

# Simulation Study of Three - phase PWM Rectifier with Square of the Voltage Double Closed Loop Control

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**Abstract.** Based on the rotating coordinate system model, the mathematic analysis of the three-phase voltage-sourced PWM rectifier is carried out, and its feed-forward decoupling control strategy is studied. In order to realize the fast response of DC voltage and improve the dynamic and static performance of the rectifier, the double-closed-loop control of outer square of voltage loop and inner current loop is established. The control system of three-phase voltage-sourced PWM rectifier based on space vector modulation algorithm is designed. The simulation results demonstrate the designed model is accurate, and can achieve unity power factor control, DC voltage fast responses, and has not steady-state error.

## 1. Introduction

Three-phase voltage-sourced PWM rectifiers are widely used and researched due to numerous advantages, which can achieve small AC current distortion, controllable unit power factor, a constant output DC voltage, energy two-way flow and so on[1-3]. At present, the commonly used control strategies are mainly divided into voltage and current double closed-loop control, direct power control and the new control strategies based on control theory. The literature [4] proposed a direct voltage control algorithm, the voltage error is a direct input control variable to realize the fast response of the voltage under the large disturbance of load, but the good control effect resulted from a more complicated algorithm. In the literature [5], the control strategy was put forward, which included slider mode control adopted in the outer voltage loop, the hybrid mode of decoupling current control without exact value of the boost inductor and internal model control was adopted in the current inner loop, and can achieve a fast dynamic response, good accuracy and stable output DC voltage in the case of inductance parameter variation. But the sliding mode control was not strongly sensitive to the input error of the model and the external destabilization. In the literature [6], the voltage outer sliding mode control was used. The inner loop combined the input and output linearization with space vector modulation, Which could transform the nonlinear control into quasi-linear control, realized the unit power factor control in harsh environment, and enhanced the robustness of the system. The literature [7] united the non-linear PI controller and decoupling current control without inductance L parameter, the new control strategy had a better control accuracy.

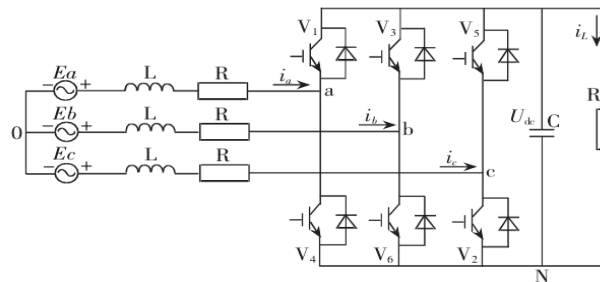
Based on the research of domestic and foreign scholars, this paper analyzes the mathematical model of three-phase voltage-sourced PWM rectifier in static and dynamic coordinate system, and



puts forward the double closed-loop control strategy of outer square of voltage, and inner current combines with space vector control to improve voltage utilization. Finally the simulation model is established and the accuracy and feasibility of the model are verified.

## 2. PWM rectifier mathematical model

General topology of the three-phase voltage-sourced PWM rectifier (VSR) is shown in figure 1.



**Figure 1.** Topology of the three-phase voltage-sourced PWM rectifier

Ignoring the linear unsaturated and asymmetric of inductance, and assuming the three-phase voltage is ideal sine wave, we can get the mathematical model of three-phase voltage-sourced PWM rectifier in the three-phase stationary coordinates.

$$\begin{aligned}
 L \frac{di_a}{dt} + Ri_a &= U_a - (S_a U_{dc} + U_{N0}) \\
 L \frac{di_b}{dt} + Ri_b &= U_b - (S_b U_{dc} + U_{N0}) \\
 L \frac{di_c}{dt} + Ri_c &= U_c - (S_c U_{dc} + U_{N0}) \\
 C \frac{dU_{dc}}{dt} + \frac{U_{dc}}{R} &= i_a S_a + i_b S_b + i_c S_c \\
 U_{N0} &= -\frac{U_{dc}(S_a + S_b + S_c)}{3}
 \end{aligned} \tag{1}$$

Where  $S_a$ ,  $S_b$  and  $S_c$  = switching state of the converter, it can be seen from the above formula, PWM rectifier has multi-input multi-output nonlinear coupling characteristics, we can reduce the amount of input-output and decrease the coupling between the system state quantity based on the equal power Park transformation, transforming the three-phase sine variable into two DC variable to achieve accurate tracking of current. Therefore, two-phase d-q rotation coordinate system Mathematical model is as follow:

$$\begin{aligned}
 L \frac{di_d}{dt} &= U_d - Ri_d + \omega Li_q - S_d U_{dc} \\
 L \frac{di_q}{dt} &= U_q - Ri_q - \omega Li_d - S_q U_{dc} \\
 C \frac{dU_{dc}}{dt} &= -\frac{U_{dc}}{R} + \frac{3}{2}(i_d S_d + i_q S_q)
 \end{aligned} \tag{2}$$

Where,  $i_d$  = the current active component,  $i_q$  = the current reactive component.

