

# On the impact index of synchronous generator displaced by DFIG on power system small-signal stability

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**Abstract.** As the maturity of wind power technology and the ageing and retirement of conventional synchronous generators, the displacement of synchronous generators by wind power generators would be a trend in the next few decades. The power system small-signal angular stability caused by the displacement is an urgent problem to be studied. The displacement of the SG by the DFIG includes withdrawing the dynamic interactions of the displaced SG and adding the dynamic interactions of the displacing DFIG. Based on this fact, a new index is proposed to predict the impact of the SG to be displaced by the DFIG on power system oscillation modes. The sensitivity index of the oscillation modes to the constant inertia of the displaced SGs, proposed in early literatures to estimate the dynamic impact of the SG being displaced by the DFIG, is also compared with the proposed index. The modified New England power system is adopted to show various results and conclusions. The proposed index can correctly identify the most dangerous and beneficial displacement to power system small-signal angular stability, and is very useful in practical applications.

## 1 Introduction

The wind power generation has been developing rapidly for the last few decades [1]-[2]. It has been well recognized that the essential difference of the variable-speed wind generators (VSWGs), such as DFIGs, to the conventional synchronous generators (SGs) is their “less inertial” response to the dynamic changes occurred in power systems. This makes the impact of the grid-connected VSWGs on power system angular stability different to that of the SGs. For over a decade, great effort has been spent to investigate power system angular stability as affected by grid connection of VSWGs. Among them, many papers [3]-[14] research the cases of SG displacement by VSWG with same power at same site. Although the new-built wind farm displacing the conventional power plant at the same site is not a common situation, this method is useful in researching the different effect of dynamic interaction on power system small-signal angular stability between SG and VSWG, excluding the impact of other factors, such as the power flow.

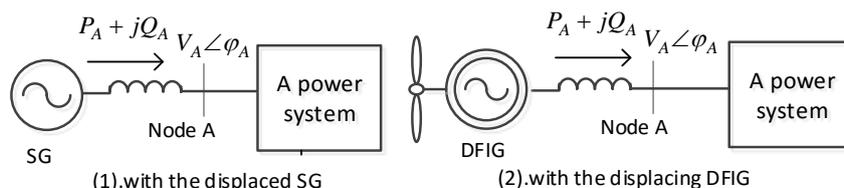
According to the study in [15], the displacement of the SG by the DFIG includes withdrawing the dynamic interactions of the displaced SG and adding the dynamic interactions of the displacing DFIG. Based on this fact, a new index is proposed to predict the impact of the SG to be displaced by the DFIG on power system oscillation modes firstly in this paper. Then the sensitivity index of the



oscillation modes to the constant inertia of the displaced SGs, proposed in [12] to estimate the dynamic impact of the SG being displaced by the DFIG, is also compared with the proposed index. Finally, conclusions of discussion and investigations made are drawn in the paper.

## 2 Dynamic importance index of SGs

### 2.1 Accurate impact of SG displacement by DFIG on power system small-signal angular stability



**Figure 1.** A SG is displaced by a DFIG in a power system

Fig. 1 illustrates the strategy of displacing a SG by a DFIG at node A in a power system. The physical cause of the dynamic interactions of the displaced SG with the rest of the SGs in the power system is the dynamic power exchange of the displaced SG with the power system,  $\Delta P_A + j\Delta Q_A$ . Hence take  $\Delta P_A$  and  $\Delta Q_A$  as the output signals from the displaced SG and the magnitude and phase of the terminal voltage at node A in Fig. 1,  $\Delta V_A$  and  $\Delta \varphi_A$ , as the input signals. The following state-space model of the displaced SG can be established

$$\begin{aligned} \frac{d}{dt} \Delta \mathbf{X}_N &= \mathbf{A}_N \Delta \mathbf{X}_N + \mathbf{b}_{NV} \Delta V_A + \mathbf{b}_{N\varphi} \Delta \varphi_A \\ \Delta P_A &= \mathbf{c}_{NP}^T \Delta \mathbf{X}_N + d_{NPV} \Delta V_A + d_{NP\varphi} \Delta \varphi_A \\ \Delta Q_A &= \mathbf{c}_{NQ}^T \Delta \mathbf{X}_N + d_{NQV} \Delta V_A + d_{NQ\varphi} \Delta \varphi_A \end{aligned} \quad (1)$$

