Interval Reliability Assessment of Power System under Epistemic Uncertainty Based on Belief UGF Method

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Abstract. At present, reliability assessment plays an important role in power systems. When the component failure probabilities are interval valued, common methods fail to achieve reasonable interval reliability assessment result of power systems. In this paper a novel approach based on the belief universal generating function (BUGF) is proposed to calculate the reliability indexes of power systems. Instead of giving a single-valued assessment result, a belief function and a plausibility function are exploited to calculate the lower and upper bounds of the loss of load probability (LOLP), the loss of load expectation (LOLE), the expected unsupplied load (EUL) and the expected unsupplied energy (EUE) in UGF, respectively. The proposed approach can track the correlation of the original data well and keep it to the end of the calculation. By using BUGF compared to other common methods to calculate the interval LOLP, LOLE, EUL, and EUE of IEEE-RTS 79, the results show the BUGF method can track the correlation of the original data well, and can get narrower and more accurate interval reliability indexes of the power generation system, which illustrates the effectiveness of the proposed approach.

1.Introduction

Common power system reliability assessment [1] is based on the theory of probabilities and statistics, to calculate the exact probability or probability distribution of the event of the power system. However, there are other uncertainties in the power systems, such as uncertainties caused by the lack of statistical data or statistical errors, and the limited cognitive ability of the decision maker [2]. Moreover, with the new energy accessing to the grid and the new technology widely used in the power system [3], quantities of new elements are lack of or have no failure statistical data Therefore，the common method of point estimate and interval estimation based on large number law is no longer applicable, and accurate component reliability indexes are difficult to obtain, the interval probability for component reliability modelling is more reasonable [4]. According to the theory of interval probability [5], the probability of occurrence of an event is no longer a precise point probability, but an imprecise interval probability. After obtaining the component -level reliability index based on interval probability [4], the next problem is how to evaluate the system reliability on the basis of the original data, and its essence is how to deal with the power system uncertain problems.

At present, the methods of dealing with power system under epistemic uncertainty are fuzzy set theory [6] and interval analysis theory [7-9]. The fuzzy set theory needs to know the membership function of uncertain parameters, but these functions are difficult to determine in practice. The interval analysis theory applied to reliability assessment of power system can reasonably transform reliability index into interval number. The results contain the risk factors, so as to provide decision makers with more scientific assessment data.

In this paper, the reliability assessment of power systems based on interval probability and interval number operation is studied. The interval number in this paper is not normal interval parameters but interval probabilities. Therefore, the unique properties of interval probabilities need to be considered in computations. Moreover, complex state enumeration method is no longer used in the interval for reliability index of power system. Instead, belief universal generating function (BUGF) method combined of belief function theory [9] and universal generating function method (UGF) [10] is adopted. The BUGF method [11] can track the correlation of the original data well and keep it to the end of the calculation, and can get narrower and more accurate interval reliability indexes of the power generation system.

The remainder of this paper is organized as follows. In Section 2, we review interval probability, the UGF and the reliabilities index of power system. We present an extension of UGFs using belief function theory in Section 3. In Section 4, we calculate the power system reliability index by using BUGF method. A case study of the proposed method is presented to illustrate how to calculate the power system reliability index by using BUGF method and its advantages in Section 5. Section 6 concludes this paper.

2. Basic theory

2.1. UGF method
In UGF method, the state probability of each component is represented by u-transform as follows:

$$u_j = \sum_{i_j=1}^{k_j} p_{ji_j} z^{g_{ji_j}} \tag{1}$$

Accordingly, the system probability distribution can be represented as follows:

$$u = \sum_{j=1}^{k} p_j z^{g_j} \tag{2}$$

The mapping between the component state probability and the system probability is realized by specially defined operator Ω_ϕ :

$$\Omega_\phi(u_1, u_2, ..., u_n) = \Omega_\phi(\sum_{i_1=1}^{k_1} p_{1i_1} z^{g_{1i_1}}, \sum_{i_2=1}^{k_2} p_{2i_2} z^{g_{2i_2}}, ..., \sum_{i=1}^{k} p_{ni_n} z^{g_{ni_n}}) = \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} ... \sum_{i_n=1}^{k_n} ((\prod_{j=1}^{n} p_{ji_j}) z^{\phi(g_{1i_1}, g_{2i_2}, ..., g_{ni_n})} \tag{3}$$