## 3. Double closed loop control strategy

### 3.1. Current loop control design

As can be seen from equation (2), the axial current component  $i_d$ ,  $i_q$  are not only affected by the control variable  $S_d U_{dc}$ ,  $S_q U_{dc}$ , but also the disturbance of cross-coupling voltage  $\omega Li_q$ ,  $\omega Li_d$ , and AC

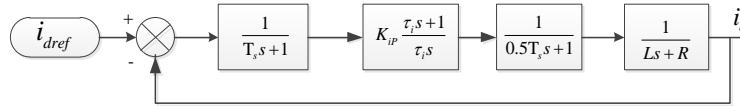
side voltage  $U_d, U_q$ . Therefore, it is necessary to introduce the feed-forward decoupling control so that current  $i_d, i_q$  can be directly controlled by the corresponding  $U_d, U_q$  to improve the current control performance. When the current loop adopts PI control, the steady-state static performance of the system can be realized. There is

$$\begin{aligned} U_d &= -(K_{ip} + \frac{K_{il}}{s})(i_{dref} - i_d) + \omega L i_q + S_d U_{dc} \\ U_q &= -(K_{ip} + \frac{K_{il}}{s})(i_{qref} - i_q) - \omega L i_d + S_q U_{dc} \end{aligned} \quad (3)$$

Bring into equation (2) we can get:

$$\begin{aligned} L \frac{di_d}{dt} &= (K_{ip} + \frac{K_{il}}{s})i_{dref} - [R + (K_{ip} + \frac{K_{il}}{s})]i_d \\ L \frac{di_q}{dt} &= (K_{ip} + \frac{K_{il}}{s})i_{qref} - [R + (K_{ip} + \frac{K_{il}}{s})]i_q \end{aligned} \quad (4)$$

Thus, the axial current component can be achieved independent control based on above equations, making  $i_{qref}=0$  to achieve the unit power factor operation. The current loop achieved feed-forward decoupling changes into closed-loop system with a PI control. Combined with the actual engineering design, the inner current loop can be simplified as shown in figure 2.



**Figure 2.** The current loop diagram

$T_s$  is current sampling period (PWM switching period),  $(T_s s + 1)^{-1}$  is the current sampling delay,  $(0.5 T_s s + 1)^{-1}$  is the PWM wave output delay, in order to improve the inner loop current performance, the current regulation can be designed according to Typical I system. Making  $\tau_i = L \cdot R^{-1}$ , we can get the corrected current loop transfer function:

$$W_{ci}(s) = \frac{1}{1 + \frac{R\tau_i}{K_{ip}}s + \frac{1.5T_s R\tau_i}{K_{ip}}s^2} \quad (5)$$

According to the design relationship of Typical I system parameters, in order to obtain a good system regulation performance, making the system damping ratio  $\zeta = 0.707$ , and then

$$\frac{1.5T_s K_{ip}}{R\tau_i} = \frac{1}{2} \quad (6)$$

We can get

$$K_{ip} = \frac{R\tau_i}{3T_s}, \quad K_{il} = \frac{R}{3T_s} \quad (7)$$

If the switching frequency is high enough, the  $s^2$  can be ignored, then  $W_{ci}(s)$  is simplified as follow:

$$W_{ci}(s) \approx \frac{1}{1 + 3T_s s} \quad (8)$$

Equation (8) shows that the corrected current loop can equal to an inertia element, as long as the switching frequency was high enough, the current loop can have a faster response.