Where  $\Delta \mathbf{X}_N$  is the vector of state variables of the displaced SG. Similarly, the state-space model of the displacing DFIG can be derived to be

$$\begin{aligned} \frac{d}{dt} \Delta \mathbf{X}_W &= \mathbf{A}_W \Delta \mathbf{X}_W + \mathbf{b}_{WV} \Delta V_A \\ \Delta P_A &= \mathbf{c}_{WP}^T \Delta \mathbf{X}_W + d_{WP} \Delta V_A \\ \Delta Q_A &= \mathbf{c}_{WQ}^T \Delta \mathbf{X}_W + d_{WQ} \Delta V_A \end{aligned} \quad (2)$$

Where  $\Delta \mathbf{X}_W$  is the vector of state variables of the displacing DFIG. And the state-space model of the rest of power system can be derived to be

$$\begin{aligned} \frac{d}{dt} \Delta \mathbf{X}_{N-1} &= \mathbf{A}_{N-1} \Delta \mathbf{X}_{N-1} + \mathbf{b}_P \Delta P_A + \mathbf{b}_Q \Delta Q_A \\ \Delta V_A &= \mathbf{c}_V^T \Delta \mathbf{X}_{N-1} + d_{VP} \Delta P_A + d_{VQ} \Delta Q_A \\ \Delta \varphi_A &= \mathbf{c}_\varphi^T \Delta \mathbf{X}_{N-1} + d_{\varphi P} \Delta P_A + d_{\varphi Q} \Delta Q_A \end{aligned} \quad (3)$$

Where  $\Delta \mathbf{X}_{N-1}$  is the vector of state variables of all the other SGs.

Thus, the complete close-loop power system is divided to two parts: one is the open-loop power system, and the other is the “feedback controller” representing the displaced SG or the displacing DFIG.

Denote the linearized model of the power system with the displaced SG to be

$$\frac{d\mathbf{X}_G}{dt} = \mathbf{A}_G \mathbf{X}_G \quad (4)$$

Equation (4) can be obtained by combining (1) and (3). Denote  $\bar{\lambda}_G$  the electromechanical oscillation mode of interests of system (4). With the SG being displaced by the DFIG, system load flow does not change. However, the vector of state variables changes with the displacement. The linearized model of the power system after the displacement becomes

$$\frac{d\mathbf{X}_D}{dt} = \mathbf{A}_D \mathbf{X}_D \quad (5)$$

Equation (5) can be obtained by combining (2) and (3). Denote  $\bar{\lambda}_D$  the oscillation mode of system (5) corresponding to  $\bar{\lambda}_G$ . Obviously,  $\bar{\lambda}_D - \bar{\lambda}_G$  gives the accurate assessment of impact of the displacement. If the real part of  $\bar{\lambda}_D - \bar{\lambda}_G$  is positive, i.e.,  $\text{Re}(\bar{\lambda}_D - \bar{\lambda}_G) > 0$ , the displacement reduces the damping of the electromechanical oscillation mode and is detrimental to power system small-signal angular stability, and vice versa.

## 2.2 Dynamic importance index of SG

Consider an assumed case that  $\Delta P_A + j\Delta Q_A = 0$ . In this case there are no dynamic interactions between the displaced SG and the rest of power system and the effect of the dynamics of the displaced SG is excluded. Obviously this is the case that the displaced SG is degraded into a constant power source  $P_{A0} + jQ_{A0}$  and the model of rest of power system is degraded from (3) to

$$\frac{d}{dt} \Delta \mathbf{X}_{N-1} = \mathbf{A}_{N-1} \Delta \mathbf{X}_{N-1} \quad (6)$$

Denote  $\bar{\lambda}_0$  the oscillation mode corresponding to  $\bar{\lambda}_G$  when the displaced SG is modelled as the constant power source. Obviously  $\bar{\lambda}_0 - \bar{\lambda}_G$  measures the impact of withdrawing the displaced SG from the power system on the oscillation mode of interests. Similarly it is easy to conclude that  $\bar{\lambda}_D - \bar{\lambda}_0$  measures the impact of adding the dynamics of the displacing DFIG on the oscillation mode of interests such that the total impact is  $(\bar{\lambda}_0 - \bar{\lambda}_G) + (\bar{\lambda}_D - \bar{\lambda}_0) = \bar{\lambda}_D - \bar{\lambda}_G$ .