For a general series parallel system, if only the connectivity is considered, we can define series operator ϕ_s and parallel operator ϕ_p , which are shown as follows:

$$\begin{cases} \phi_s(g_1, g_2, ..., g_n) = \min(g_1, g_2, ..., g_n) \\ \phi_p(g_1, g_2, ..., g_n) = \max(g_1, g_2, ..., g_n) \end{cases} \tag{4}$$

If only the transfer capability is considered, the series operator   and parallel operator   can be defined as :

$$\begin{cases} \phi_s(g_1, g_2, ..., g_n) = \min(g_1, g_2, ..., g_n) \\ \phi_p(g_1, g_2, ..., g_n) = \sum_{i=1}^{n} g_i \end{cases} \tag{5}$$

2.2. Belief function theory

There is only one true value of the variable of the frame of discernment D , which is denoted as X, the element in D. That we are able to make sure the *X* is fully certain. Otherwise, there is no information but *X* in D, that is completely uncertain. In reality, the common scene is that we can only know a part of the information about the real value of the variable. We can build a model by defining $m(X) : 2^D \rightarrow [0,1]$,which satisfies two conditions:

$$\begin{cases} m_X(\varnothing) = 0 \\ \sum_{A \subseteq D} m_X(A) = 1 \end{cases} \tag{6}$$

We can get the belief function and plausibility function when *m(X)* is given.

$$Bel(X) = \sum_{\substack{Y \subseteq X \\ X \subseteq D}} m(Y) \tag{7}$$

Belief function expresses the degree of support for propositional X, and provides the lower bound of the degree of trust.

$$Pl(X) = \sum_{Y \cap X \neq 0} m(Y) \tag{8}$$

Plausibility function expresses the possible support for propositional X, and provides the upper bound of the degree of support. Then, the pair $[Bel(X), Pl(X)]$ of functions composes an integrated uncertainty interval of event , which can be used to represent the interval state probability of focal element[11].

2.3. The reliability indexes of power systems

We define the available capacity of the whole power generation as random variable *G* and the load random variable *L*.

The definition of load loss function(LLF) is as follows:

$$f_{LLF}(G, L) = 1(G{<}L) = \begin{cases} 1, G{<}L; \\ 0, G \geq L \end{cases} \tag{9}$$

The definition of unsupplied load function(ULF) is as follows:

$$f_{ULF}(G, L) = \max(L - G, 0) \tag{10}$$

The definition of reliability index of power system is as follows:

a. the loss of load probability is probability of available capacity of power generation dissatisfying load demand during the study period, which is as follows:

$$LOLP = E(f_{LLP}) \tag{11}$$

b. the loss of load expectation(LOLE) is the expectation time of system capacity being less than the value of daily maximum load, which is as follows

$$LOLE = T \cdot LOLP \tag{12}$$

c. the expected unsupplied load(EUL) is the expectation of load without power supply due to the outage of power generation equipment during the study period, which is as follows:

$$EUL = E(f_{FUL}) \tag{13}$$

d. the expected unsupplied energy (EUE) is the lack of electricity due to power failure of load caused by the outage of power generation equipment during the study period, which is as follows:

$$EUE = EUL \, LOLE \tag{14}$$

where T is running time and $E(\cdot)$ is the expectation value.

3. Belief Universal Generating Function Model

3.1. Interval UGF model

Supposing the power generation system is composed of n generators, and 2-state model (the subscript 0 and 1 is represented for outage and running state) is used. For generator unit i, its probable value of power generation capacity G_i is $\{g_{i0}, g_{i1}\}$, where g_{i1} is rated capacity g_i of the generator, g_{i0} is 0, and their interval state probability is $p_i = \{[p_{i0}], [p_{i1}]\}$. We assume that the probable value of load L is $\{l_1, l_2, ..., l_m\}$, and their interval state probability is $\{[q_1], [q_2], ..., [q_m]\}$. $\{[q_1], [q_2], ..., [q_m]\}$ can be represented as $\{[\overline{q}_1, \underline{q}_1], [\overline{q}_2, \underline{q}_2], ..., [\overline{q}_m, \underline{q}_m]\}$.