### 3.2. Outer voltage loop control design

Similarly according to the equation (2), the DC voltage  $U_{dc}$  is also affected by the interaction of current  $i_d$  and  $i_q$ , when adopting linear PI control, using  $i_d$  to dynamically track DC voltage cannot gain a better control performance. At the same time, if the PWM rectifier connected the load (such as

flywheel energy storage, high-energy weapons, etc.), the DC voltage immunity will be severely tested. Active power and reactive power in the rotating coordinate d-q system can be expressed as:

$$\begin{aligned} P &= \frac{3}{2}(U_d i_d + U_q i_q) \\ Q &= \frac{3}{2}(U_q i_d - U_d i_q) \end{aligned} \quad (9)$$

Making the d-axis for the exchange side of the voltage synthesis vector direction, there is  $U_q = 0$ , then

$$\begin{aligned} P &= \frac{3}{2} U_d i_d \\ Q &= -\frac{3}{2} U_d i_q \end{aligned} \quad (10)$$

The expression about resistance load and capacitor instantaneous active power of DC voltage is

$$P_0 = \frac{3}{2} U_d i_d = C U_{dc} \frac{dU_{dc}}{dt} + U_{dc} \frac{U_{dc}}{R_{L_1}} \quad (11)$$

If the DC voltage  $U_{dc}^*$  was given, and the control system is linearized at operating point, making

$$\begin{cases} U_{dc} = U_{dc}^* + \Delta U_{dc} \\ P_0 = P_0^* + \Delta P_0 \end{cases} \quad (12)$$

Bring equation (12) into equation (11), we can get

$$\frac{P_0}{U_{dc}^*} + \frac{\Delta P_0}{U_{dc}^*} = C(1 + \frac{\Delta U_{dc}}{U_{dc}^*}) \frac{d\Delta U_{dc}}{dt} + (U_{dc}^* + 2\Delta U_{dc} + \frac{\Delta U_{dc}^2}{U_{dc}^*}) / R_L \quad (13)$$

When the system is dynamic,  $\Delta U_{dc} \ll U_{dc}^*$ , then the equation (13) instantaneous power can be approximated as

$$\frac{\Delta P_0}{U_{dc}^*} \approx C \frac{d\Delta U_{dc}}{dt} + \frac{2\Delta U_{dc}}{R_L} \quad (14)$$

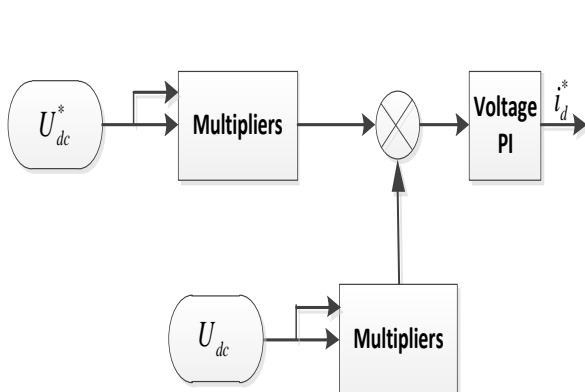
Further approximation can be obtained

$$\frac{\Delta P_0}{U_{dc}^*} = \frac{\frac{3}{2} U_q \Delta i_d}{U_{dc}^*} \approx \frac{2 \Delta U_{dc}}{R_L} \quad (15)$$

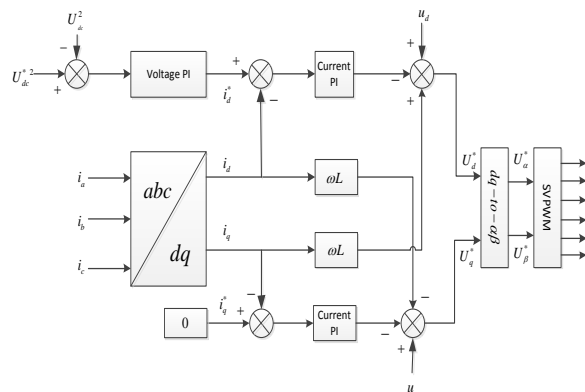
According to equation (15),  $U_{dc}^* \cdot \Delta U$  and  $\Delta i_d$  is linear, when system is steady state, there is

$$R_L C \frac{dU_{dc}^2}{dt} + 2U_{dc}^2 = 3R_L U_d i_d \quad (16)$$

Where  $\frac{dU_{dc}^2}{dt} = 0$ , leading that the  $U_{dc}^2$  and  $i_d$  becomes linear, so we can consider the square of the DC voltage as the input control variable, to achieve rapid tracking of DC side voltage, its design is shown in figure 3.