As is stated in II.A, the displaced SG and the displacing DFIG are all regarded as feedback controllers. From (2), it can be seen that the transfer functions of DFIG is only related to the magnitude of the terminal voltage at node A  $\Delta V_A$ , which is similar to that of Static VAR Compensator (SVC). Previous researches [16] have proved that the impact of voltage control function of SVC on the electromechanical oscillation mode is small, because the variation of voltage magnitude of every node in power system is limited. The damping torque of SVC to the low frequency oscillation is near to zero. Similarly, the dynamic interactions between the DFIG and power system normally are very weak, thus  $\bar{\lambda}_D - \bar{\lambda}_0 \approx 0$  stands approximately, which has also been proved in [15].

From Eq. (1), it can be seen that the transfer functions of SG is related to both the magnitude of the terminal voltage at node A  $\Delta V_A$  and the phase  $\Delta \varphi_A$ . The variation of voltage phase of some nodes in power system is likely very large in some cases. Thus the dynamic interactions between the SG and power system are likely very strong. If the dynamic interactions between a displaced SG and power system are strong and the impact is significant, it can have

$$\bar{\lambda}_D - \bar{\lambda}_G = (\bar{\lambda}_0 - \bar{\lambda}_G) + (\bar{\lambda}_D - \bar{\lambda}_0) \approx (\bar{\lambda}_0 - \bar{\lambda}_G) = \Delta \bar{\lambda}_G \quad (7)$$

If the dynamic interactions between the displaced SG and power system are weak and the impact is significant.  $\bar{\lambda}_0 - \bar{\lambda}_G \approx 0$  stands approximately. It can have

$$(\bar{\lambda}_D - \bar{\lambda}_G) = (\bar{\lambda}_0 - \bar{\lambda}_G) + (\bar{\lambda}_D - \bar{\lambda}_0) \approx 0 \approx \Delta \bar{\lambda}_G \quad (8)$$

From (7) and (8), it can be seen that whether the dynamic interactions between the displaced SG and power system are weak or strong, the index  $\Delta\bar{\lambda}_G = \bar{\lambda}_0 - \bar{\lambda}_G$  can always be used to predict the impact of various SGs displacement by DFIG on the power system small-signal angular stability. The index  $\Delta\bar{\lambda}_G = \bar{\lambda}_0 - \bar{\lambda}_G$  proposed here is called the dynamic importance index of SG.

Compared to the accurate method stated in II.A, the computation of the proposed index,  $\Delta\bar{\lambda}_G = \bar{\lambda}_0 - \bar{\lambda}_G$ , does not need to know the dynamic model of the displacing DFIG. This is useful in practical applications. For example, at the stage of planning the connections of wind power generation, it is likely that the dynamic model of the DFIG to be connected is unknown. Also it is often that the SG would be displaced by a wind farm rather than a DFIG and it is not a straightforward job to work out the equivalent dynamic model of the wind farm with many DFIGs.

### 2.3 Modal sensitivity to the constant of inertia of SG

Considering the fact that with a DFIG displacing a SG in a power system, the equivalent inertia is reduced, the modal sensitivity to the SG's constant of inertia,  $-\partial\bar{\lambda}_G/\partial H$ , is proposed in [12] to estimate the dynamic impact of the SG being displaced by the DFIG. The negative sign indicates the trend of withdrawing the displaced SG with the constant of inertia reduced.

However, the impact of the SG displacement by DFIG is not only depend on the reduced equivalent inertia. Other factors, such as the different damping factor, or the plug in/off of the PSS, also have an impact on the oscillation modes in some cases, but they are not considered in the index  $-\partial\bar{\lambda}_G/\partial H$  at all.