3.2. BUGF model

In the BUGF model, the mass function should be obtained first; we can use the following method to obtain the mass function from probability intervals.

$$m(S) = \begin{cases} \underline{p}_{i,0}, S = g_{i,0} \\ \underline{p}_{i,1}, S = g_{i,1} \\ 1 - \underline{p}_{i,0} - \underline{p}_{i,1}, S = \{g_{i,0}, g_{i,1}\} \\ 0, else \end{cases} \tag{15}$$

where S represents the set which concludes all the system state of the element i of the power system. Assuming a power system containing n power generator units, when the precise value of power generation capacity is set and expressed as $[g]_i, (1 \le i \le n, [g]_i \subseteq \{g_i\})$, and then the relationship of the power system before and after the mapping is obtained and can be expressed as:

$$\phi([g]_1, [g]_2, ..., [g]_n) = \{\phi(g_1, g_2, ..., g_n) | g_i \in [g]_i\} \tag{16}$$

We can define m_{ij} to be the mass of component i with state j of the power system, and then we transform the state and probability of component i into the z-function, that is the Belief Universal Generating Function Model, which can be expressed as:

$$U_i^B(z) = \sum_{j=1}^{n} m_{i,j} z^{[g]_{i,j}} \tag{17}$$

We define operator Ω_ϕ to represent mapping between the component state probability and the system probability with n components, which can be expressed as:

$$U_s^B(z) = \Omega_\phi(U_1^B(z), ..., U_n^B(z)) = \sum_{j_1=1}^{2} \cdots \sum_{j_n=1}^{2} \prod_{i=1}^{n} m_{i,j_i} z^{\phi([g]_{1,j_1}, ..., [g]_{n,j_n})} \tag{18}$$

From the discussion above, the bounds of interval value can be obtained by the belief function and the plausibility function. Assuming that the demand of the power system is w, the two following operators are defined to obtain the plausibility and belief values:

$$1_w^+(z^\phi) = \begin{cases} 1, \phi([g]_1, ..., [g]_n) \cap w \ne 0 \\ 0, else \end{cases} \tag{19}$$

$$1_w^-(z^\phi) = \begin{cases} 1, \phi([g]_1, ..., [g]_n) \subseteq w \\ 0, else \end{cases} \tag{20}$$

where $\phi([g]_1,...,[g]_n) \cap w \neq 0$ used in(19) means that at least one element of the interval-valued performance ϕ is above or equal to demand w and $\phi([g]_1,...,[g]_n) \subseteq w$ used in(20)means that that all elements in ϕ are above or equal to w. According to the two operators 1_w^+ and 1_w^- above ,when the demand of the power system w is known, the belief function and the plausibility function can be calculated by(19) and(20) to represent the lower bound and the upper bound of the interval probabilities[11].

$$Pl(w) = 1_w^+(U(z)) = 1_w^+(\sum_{j_1=1}^{2} \cdots \sum_{j_n=1}^{2} \prod_{i=1}^{n} m_{ij_i} z^\phi) = \sum_{j_1=1}^{2} \cdots \sum_{j_n=1}^{2} \prod_{i=1}^{n} m_{ij_i} 1_w^+ \qquad (21)$$

$$Bel(w) = 1_w^-(U(z)) = 1_w^-(\sum_{j_1=1}^{2} \cdots \sum_{j_n=1}^{2} \prod_{i=1}^{n} m_{ij_i} z^\phi) = \sum_{j_1=1}^{2} \cdots \sum_{j_n=1}^{2} \prod_{i=1}^{n} m_{ij_i} 1_w^- \qquad (22)$$

4. Calculating Power System Reliability Indexes Using BUGF
We regard the available capacity of power generation of the generator i as discrete random variable, and its BUGF is

$$U_i^B(z) = \underline{p}_{i,1} z^{h_i} + \underline{p}_{i,0} z^0 + (1 - \underline{p}_{i,1} - \underline{p}_{i,0}) z^{\{0,h_i\}} \qquad (23)$$

where h_i is the available capacity of power generation of the generator i, and $\underline{p}_{i,1}$ is the lower bound of the probability interval value of the running of generator i, $\underline{p}_{i,0}$ is the lower bound of the probability interval value of the outage of generator i.

The available capacity of power generation containing n generators of power system is the sum of the available capacity of power generation of all generator units, and its BUGF is

$$U_G^B(z) = \Omega_\phi(U_1^B(z),...,U_n^B(z)) = \sum_{s_1=1}^{2} \cdots \sum_{s_n=1}^{2} \prod_{i=1}^{n} m_{i,s_i} z^{\phi([h]_{1,s_1},...,[h]_{n,s_n})} \qquad (24)$$

where $[h]_{i,s_i}$ is the state performance of generator i and m_{i,s_i} is its mass function.