**Figure 3.** Voltage outer loop control block diagram



**Figure 4.** The overall control strategy for VSR rectifier block diagram

Taking into account to improve the voltage utilization, it needs to design a reasonable SVPWM modulation algorithm to generate the corresponding 6-way pulse to achieve the PWM rectifier bridge thyristor on or off. Here we directly reference [7-10] design method. Thus, the overall controls strategy for the three-phase voltage-sourced PWM rectifier is shown in figure 4.

#### 4. Simulation analysis

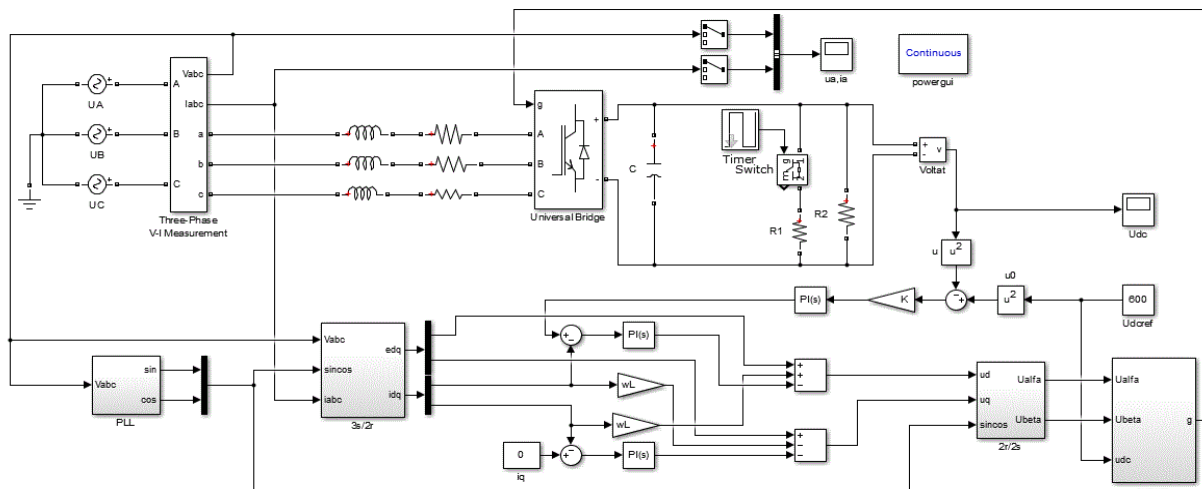
##### 4.1. System simulation parameters

The design of the system is simulated by Matlab /Simulink software, the relevant parameters of the three-phase voltage-sourced PWM rectifier can be found in table 1.

**Table 1.** The relevant parameters of the three-phase voltage-sourced PWM rectifier

Parameter	Value
Three-phase AC voltage amplitude	311V
Three-phase AC voltage frequency	50Hz
Given DC voltage	600V
Inductance L, Resistance R	$5e^{-3}H, 0.5\Omega$
Switching frequency $f_s$	10kHz
Capacitor C	$3300 \mu F$
Load resistance R	$10 \rightarrow 50\Omega$

Then the system simulation is shown in figure 5.