The analysis below tries to find the relationship between the proposed index in this paper and the sensitivity index. As stated in the subsection II.B, the constant power source is introduced to estimate the impact of displaced SGs. The characteristics of constant power source is  $\Delta P_A + j\Delta Q_A = 0$ . According to the rotational speed equation of SG

$$\frac{d\omega}{dt} = \frac{1}{2H}(P_m - P_e) \quad (9)$$

If the constant of inertia H is set to 0 (this is the usual assumptions for DFIG as the equivalent inertia of DFIG is very small), the following equation stands approximately.

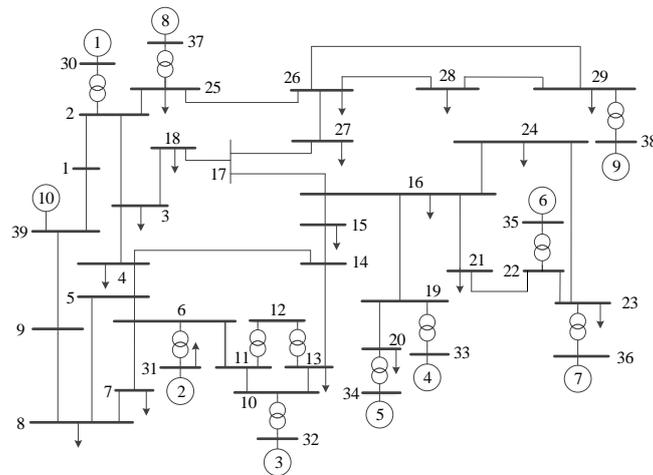
$$P_e = P_m - 2H \frac{d\omega}{dt} \approx P_m \quad (10)$$

That is to say,  $H = 0$  is equal to  $\Delta P_A = 0$ , but  $\Delta Q_A \neq 0$ . So the difference between the index  $\Delta\bar{\lambda}_G = \bar{\lambda}_0 - \bar{\lambda}_G$  and  $\bar{\lambda}_G(H=0) - \bar{\lambda}_G$  relies in the impact caused by  $\Delta Q_A$ . Generally, the values of  $\bar{\lambda}_G(H=0) - \bar{\lambda}_G$  and  $-\partial\bar{\lambda}_G/\partial H$  are almost equal, because the relationship between the oscillation mode and the constant of inertia of the displaced SG is linear in most cases. Thus the main difference between the proposed index in this paper and the sensitivity index comes from the impact of reactive power change  $\Delta Q_A$  to the oscillation mode.

Furthermore, the equipment of PSS would greatly reduce the effect of the reactive power change of displaced SG on the oscillation mode of interests. This is because the power system stabilizers can help SGs to restore to synchronization and the output power of SGs equipped with PSS changes more slightly than those without PSS. The example below shows the influence of PSS on the accuracy of the sensitivity index.

## 3 Examples

### 3.1 The test system



**Figure 2.** Configuration of test power system

Fig. 2 shows the configuration of modified New England power system where  $G_k, k = 1, 2, \dots, 10$  are ten SGs in the original New England system. For  $G_k, k = 1, 2, \dots, 10$ , a third-order model of the synchronous generator and a simple first-order model of the AVR are used. They are all equipped with PSS which adopt a simple second-order model. The loads are modelled as constant impedance. Bus 6 is the slack bus and  $G_{10}$  is the equivalent SG to an external power system. Thus  $G_2$  connected at node 6 and  $G_{10}$  will not be displaced by the DFIGs in the test. The model and parameters of SGs and DFIG are given in [16].

The modification to the original New England system is the addition of eight more SGs in parallel to  $G_k, k = 1, 3, 4, 5, 6, 7, 8, 9$ . Thus  $G_k, k = 1, 3, 4, 5, 6, 7, 8, 9$  are exactly same to  $G_{ak}, k = 1, 3, 4, 5, 6, 7, 8, 9$  respectively except that indicated in Table 1.

