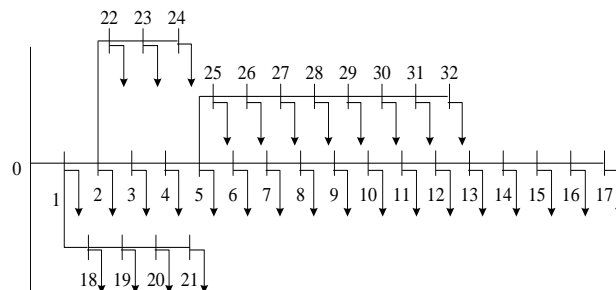
Where  $V_i$  represents the voltage amplitude of node  $i$ ,  $V_{\max}$  and  $V_{\min}$  are the upper and lower bounds of the allowable voltage amplitude for the node  $i$ . It assumes in this paper that the upper and lower limits are 1.05 and 0.95 respectively.

Severity indicators are used to account for small probability events that have a severe impact on the system, which gives the most severe state of the node voltage at the current grid operating conditions. The expression is as follows.

$$Sev(V_i) = \max\left\{\left|\frac{V_i - V_{i\max}}{V_{i\max}}\right|, \left|\frac{V_i - V_{i\min}}{V_{i\min}}\right|\right\} \quad (19)$$

## 4. Case study

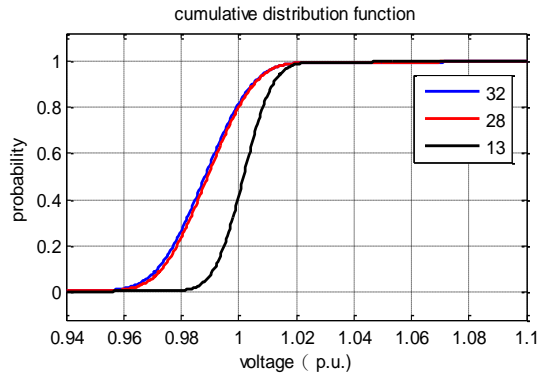
In this paper, the system static security voltage risk indicators of IEEE33 node system are calculated by the use of MATLAB programming simulation. The IEEE 33 nodes distribution system structure is shown in Fig.1. The risk index of different distributed power access system is calculated. Assuming that the node load obeys the normal distribution, the expected value is the system original load value, and the standard deviation is taken the expected value 10%. The cut in, rated and cut out wind speed of the wind power generator are 3, 13.5 and 20m/s respectively. The rated power of the wind power generator is 100 kW. The two parameters of Weibull distribution  $c$  and  $k$  are 10.5958 and 8.3664. The maximum irradiance intensity, total cell area and photoelectric conversion efficiency  $\eta$  of PV system are respectively  $1000W/m^2$ ,  $2m^2$  and 14%. The shape parameter of Beta distribution  $\alpha$  and  $\beta$  are 0.95.



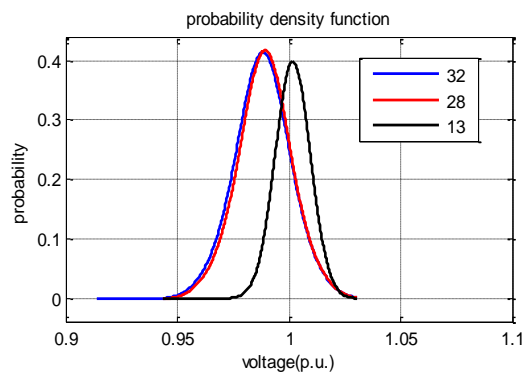
**Figure 1.** IEEE33 node distribution system

When the wind power generator with 2 rated power of 100KW is added to the 32 node, the wind generator access point node 32, the node 28 and the node 13 away from the wind power generator are selected as examples. The voltage cumulative distribution function and the probability density

function of different nodes are shown in Fig.2 and Fig.3. The voltage over-limit probability, severity index and risk index of different nodes are calculated in the Table 1.



**Figure 2.** cumulative distribution function curve

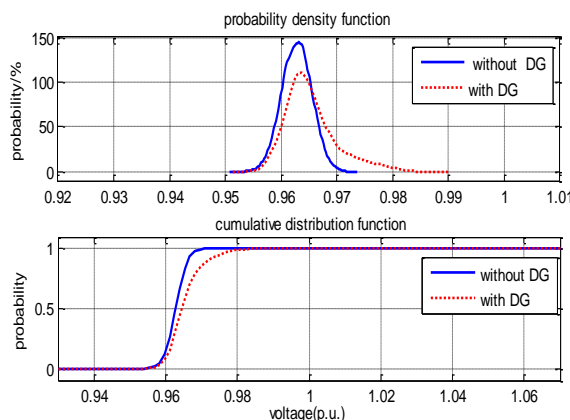


**Figure 3.** probability density function curve

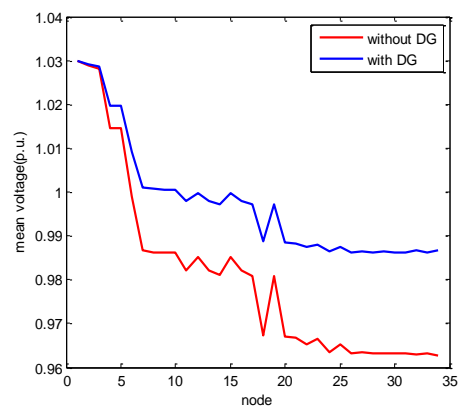
**Table 1.** Partial node risk results

Node number	32	28	13
Voltage over-limit probability	0.001607	0.001221	0
Severity index	0.03768	0.007158	0
Risk index	6.0552e-5	8.7399e-6	0

From Fig.2, when a certain amount of wind generator is added to the distribution system, the voltage of different nodes is different. The wind turbine access point node 32 is affected by the output power of the wind turbine. The voltage fluctuation range of node32 is the most extensive, and it deviates from the expected value of the voltage, which has a large risk indicator. The node 28 near the wind turbine node is secondarily affected by wind power fluctuation, and its fluctuation range becomes slightly smaller. While the node 13 far away from the node 32 is very little affected by the fluctuation of the wind power output, so the voltage fluctuation is small and the risk of the voltage over limit is zero.



**Figure 4.** The voltage function curve



**Figure 5.** The mean voltage of each node with and without DG

When the wind power generator with 2 rated power of 100KW is added to the 32 node, it is the cumulative distribution function curve and probability density function curve of node 31 with and without wind power generator in Fig.4. The mean voltage of each node with and without wind power generator in the system is shown in Fig.5. With distributed generation the voltage fluctuation range becomes wider.

From Fig.5, the access of the distributed power supply can improve voltage level of each node, and improve the quality of power system, but also bring the risk of over limit to the node voltage. When there is no distributed generation in the system, the probability of the voltage over limit is almost zero.

After adding the distributed generation, the voltage of node increases, and the probability of voltage over-limit increases.

## 5. Conclusions

The node voltage will be affected by the fluctuant output power of distributed power in distribution network. A stochastic model of distributed generation is established in this paper. Based on stochastic power flow algorithm, the influence of the randomness of the output power flow on the static voltage of the power network is studied. Taking IEEE-33 node system as an example, the risk assessment of distribution network with distributed generation is carried out scientifically in this method. Different distributed power access schemes have different effects on the static safety voltage risk of the distribution network.

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