To calculate the power system reliability index LOLP according to (13)and(15), the belief function and the plausibility function are

$$Pl(LOLP) = \sum_{j=1}^{n} p_{l_j} 1_{l_j}^+(U_G^B(z)) \qquad (25)$$

$$Bel(LOLP) = \sum_{j=1}^{n} p_{l_j} 1_{l_j}^-(U_G^B(z)) \qquad (26)$$

where l_j is the state of the whole power system load and p_{l_j} is its probability. Therefore, the LOLP interval value of the power system is$[Bel(LOLP), Pl(LOLP)]$.

To calculate the power system reliability index EUL according to (10)and(13), the belief function and the plausibility function are

$$Pl(EUL) = \sum_{s=1}^{N} \sum_{j=1}^{n} p_{l_j} 1_{l_j}^+(U_G^B(z)) f_{ULF}(h_s, l_j) \qquad (27)$$

$$Bel(EUL) = \sum_{s=1}^{N} \sum_{j=1}^{n} p_{l_j} 1_{l_j}^-(U_G^B(z)) f_{ULF}(h_s, l_j) \qquad (28)$$

where n is the number of state of load, l_j is the state of the whole power system load and p_{l_j} is its probability, M is the number of state of available capacity of power generating system, h_s is the state of

power generating system. Therefore, the EUL interval value of the power system is $[Bel(EUL), Pl(EUL)]$.

As the same as above, LOLE and EUE can be calculated by (12) and (14).

5.Cases

5.1.Texting system

We use the raw data of IEEE-RTS 79; there are 32 generator units of 79 nodes. We can assume that the failure probability of generator units numbered 30,31,32 is imprecise while state probability of other generator units keep unchanged. Of the uncertain generator units numbered 30,31 and 32, we assume that the failure probability changes in the range of ±5%. The failure probability interval and power ratings of uncertain generator units numbered 30, 31 and 32 in the power generation system are shown in Table 1.

Table 1. Failure probability and power ratings of uncertain generator units.

Number	Power ratings/MW	Failure probability (±5%)
30	350	[0.0340,0.1260]
31	400	[0.0760,0.1640]
32	400	[0.0760.0.1640]

After statistical analysis of the original data of load, we can get the load and its state probability, which is shown in Table 2.

Table 2. Load and its state probability

Load/MW	State probability	Load/MW	State probability
1566	0.0823	2240	0.1126
1698	0.0578	2371	0.1511
1846	0.0825	2503	0.1044
1963	0.2114	2647	0.0384
2097	0.1484	2786	0.0111

5.2.Results and analysis

According to the failure probability interval of uncertain generator units numbered 30, 31 and 32 in the power generation system which is discussed above, we use the BUGF method to calculate interval reliability indexes of power systems, and compare the results with the interval UGF (IUGF) method [6]. The results are shown in Table 3.

Table 3. Interval Reliability of power system containing 3 uncertain generator units

Reliability Index	BUGF Method	IUGF Method
LOLP	[0.00126,0.00704]	[0.00125,0.00781]
LOLE(d•a^{-1})	[0.49136,2.47106]	[0.44191,2.73372]
EUL(MW•a^{-1})	[0.17622,1.17010]	[0.14767,1.06859]
EUE(MW•h•a^{-1})	[1.80256,60.3510]	[1.55654,72.3112]

As is seen in Table 3, compared to the IUGF method obtained by Li Chunyang & Chen Xun[6], the upper and lower bounds of the interval reliability index of power system which we use the BUGF method to obtain narrows evidently than the interval from the IUGF method. What's more, the computational procedure of the BUGF method is much easier, and the calculation quantity and speed are mainly determined by the quantity of focal elements, which leads the computation quantity and speed much more acceptable.

6. Conclusions

In this paper, a new method which is based on universal generating function and belief function theory is proposed to analyse interval-valued reliability of power systems. When the available data of components are insufficient, we cannot obtain the precise probability of the component of the power systems, therefore, the non-probabilistic method is used to present the component state probabilities. Because of its advantages in dealing with problems under epistemic uncertainties, the belief function theory is introduced into the universal function method to obtain the interval valued probability of power system. The results which are calculated in the cases show that the upper and lower bounds of the power system reliability interval which is obtained by the BUGF method is narrower than the previous IUGF method and the calculation of BUGF method is much easier. However, there are still great difficulties in obtaining the state probabilities and the reliability parameters of the electrical component directly considering epistemic uncertainty, which should be solved in the future work.

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