**Table 1.** Difference of SGs connected to a same node

SGs	parameters	SGs	parameters
$G_1$	H=74	$G_{a1}$	H=94
$G_3$	P=3.05	$G_{a3}$	P=1.05
$G_4$	The 6 <sup>th</sup> -order model	$G_{a4}$	The 2 <sup>nd</sup> -order model without AVR and PSS
$G_5$	H=52	$G_{a5}$	H=32
$G_6$	The 6 <sup>th</sup> -order model	$G_{a6}$	The 4 <sup>nd</sup> -order model without PSS
$G_7$	The 6 <sup>th</sup> -order model	$G_{a7}$	The 2 <sup>nd</sup> -order model without AVR and PSS
$G_8$	P=2.7	$G_{a8}$	P=8.7
$G_9$	The 6 <sup>th</sup> -order model	$G_{a9}$	The 4 <sup>nd</sup> -order model without PSS

For the test power system of Fig. 2, the electromechanical oscillation mode of interests,  $\bar{\lambda}_G$ , is an inter-area oscillation mode of  $G_{10}$  in respect to all the other SGs.

**3.2 The accurate results of the impact of SG displacement by DFIG on the inter-area oscillation mode**  
 According to (4),(5) and the method presented in subsection II.A, results of impact assessment with the SGs being displaced by the DFIGs are presented in Table 2.

**Table 2.** Accurate impact of the SGs being displaced by the DFIGs

SGs	Accurate impact	SGs	Accurate impact
$G_1$	$-0.0030 + j0.0154$	$G_{a1}$	$-0.0045 + j0.0216$
$G_3$	$0.0004 + j0.0208$	$G_{a3}$	$0.0018 + j0.0235$
$G_4$	$0.0044 + j0.0386$	$G_{a4}$	$-0.0078 + j0.0473$
$G_5$	<b><math>0.0081 + j0.0518</math></b>	$G_{a5}$	$0.0037 + j0.0274$
$G_6$	$-0.0014 + j0.0353$	$G_{a6}$	$-0.0063 + j0.0455$
$G_7$	$-0.0011 + j0.0450$	$G_{a7}$	<b><math>-0.0108 + j0.0523</math></b>
$G_8$	$-0.0001 + j0.0130$	$G_{a8}$	$-0.0013 + j0.0145$
$G_9$	$-0.0025 + j0.0523$	$G_{a9}$	$-0.0102 + j0.0660$

From Table 2, it can be seen that the impact of displacement of  $G_5$  is biggest with  $\text{Re}(\bar{\lambda}_D - \bar{\lambda}_G) > 0$ . This means that displacement of  $G_5$  is the most dangerous scenario for the test power system as far as the inter-area oscillation mode of interests is concerned. Similarly, the impact of the displacement of  $G_{a7}$  is smallest with  $\text{Re}(\bar{\lambda}_D - \bar{\lambda}_G) < 0$ . This indicates that the displacement of  $G_{a7}$  will be most beneficial to the system small-signal angular stability as far as the inter-area oscillation mode of interests is concerned.

### 3.3 The Results of the Proposed Index $\Delta\bar{\lambda}_G$

Each of sixteen SGs in the test power system of Fig. 1,  $G_k, k=1,3,4,5,6,7,8,9$  and  $G_{ak}, k=1,3,4,5,6,7,8,9$  is modelled as a constant power source,  $P_{A0} + jQ_{A0}$ , which is the power output of the SG at the steady-state operation. Then the inter-area mode of interest is computed as  $\bar{\lambda}_0$  with the SG's model of constant power source. Results of  $\bar{\lambda}_0 - \bar{\lambda}_G$  for each of the SGs are presented in Table 3. By comparing Table 2 and Table 3 it can be seen that the corresponding data in two tables are almost same, which verifies the accuracy of the dynamic importance index proposed in this paper.

**Table 3.** Impact assessment of the SGs being displaced with  $\bar{\lambda}_0 - \bar{\lambda}_G$ 

SGs	Impact assessment	SGs	Impact assessment
$G_1$	$-0.0029 + j0.0154$	$G_{a1}$	$-0.0045 + j0.0216$
$G_3$	<b><math>-0.0003 + j0.0207</math></b>	$G_{a3}$	$0.0018 + j0.0235$
$G_4$	$0.0041 + j0.0387$	$G_{a4}$	$-0.0083 + j0.0475$
$G_5$	$0.0080 + j0.0519$	$G_{a5}$	$0.0035 + j0.0274$
$G_6$	$-0.0016 + j0.0351$	$G_{a6}$	$-0.0066 + j0.0455$
$G_7$	$-0.0013 + j0.0449$	$G_{a7}$	$-0.0113 + j0.0523$
$G_8$	<b><math>-0.0002 + j0.0130</math></b>	$G_{a8}$	$-0.0015 + j0.0146$
$G_9$	$-0.0026 + j0.0520$	$G_{a9}$	$-0.0109 + j0.0660$