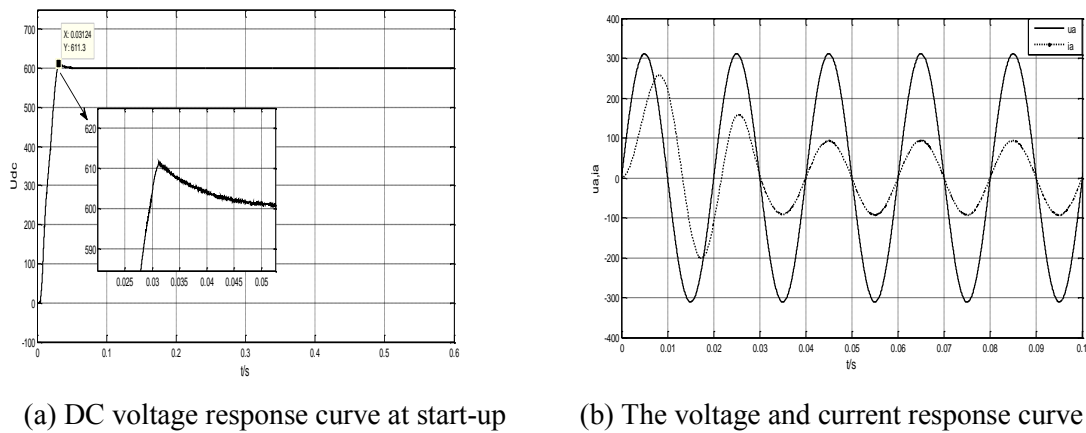


**Figure 5.** The three-phase voltage-sourced PWM rectifier simulation block diagram

##### 4.2. System start-up response

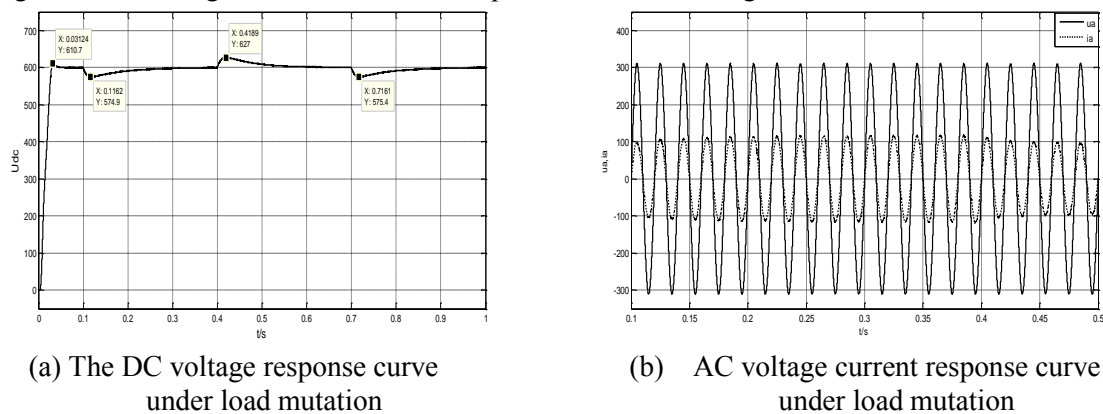
System is operated on unit power factor and full load, the DC voltage and A phase voltage current partial amplification waveform response are shown in figure 6.

The voltage can be fast response, overshoot  $\sigma = 1.9\%$  meet system requirements, and system achieves steady-state error free, the voltage current is smooth sine waveform, and keeps the same phase. The system starts to reach a stable at  $0.04s$ ,  $i_{THD} = 0.68\%$ , the rectifier has quick response speed.

**Figure 6.** Full load response curve

#### 4.3. Load mutation response

Considering the actual operation of the system, which starts up with load  $10\Omega$ , suddenly increasing  $50\Omega$  at  $0.1s$ , suddenly decreasing  $50\Omega$  at  $0.4s$ , and then suddenly increasing  $50\Omega$  at  $0.7s$ , the DC voltage and AC voltage current waveform response are shown in figure 7.

**Figure 7.** Response curve under load mutation

The system enters steady state at  $0.04s$ , the DC voltage quickly reaches the set value, and overshoot  $\sigma = 1.78\%$  meets the requirements. When load suddenly increases, the voltage falls to  $574.9V$ , the transient rate of change  $\varepsilon = 4.18\%$ , the voltage restores the initial value with no oscillation by about  $1.92s$ , and the output voltage keeps steady. Voltage increases to  $627V$  after the load suddenly decreases, the transient rate of change  $\varepsilon = 4.5\%$ , voltage restores the original stability of  $600V$  with no oscillation by about  $1.61s$ . When load suddenly increases, A phase current gradually increased, the current enters stability after about  $1.92s$ . When the load is suddenly reduced, A phase current decreases gradually, and remains stable after about  $1.47s$ . The phase voltage and current are all sinusoidal and in phase,  $i_{THD} = 2.81\%$ .

It can be found from the simulation data, the design of the outer loop voltage square control can maintain DC voltage stability in the set value when the load frequently changes, while the system has a quick response speed, and can effectively suppress the voltage fluctuations, quickly restore the original value, keep the unit power factor running, reduce the current distortion.

## 5. Concluding remarks

Based on the mathematical model of the three-phase voltage-sourced PWM rectifier d-q coordinate system, the feed-forward decoupling control of the active and reactive components of the current is realized. The double closed loop control strategy of the outer voltage square and the inner current is used to realize the fast no difference tracking control, combining with the space vector modulation algorithm to improve the voltage utilization. At the same time, the simulation model of three-phase voltage-sourced PWM rectifier is verified by Matlab simulation platform. The simulation results show that the control system can operate with the unit power factor, the DC voltage is fast, the steady-state is error free and the current distortion rate is small, Under the frequent load mutation the system has a strong anti-interference ability and strong robust.

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