

# All-dielectric microwave devices for controlling the path of electromagnetic waves

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**Abstract.** All-dielectric devices are designed using Quasi-Conformal Transformation Optics (QCTO) concept and fabricated by additive manufacturing for the control of wave propagation. Three lenses are studied; the first one is used to compensate for the curvature of a non-planar antenna array, the second one to steer an electromagnetic beam to a desired direction and the last one to taper an electromagnetic field between two sections of different dimensions.

## 1. Introduction

Transformation Optics (TO) is a powerful tool that enables to control electromagnetic (EM) fields in unprecedented and unbelievable ways through the use of judiciously engineered materials with parameters that vary spatially [1, 2]. The TO formalism is based on the invariance of Maxwell's equations over space-time coordinate transformations. Such flexibility in controlling EM waves appears to be convenient in the design of innovative devices with performances or special properties difficult to achieve from classical methods and has therefore inspired considerable research interests in wave propagation. The electromagnetic cloak is one of the most striking devices realized with TO concept [3].

In order to avoid anisotropy in material parameters that arises during coordinate transformation, an equivalence between coordinate transformation and spatial deformation by using Laplace's equation to determine the deformation of coordinate grids during the transformation has been proposed [4]. So the transformed material parameters are determined by the solutions of the Laplace's equation under proper boundary conditions. Such mapping known as quasi-conformal transformation optics (QCTO) has been proposed to simplify the electromagnetic distributions in the new transformed coordinate space. QCTO helps to minimize the anisotropy of the constitutive materials, giving the possibility to implement all-dielectric materials in the transformed medium. As a result, nearly-isotropic gradient index (GRIN) materials with broad frequency bandwidth and low losses can be employed, opening the way to broadband devices.

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In this work, QCTO is applied to design microwaves devices for controlling the path of electromagnetic waves. Two broadband lenses; one capable of restoring in-phase emissions from an array of radiators fixed on a non-planar ground plane and another one capable of steering the radiated beam of an antenna to an off-normal direction and a waveguide taper are designed, fabricated and experimentally validated in this work.

## 2. QCTO formulations

The design of a QCTO-based device is associated to the calculation of the 2D Laplace's equation so as to evaluate the coordinate grids deformation during the transformation from the initial virtual space filled with vacuum in the coordinate system  $(x, y, z)$  to the real final physical space filled with transformed medium in the coordinate system  $(x', y', z')$  [4]. The method then implies that the calculation of the electromagnetic material parameters of the transformation medium is equivalent to the computation of spatial deformation field governed by the 2D Laplace's equation with proper boundary conditions, whatever the shape of the device.

The conformal mapping in the physical space is obtained by solving Laplace's equation with predefined boundary conditions. Let's suppose that the coordinate transformation between the physical and virtual spaces is  $x = f(x', y')$ ,  $y = g(x', y')$ . The mathematical equivalence of the transformation is expressed by the Jacobian matrix  $J = \partial(x, y)/\partial(x', y')$ . By solving Laplace's equations in the virtual space with respect to specific boundary conditions

$$\frac{\partial^2 x}{\partial x'^2} + \frac{\partial^2 x}{\partial y'^2} = 0, \quad \frac{\partial^2 y}{\partial x'^2} + \frac{\partial^2 y}{\partial y'^2} = 0 \quad (1)$$

the Jacobian matrix  $J$  of the mapping can be obtained.

The physical space performs an inverse function of the virtual space. Thus the Jacobian matrix of this inverse transformation from  $(x', y')$  to  $(x, y)$  is represented by  $J^{-1}$ . The properties of the intermediate medium are calculated such that the constitutive tensors have only diagonal components. It can then be found that the mapping satisfies the vector form of Laplace's equation. This leads to the simple material parameters given as

$$\varepsilon = \mu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & |J|^{-1} \end{bmatrix} \quad (2)$$

For fields' equivalence at the outer boundaries with the virtual space for which the transformation is carried out, Neumann and Dirichlet sliding boundary conditions are applied to fix the geometry of the transformed domain and to maintain orthogonality on the boundary. The determination of the mapping is performed using COMSOL Multiphysics Partial Differential Equation (PDE) solver. In all the designed lenses, the transformation deals with a two-dimensional (2D) model with incident transverse electric (TE) polarized wave. In such a configuration, the electric field has only a component directed along the  $z$  direction. The material properties of the lenses are then simplified and

possess only dielectric properties  $\varepsilon = \frac{\varepsilon_r}{\det(J^{-1})}$ .

## 3. QCTO-based devices

First, we study the enhancement of directivity through the use of a QCTO-based lens conformed on a non-planar array of radiating elements that follows a cylindrical shape. The lens is capable of restoring in-phase emissions from a conformal array of antennas so as to obtain performances similar to that of

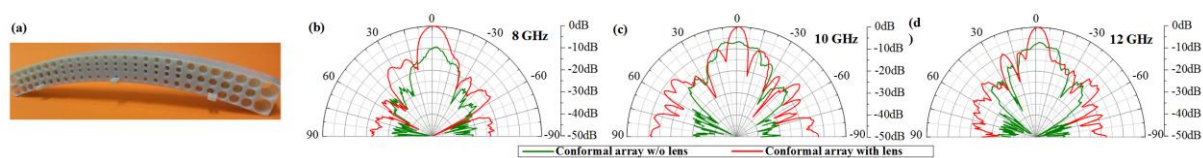
a linear one. The design of the conformal lens-antenna is presented in Fig. 1(a). Calculations done in [5-6] have shown that the effective permittivity varies from 1 to 2.76 in the lens.