In order to verify the conclusions drawn in subsection II.B further, results of  $\bar{\lambda}_D - \bar{\lambda}_0$  are presented in Table 4. From Table 3 and Table 4, it can be seen clearly that for all SGs except  $G_3$  and  $G_8$ , the value of  $\bar{\lambda}_D - \bar{\lambda}_0$  is much smaller than that of  $\bar{\lambda}_0 - \bar{\lambda}_G$  (corresponding to (7)). For  $G_3$  and  $G_8$ , the values of  $\bar{\lambda}_D - \bar{\lambda}_0$  and  $\bar{\lambda}_0 - \bar{\lambda}_G$  are all near to zero (corresponding to (8)). Thus, the proposed index can always predict the dynamic importance of the SGs correctly with respect to the power system small-signal angular stability.

**Table 4.** Computational results of  $\bar{\lambda}_D - \bar{\lambda}_0$

SGs	Computational results	SGs	Computational results
$G_1$	0.0000 - j0.0000	$G_{a1}$	0.0000 - j0.0000
$G_3$	-0.0001 - j0.0000	$G_{a3}$	-0.0000 - j0.0000
$G_4$	-0.0003 + j0.0001	$G_{a4}$	-0.0006 + j0.0001
$G_5$	-0.0001 + j0.0000	$G_{a5}$	-0.0002 + j0.0001
$G_6$	-0.0002 - j0.0002	$G_{a6}$	-0.0004 + j0.0000
$G_7$	-0.0002 - j0.0000	$G_{a7}$	-0.0006 + j0.0000
$G_8$	-0.0000 - j0.0000	$G_{a8}$	-0.0003 + j0.0001
$G_9$	-0.0002 - j0.0003	$G_{a9}$	-0.0006 + j0.0000

### 3.4 The Results of the Sensitivity Index $-\partial\bar{\lambda}_G/\partial H$

Table 5 presents the computational results of  $-\partial\bar{\lambda}_G/\partial H$  for the test power system of Fig. 1.

**Table 5.** Impact assessment of the SGs being displaced with  $-\partial\bar{\lambda}_G/\partial H$

SGs	Impact assessment	SGs	Impact assessment
$G_1$	-0.0001 + j0.0003	$G_{a1}$	-0.0001 + j0.0004
$G_3$	-0.0000 + j0.0005	$G_{a3}$	-0.0000 + j0.0007
$G_4$	0.0001 + j0.0009	$G_{a4}$	-0.0001 + j0.0010
$G_5$	0.0003 + j0.0017	$G_{a5}$	0.0002 + j0.0012
$G_6$	-0.0000 + j0.0010	$G_{a6}$	-0.0001 + j0.0011
$G_7$	-0.0000 + j0.0011	$G_{a7}$	-0.0002 + j0.0012
$G_8$	-0.0000 + j0.0004	$G_{a8}$	-0.0000 + j0.0004
$G_9$	-0.0001 + j0.0012	$G_{a9}$	-0.0001 + j0.0013

As the sensitivity index proposed in [12] is only used to predict the change of the oscillation damping, the real part of the index  $\text{Re}(-\partial\bar{\lambda}_G/\partial H)$  is used here. The rank of dynamic importance of the SGs when they are displaced by the DFIGs according to the value of  $\text{Re}(\bar{\lambda}_D - \bar{\lambda}_G)$  (the accurate impact) and  $\text{Re}(-\partial\bar{\lambda}_G/\partial H)$  from the biggest to smallest can be produced and listed in the 1st and 2nd row in Table 6. The result in Table 6 shows that the rank obtained from  $\text{Re}(-\partial\bar{\lambda}_G/\partial H)$  is almost the same to that from  $\text{Re}(\bar{\lambda}_D - \bar{\lambda}_G)$  except  $G_{a4}$  and  $G_{a6}$ .

**Table 6.** Rank of dynamic importance of SGs from the biggest o smallest using different methods

the accurate impact	$G_5$ $G_4$ $G_{a5}$ $G_{a3}$ $G_3$ $G_8$ $G_7$ $G_{a8}$ $G_6$ $G_9$ $G_1$ $G_{a1}$ $G_{a6}$ $G_{a4}$ $G_{a9}$ $G_{a7}$
the predicted impact	$G_5$ $G_4$ $G_{a5}$ $G_{a3}$ $G_3$ $G_8$ $G_7$ $G_{a8}$ $G_6$ $G_9$ $G_{a6}$ $G_{a4}$ $G_1$ $G_{a1}$ $G_{a9}$ $G_{a7}$

To examine the impact of reactive power change  $\Delta Q_A$  to the oscillation mode, Table 7 gives the computational results of  $\bar{\lambda}_G(H=0) - \bar{\lambda}_0$  for the test power system of Fig. 1. From Table 7 it can be seen that  $G_{a4}$ ,  $G_{a6}$ ,  $G_{a7}$  and  $G_{a9}$  have relatively large impact on the damping of inter-area oscillation mode of interests compared to other SGs.

**Table 7.** Computational results of  $\bar{\lambda}_G(H=0) - \bar{\lambda}_0$

SGs	Computational results	SGs	Computational results
$G_1$	$-0.0019 + j0.0051$	$G_{a1}$	$-0.0019 + j0.0050$
$G_3$	$-0.0008 + j0.0024$	$G_{a3}$	$-0.0018 + j0.0040$
$G_4$	$-0.0007 + j0.0035$	$G_{a4}$	<b><math>0.0056 - j0.0023</math></b>
$G_5$	$-0.0006 + j0.0019$	$G_{a5}$	$-0.0003 + j0.0012$
$G_6$	$-0.0008 + j0.0058$	$G_{a6}$	<b><math>0.0032 - j0.0011</math></b>
$G_7$	$-0.0010 + j0.0039$	$G_{a7}$	<b><math>0.0037 - j0.0008</math></b>
$G_8$	$-0.0005 + j0.0017$	$G_{a8}$	$-0.0003 + j0.0010$
$G_9$	$-0.0019 + j0.0062$	$G_{a9}$	<b><math>0.0044 - j0.0022</math></b>

It is interesting to note that  $G_{a4}, G_{a6}, G_{a7}$  and  $G_{a9}$  are all equipped without PSS as seen in Table 1. From other various examples, it is a common conclusion that the equipment of PSS would greatly reduce the effect of the reactive power change of displaced SG on the oscillation mode of interests.

The impact of the reactive power of displaced SGs does influence the accuracy of the index  $-\partial\bar{\lambda}_G/\partial H$ . The rank of  $G_{a4}$  and  $G_{a9}$  is misjudged because of lack of the impact of  $\Delta Q_A$  in the index  $-\partial\bar{\lambda}_G/\partial H$ . The rank of  $G_{a3}$  and  $G_{a5}$  is unchanged because for  $G_{a3}$  and  $G_{a5}$ , the impact by active power change is the smallest of all SGs, and the change direction of active and reactive power is consistent. For other generators equipped with PSS, the impact of the change of reactive power is very small, and  $\Delta\bar{\lambda}_G \approx \bar{\lambda}_G(H=0) - \bar{\lambda}_G$  stands approximately. Thus, it can be seen from the result that the accuracy of the sensitivity index is dependent on the equipment of PSS, weakening the impact of  $\Delta Q_A$  on the power system small-signal angular stability, which affects its availability in applications without PSSs.

#### 4 Discussion and prospect

With a DFIG displacing a SG in a power system, the total impact of the displacement is equal to that of withdrawing the SG plus that of adding the DFIG. In most instances, the influence of the latter is much smaller than that of the former. Thus the proposed index,  $\Delta\bar{\lambda}_G = \bar{\lambda}_0 - \bar{\lambda}_G$ , can always rank the dynamic importance of the SGs correctly with respect to the power system small-signal angular stability. This is useful in practical applications in that no detailed dynamic models of displacing DFIG needed and its high accuracy. Modal sensitivity to the constant of inertia of displaced SG is accurate under the conditions where the PSS must be equipped with, which affects its availability in applications.

#### 5 Acknowledgement

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