The second lens is designed to steer an antenna's radiated beam to an off-normal direction. The lens-antenna structure is shown in Fig. 2(a). The permittivity values range from 1 to 6.4 in the designed lens [7-8]. A linear array of 4 patch antennas is used as primary source.

The third device is a coupling device capable of tapering the electromagnetic fields between two sections of different dimensions (Fig. 3(a)). The electromagnetic waves exiting from a large section or aperture are properly guided through the taper to a smaller one. The permittivity ( $\epsilon_{zz}$ ) distribution ranges from 1 to 5.02 for the calculated taper.

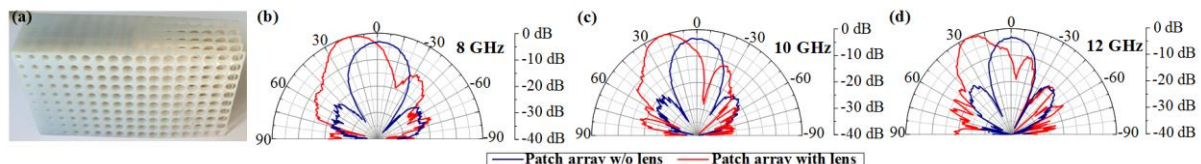
#### 4. Implementations and experimental validations

In this work, we favor the realization of the permittivity values by using a dielectric with air holes. We consider 3D printing additive fabrication technique. The polymer used in the printing process presents a relative permittivity  $\epsilon_d = 2.8$  [9].



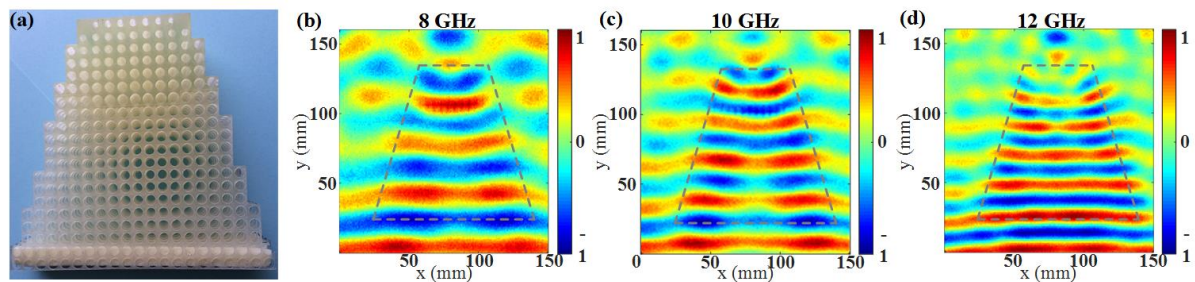
**Figure 1.** (a) Conformal lens for in-phase emission restoring. (b)-(d) Measured performances in the 8 GHz – 12 GHz frequency range.

Taking in consideration the minimum and maximum value of hole diameter that can be realized, we fix the variation of the effective permittivity from 1 to 2.8 in the discrete model of the devices. Such variation is possible by varying the diameter of the holes.



**Figure 2.** (a) Beam steering lens for off-normal radiation. (b)-(d) Measured performances in the 8 GHz – 12 GHz frequency range.

The performances measured from the transformed devices illustrate clearly the control of the path of electromagnetic waves. The all-dielectric devices present the advantage of having broadband characteristics. Theoretically, the achieved bandwidth can be very broad since we are making use of all-dielectric non resonant materials. However, in physically fabricated prototypes, the bandwidth will definitely depend on the size of the devices and also on the operating frequency. At low frequencies, the size of the device must be large enough since wavelength is large and at high frequencies, we are limited by small wavelength. Therefore at high frequencies the unit cells (holes in dielectric) used to tailor the material parameters must be engineered with respect to the wavelength so as to be consistent with the effective medium theory. Therefore, a trade-off has to be necessarily made between the size of the lens and the desired bandwidth so that the transformed medium can be considered homogeneous in such frequency range and so that the dimensions of the device are large enough at the lowest frequency of operation.



**Figure 3.** (a) Taper to control the flow of electromagnetic waves between two waveguides of different cross-sections. (b)-(d) Measured performances in the 8 GHz – 12 GHz frequency range.

## 5. Conclusion

To summarize, we have presented the design and experimental realization of compact all-dielectric microwave devices operating on a wide frequency range. The devices are designed through the transformation optics concept in order to control the direction of EM wave propagation. Furthermore, the design concept has been combined with additive fabrication technology for the realization of the devices. The concept can be readily applied at any frequency with available fabrication techniques.

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